Your Name

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 5, 2016
Prof. Alan Guth

QUIZ 1

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Please answer all questions in this stapled booklet.

PROBLEM 1: DID YOU DO THE READING? *(35 points)*

(a) *(5 points)* The Milky Way has been known since ancient times as a band of light stretching across the sky. We now recognize the Milky Way as the galaxy of stars in which we live, with a large collection of stars, including our sun, arranged in a giant disk. Since the individual stars are mostly too small for our eyes to resolve, we observe the collective light from these stars, concentrated in the plane of the disk. The idea that the Milky Way is actually a disk of stars was proposed by

(i) Claudius Ptolemy, in the 2nd century AD.

(ii) Johannes Kepler, in 1610.

(iii) Isaac Newton, in 1695.

(iv) Thomas Wright, in 1750.

(v) Immanuel Kant, in 1755.

(vi) Edwin Hubble, in 1923.

(b) *(5 points)* Once it was recognized that we live in a galaxy, it was initially assumed that ours was the only galaxy. The suggestion that some of the patches of light known as nebulae might actually be other galaxies like our own was made by

(i) Claudius Ptolemy, in the 2nd century AD.

(ii) Johannes Kepler, in 1610.

(iii) Isaac Newton, in 1695.

(iv) Thomas Wright, in 1750.

(v) Immanuel Kant, in 1755.

(vi) Edwin Hubble, in 1923.

— Problem 1 continues on next page. —

(c) *(5 points)* The first firm evidence that there is more than one galaxy stemmed from the ability to observe the Andromeda Nebula with high enough resolution to distinguish its individual stars. In particular, the observation of Cepheid variable stars in Andromeda allowed a distance estimate that placed it well outside the Milky Way. The observation of Cepheid variable stars in Andromeda was first made by

(i) Johannes Kepler, in 1610.

(ii) Isaac Newton, in 1695.

(iii Thomas Wright, in 1750.

(iv) Immanuel Kant, in 1755.

(v) Henrietta Swan Leavitt and Harlow Shapley in 1915.

(vi) Edwin Hubble, in 1923.

(d) *(5 points)* The first hint that the universe is filled with radiation with an effective temperature near 3 K, although not recognized at the time, was an observation of absorption lines in cyanogen (CN) by Adams and McKellar in 1941. They observed dark spectral lines which they interpreted as absorption by the cyanogen of light coming from the star behind the gas cloud. Explain in a few sentences how these absorption lines can be used to make inferences about the cosmic background radiation bathing the cyanogen gas cloud.

— Problem 1 continues on next page. —

(e) *(5 points)* As the universe expands, the temperature of the cosmic microwave background

(i) goes up in proportion to the scale factor $a(t)$.

(ii) stays constant.

(iii) goes down in proportion to $1/a(t)$.

(iv) goes down in proportion to $1/a^2(t)$.

(f) *(5 points)* When Hubble measured the value of his constant, he found $H^{-1} \approx$ 100 million years, 2 billion years, 10 billion years, or 20 billion years?

(g) *(5 points)* Explain in a few sentences what is meant by the equivalence principle?

— End of Problem 1. —

PROBLEM 2: OBSERVING A DISTANT GALAXY IN A MATTER-DOMINATED FLAT UNIVERSE *(40 points)*

Suppose that we are living in a matter-dominated flat universe, with a scale factor given by

$$a(t) = bt^{2/3} ,$$

where b is a constant. The present time is denoted by t_0.

(a) *(5 points)* If we measure time in seconds, distance in meters, and coordinate distances in notches, what are the units of b?

(b) *(5 points)* Suppose that we observe a distant galaxy which is one half of a "Hubble length" away, which means that the physical distance today is $\ell_p = \frac{1}{2}cH_0^{-1}$, where c is the speed of light and H_0 is the present value of the Hubble expansion rate. What is the proper velocity $v_p \equiv \frac{\mathrm{d}\ell_p(t)}{\mathrm{d}t}$ of this galaxy relative to us?

(c) *(5 points)* What is the coordinate distance ℓ_c between us and the distant galaxy? Express your answer in terms of b, t_0, and c (but not H_0).*

If you did not answer the previous part, you may still continue with the following parts, using the symbol ℓ_c for the coordinate distance to the galaxy.

(d) *(5 points)* At what time t_e was the light that we are now receiving from the galaxy emitted?

(e) *(5 points)* What is the redshift z of the light that we are now receiving from the distant galaxy?

(f) *(10 points)* Consider a light pulse that leaves the distant galaxy at time t_e, as calculated in part (d), and arrives here at the present time, t_0. Calculate the physical distance $r_p(t)$ between the light pulse and us. Find $r_p(t)$ as a function of t for all t between t_e and t_0.

(g) *(5 points)* If we send a radio message now to the distant galaxy, at what time t_r will it be received?

— *End of Problem 2.* —

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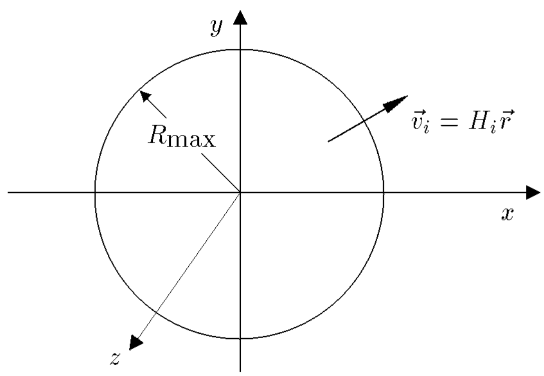
* This sentence was not part of the original printed quiz, but was written on the board during the quiz.

PROBLEM 3: A POSSIBLE MODIFICATION OF NEWTON'S LAW OF GRAVITY *(25 points)*

The following problem was Problem 4 on Problem Set 3.

In Lecture Notes 3 we developed a Newtonian model of cosmology, by considering a uniform sphere of mass, centered at the origin, with initial mass density ρ_i and an initial pattern of velocities corresponding to Hubble expansion: $\vec{v}_i = H_i \vec{r}$:



We denoted the radius at time t of a particle which started at radius r_i by the function $r(r_i, t)$. Assuming Newton's law of gravity, we concluded that each particle would experience an acceleration given by

$$\vec{g} = -\frac{GM(r_i)}{r^2(r_i,t)}\,\hat{r} \ ,$$

where $M(r_i)$ denotes the total mass contained initially in the region $r < r_i$, given by

$$M(r_i) = \frac{4\pi}{3} r_i^3 \rho_i \ .$$

Suppose that the law of gravity is modified to contain a new, repulsive term, producing an acceleration which grows as the nth power of the distance, with a strength that is independent of the mass. That is, suppose $\vec{g}$ is given by

$$\vec{g} = -\frac{GM(r_i)}{r^2(r_i,t)}\,\hat{r} + \gamma r^n(r_i,t)\,\hat{r} \ ,$$

where γ is a constant. The function $r(r_i, t)$ then obeys the differential equation

$$\ddot{r} = -\frac{GM(r_i)}{r^2(r_i,t)} + \gamma r^n(r_i,t) \ .$$

— *Problem 3 continues on the next page.* —

(a) *(5 points)* As done in the lecture notes, we define

$$u(r_i, t) \equiv r(r_i, t)/r_i \ .$$

Write the differential equation obeyed by u. *(Hint: be sure that u is the only time-dependent quantity in your equation; r, ρ, etc. must be rewritten in terms of u, ρ_i, etc.)*

(b) *(5 points)* For what value of the power n is the differential equation found in part (a) independent of r_i?

(c) *(5 points)* Write the initial conditions for u which, when combined with the differential equation found in (a), uniquely determine the function u.

(d) *(10 points)* If all is going well, then you have learned that for a certain value of n, the function $u(r_i, t)$ will in fact not depend on r_i, so we can define

$$a(t) \equiv u(r_i, t) \ .$$

Show, for this value of n, that the differential equation for a can be integrated once to obtain an equation related to the conservation of energy. The desired equation should include terms depending on a and $\dot{a}$, but not $\ddot{a}$ or any higher derivatives.

— *End of Problem 3.* —

Problem	**Maximum**	**Score**
1	35	
2	40	
3	25	
TOTAL	100	

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QUIZ 1 FORMULA SHEET

DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{observed}})}{a(t_{\text{emitted}})}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:
Trailing clock reads later by an amount $\beta \ell_0 / c$.

KINEMATICS OF A HOMOGENEOUSLY EXPANDING UNIVERSE:

Hubble's Law: $v = Hr$,
where $v =$ recession velocity of a distant object, $H =$ Hubble expansion rate, and $r =$ distance to the distant object.

Present Value of Hubble Expansion Rate (Planck 2015):

$$H_0 = 67.7 \pm 0.5 \text{ km-s}^{-1}\text{-Mpc}^{-1}$$

Scale Factor: $\ell_p(t) = a(t)\ell_c$,
where $\ell_p(t)$ is the physical distance between any two objects, $a(t)$ is the scale factor, and ℓ_c is the coordinate distance between the objects, also called the comoving distance.

Hubble Expansion Rate: $H(t) = \dfrac{1}{a(t)}\dfrac{\mathrm{d}a(t)}{\mathrm{d}t}$.

Light Rays in Comoving Coordinates: Light rays travel in straight lines with speed $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{c}{a(t)}$.

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \ , \quad \ddot{a} = -\frac{4\pi}{3}G\rho a \ ,$$

$$\rho(t) = \frac{a^3(t_i)}{a^3(t)}\,\rho(t_i)$$

$$\Omega \equiv \rho/\rho_c \ , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \ .$$

Flat ($k = 0$): $a(t) \propto t^{2/3}$, $\Omega = 1$