MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — December 21, 2016
Prof. Alan Guth

QUIZ 3

Quiz Date: December 7, 2016

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Please answer all questions in this stapled booklet.

A few corrections/clarifications announced at the quiz have been incorporated here.

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

Except for part (d), you should answer these questions by circling the one statement that is correct.

(a) *(5 points)* In the Epilogue of *The First Three Minutes*, Steve Weinberg wrote: "The more the universe seems comprehensible, the more it also seems pointless." The sentence was qualified, however, by a closing paragraph that points out that

(i) the quest of the human race to create a better life for all can still give meaning to our lives.

(ii) if the universe cannot give meaning to our lives, then perhaps there is an afterlife that will.

(iii) the complexity and beauty of the laws of physics strongly suggest that the universe must have a purpose, even if we are not aware of what it is.

(iv) the effort to understand the universe gives human life some of the grace of tragedy.

(b) *(5 points)* In the Afterword of *The First Three Minutes*, Weinberg discusses the baryon number of the universe. (The baryon number of any system is the total number of protons and neutrons (and certain related particles known as hyperons) minus the number of their antiparticles (antiprotons, antineutrons, antihyperons) that are contained in the system.) Weinberg concluded that

(i) baryon number is exactly conserved, so the total baryon number of the universe must be zero. While nuclei in our part of the universe are composed of protons and neutrons, the universe must also contain antimatter regions in which nuclei are composed of antiprotons and antineutrons.

(ii) there appears to be a cosmic excess of matter over antimatter throughout the part of the universe we can observe, and hence a positive density of baryon number. Since baryon number is conserved, this can only be explained by assuming that the excess baryons were put in at the beginning.

(iii) there appears to be a cosmic excess of matter over antimatter throughout the part of the universe we can observe, and hence a positive density of baryon number. This can be taken as a positive hint that baryon number is not conserved, which can happen if there exist as yet undetected heavy "exotic" particles.

(iv) it is possible that baryon number is not exactly conserved, but even if that is the case, it is not possible that the observed excess of matter over antimatter can be explained by the very rare processes that violate baryon number conservation.

(c) *(5 points)* In discussing the COBE measurements of the cosmic microwave background, Ryden describes a dipole component of the temperature pattern, for which the temperature of the radiation from one direction is found to be hotter than the temperature of the radiation detected from the opposite direction.

(i) This discovery is important, because it allows us to pinpoint the direction of the point in space where the big bang occurred.

(ii) This is the largest component of the CMB anisotropies, amounting to a 10% variation in the temperature of the radiation.

(iii) In addition to the dipole component, the anisotropies also includes contributions from a quadrupole, octupole, etc., all of which are comparable in magnitude.

(iv) This pattern is interpreted as a simple Doppler shift, caused by the net motion of the COBE satellite relative to a frame of reference in which the CMB is almost isotropic.

(d) *(5 points)* (CMB basic facts) Which one of the following statements about CMB is *not* correct:

(i) After the dipole distortion of the CMB is subtracted away, the mean temperature averaging over the sky is $\langle T \rangle = 2.725K$.

(ii) After the dipole distortion of the CMB is subtracted away, the root mean square temperature fluctuation is $\left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-3}$.

(iii) The dipole distortion is a simple Doppler shift, caused by the net motion of the observer relative to a frame of reference in which the CMB is isotropic.

(iv) In their groundbreaking paper, Wilson and Penzias reported the measurement of an excess temperature of about 3.5 K that was isotropic, unpolarized, and free from seasonal variations. In a companion paper written by Dicke, Peebles, Roll and Wilkinson, the authors interpreted the radiation to be a relic of an early, hot, dense, and opaque state of the universe.

(e) *(5 points)* Inflation is driven by a field that is by definition called the *inflaton* field. In standard inflationary models, the field has the following properties:

(i) The inflaton is a scalar field, and during inflation the energy density of the universe is dominated by its potential energy.

(ii) The inflaton is a vector field, and during inflation the energy density of the universe is dominated by its potential energy.

(iii) The inflaton is a scalar field, and during inflation the energy density of the universe is dominated by its kinetic energy.

(iv) The inflaton is a vector field, and during inflation the energy density of the universe is dominated by its kinetic energy.

(v) The inflaton is a tensor field, which is responsible for only a small fraction of the energy density of the universe during inflation.

PROBLEM 2: THE EFFECT OF PRESSURE ON COSMOLOGICAL EVOLUTION *(25 points)*

The following problem was Problem 2 of Problem Set 7 (2016), except that some numerical constants have been changed, so the answers will not be identical.

A radiation-dominated universe behaves differently from a matter-dominated universe because the pressure of the radiation is significant. In this problem we explore the role of pressure for several fictitious forms of matter.

(a) *(8 points)* For the first fictitious form of matter, the mass density ρ decreases as the scale factor $a(t)$ grows, with the relation

$$\rho(t) \propto \frac{1}{a^8(t)} \ .$$

What is the pressure of this form of matter? *[Hint: the answer is proportional to the mass density.]*

(b) *(9 points)* Find the behavior of the scale factor $a(t)$ for a flat universe dominated by the form of matter described in part (a). You should be able to determine the function $a(t)$ up to a constant factor.

(c) *(8 points)* Now consider a universe dominated by a different form of fictitious matter, with a pressure given by

$$p = \frac{2}{3}\rho c^2 \ .$$

As the universe expands, the mass density of this form of matter behaves as

$$\rho(t) \propto \frac{1}{a^n(t)} \ .$$

Find the power n.

PROBLEM 3: THE FREEZE-0UT OF A FICTITIOUS PARTICLE X
(25 points)

Suppose that, in addition to the particles that are known to exist, there also existed a family of three spin-1 particles, X^+, X^-, and X^0, all with masses 0.511 MeV/c^2, exactly the same as the electron. The X^- is the antiparticle of the X^+, and the X^0 is its own antiparticle. Since the X's are spin-1 particles with nonzero mass, each particle has three spin states.

The X's do not interact with neutrinos any more strongly than the electrons and positrons do, so when the X's freeze out, all of their energy and entropy are given to the photons, just like the electron-positron pairs.

(a) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c^2, what is the total energy density of the X^+, X^-, and X^0 particles?

(b) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c^2, what is the total number density of the X^+, X^-, and X^0 particles?

(c) *(10 points)* The X particles and the electron-positron pairs freeze out of the thermal equilibrium radiation at the same time, as kT decreases from values large compared to 0.511 MeV/c^2 to values that are small compared to it. If the X's, electron-positron pairs, photons, and neutrinos were all in thermal equilibrium before this freeze-out, what will be the ratio T_ν/T_γ, the ratio of the neutrino temperature to the photon temperature, after the freeze-out?

(d) *(5 points)* If the mass of the X's was, for example, 0.100 MeV/c^2, so that the electron-positron pairs froze out first, and then the X's froze out, would the final ratio T_ν/T_γ be higher, lower, or the same as the answer to part (c)? Explain your answer in a sentence or two.

PROBLEM 4: THE TIME t_d OF DECOUPLING *(25 points)*

The process by which the photons of the cosmic microwave background stop scattering and begin to travel on straight lines is called *decoupling*, and it happens at a photon temperature of about $T_d \approx 3,000$ K. In Lecture Notes 6 we estimated the time t_d of decoupling, working in the approximation that the universe has been matter-dominated from that time to the present. We found a value of 370,000 years. In this problem we will remove this approximation, although we will not carry out the numerical evaluation needed to compare with the previous answer.

(a) *(5 points)* Let us define

$$x(t) \equiv \frac{a(t)}{a(t_0)} ,$$

as on the formula sheets, where t_0 is the present time. What is the value of $x_d \equiv x(t_d)$? Assume that the entropy of photons is conserved from time t_d to the present, and let T_0 denote the present photon temperature.

(b) *(5 points)* Assume that the universe is flat, and that $\Omega_{m,0}$, $\Omega_{\rm rad,0}$, and $\Omega_{\rm vac,0}$ denote the present contributions to Ω from nonrelativistic matter, radiation, and vacuum energy, respectively. Let H_0 denote the present value of the Hubble expansion rate. Write an expression in terms of these quantities for $\mathrm{d}x/\mathrm{d}t$, the derivative of x with respect to t. *Hint: you may use formulas from the formula sheet without derivation, so this problem should require essentially no work. To receive full credit, your answer should include only terms that make a nonzero contribution to the answer.*

(c) *(5 points)* Write an expression for t_d. If your answer involves an integral, you need not try to evaluate it, but you should be sure that the limits of integration are clearly shown.

(d) *(10 points)* Now suppose that in addition to the constituents described in part (b), the universe also contains some of the fictitious material from part (a) of Problem 2, with

$$\rho(t) \propto \frac{1}{a^8(t)} .$$

Denote the present contribution to Ω from this fictitious material as $\Omega_{f,0}$. The universe is still assumed to be flat, so the numerical values of $\Omega_{m,0}$, $\Omega_{\rm rad,0}$, and $\Omega_{\rm vac,0}$ must sum to a smaller value than in parts (b) and (c). With this extra contribution to the mass density of the universe, what is the new expression for t_d?

Problem	**Maximum**	**Score**
1	25	
2	25	
3	25	
4	25	
TOTAL	100	

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QUIZ 3 FORMULA SHEET

DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{observed}})}{a(t_{\text{emitted}})}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:
Trailing clock reads later by an amount $\beta \ell_0 / c$.

Energy-Momentum Four-Vector:

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) , \quad \vec{p} = \gamma m_0 \vec{v} , \quad E = \gamma m_0 c^2 = \sqrt{\left(m_0 c^2\right)^2 + |\vec{p}|^2 c^2} ,$$

$$p^2 \equiv |\vec{p}|^2 - \left(p^0\right)^2 = |\vec{p}|^2 - \frac{E^2}{c^2} = -\left(m_0 c\right)^2 .$$

KINEMATICS OF A HOMOGENEOUSLY EXPANDING UNIVERSE:

Hubble's Law: $v = Hr$,
where v = recession velocity of a distant object, H = Hubble expansion rate, and r = distance to the distant object.

Present Value of Hubble Expansion Rate (Planck 2015):

$$H_0 = 67.7 \pm 0.5 \text{ km-s}^{-1}\text{-Mpc}^{-1}$$

Scale Factor: $\ell_p(t) = a(t)\ell_c$,
where $\ell_p(t)$ is the physical distance between any two objects, $a(t)$ is the scale factor, and ℓ_c is the coordinate distance between the objects, also called the comoving distance.

Hubble Expansion Rate: $H(t) = \dfrac{1}{a(t)} \dfrac{\mathrm{d}a(t)}{\mathrm{d}t}$.

Light Rays in Comoving Coordinates: Light rays travel in straight lines with speed $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{c}{a(t)}$.

Horizon Distance:

$$\begin{aligned}\ell_{p,\text{horizon}}(t) &= a(t) \int_0^t \frac{c}{a(t')} dt' \\ &= \begin{cases} 3ct & \text{(flat, matter-dominated),} \\ 2ct & \text{(flat, radiation-dominated).} \end{cases}\end{aligned}$$

COSMOLOGICAL EVOLUTION:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} \ , \quad \ddot{a} = -\frac{4\pi}{3} G \left(\rho + \frac{3p}{c^2}\right) a \ ,$$

$$\rho_m(t) = \frac{a^3(t_i)}{a^3(t)} \rho_m(t_i) \text{ (matter)}, \quad \rho_r(t) = \frac{a^4(t_i)}{a^4(t)} \rho_r(t_i) \text{ (radiation)}.$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \ , \quad \Omega \equiv \rho/\rho_c \ , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \ .$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

Flat ($k = 0$): $a(t) \propto t^{2/3}$

$$\Omega = 1 \ .$$

Closed ($k > 0$):

$$ct = \alpha(\theta - \sin\theta) \ , \quad \frac{a}{\sqrt{k}} = \alpha(1 - \cos\theta) \ ,$$

$$\Omega = \frac{2}{1 + \cos\theta} > 1 \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho}{c^2}\left(\frac{a}{\sqrt{k}}\right)^3 \ .$$

Open ($k < 0$):

$$ct = \alpha\left(\sinh\theta - \theta\right) \ , \quad \frac{a}{\sqrt{\kappa}} = \alpha\left(\cosh\theta - 1\right) \ ,$$

$$\Omega = \frac{2}{1 + \cosh\theta} < 1 \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho}{c^2}\left(\frac{a}{\sqrt{\kappa}}\right)^3 \ ,$$

$$\kappa \equiv -k > 0 \ .$$

ROBERTSON-WALKER METRIC:

$$ds^2 = -c^2\,d\tau^2 = -c^2\,dt^2 + a^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} \ .$$

Alternatively, for $k > 0$, we can define $r = \dfrac{\sin\psi}{\sqrt{k}}$, and then

$$ds^2 = -c^2\,d\tau^2 = -c^2\,dt^2 + \tilde{a}^2(t)\left\{d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} \ ,$$

where $\tilde{a}(t) = a(t)/\sqrt{k}$. For $k < 0$ we can define $r = \dfrac{\sinh\psi}{\sqrt{-k}}$, and then

$$ds^2 = -c^2\,d\tau^2 = -c^2\,dt^2 + \tilde{a}^2(t)\left\{d\psi^2 + \sinh^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} \ ,$$

where $\tilde{a}(t) = a(t)/\sqrt{-k}$. Note that $\tilde{a}$ can be called a if there is no need to relate it to the $a(t)$ that appears in the first equation above.

SCHWARZSCHILD METRIC:

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2$$
$$+ r^2 d\theta^2 + r^2\sin^2\theta\,d\phi^2 \ ,$$

GEODESIC EQUATION:

$$\frac{d}{ds}\left\{g_{ij}\frac{dx^j}{ds}\right\} = \frac{1}{2}\left(\partial_i g_{k\ell}\right)\frac{dx^k}{ds}\,\frac{dx^\ell}{ds}$$

$$\text{or:}\qquad \frac{d}{d\tau}\left\{g_{\mu\nu}\frac{dx^\nu}{d\tau}\right\} = \frac{1}{2}\left(\partial_\mu g_{\lambda\sigma}\right)\frac{dx^\lambda}{d\tau}\frac{dx^\sigma}{d\tau}$$

BLACK-BODY RADIATION:

$$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3} \qquad \text{(energy density)}$$

$$p = \frac{1}{3}u \qquad \rho = u/c^2 \qquad \text{(pressure, mass density)}$$

$$n = g^*\frac{\zeta(3)}{\pi^2}\,\frac{(kT)^3}{(\hbar c)^3} \qquad \text{(number density)}$$

$$s = g\frac{2\pi^2}{45}\,\frac{k^4T^3}{(\hbar c)^3}\ , \qquad \text{(entropy density)}$$

where

$$g \equiv \begin{cases} 1 \text{ per spin state for bosons (integer spin)} \\ 7/8 \text{ per spin state for fermions (half-integer spin)} \end{cases}$$

$$g^* \equiv \begin{cases} 1 \text{ per spin state for bosons} \\ 3/4 \text{ per spin state for fermions}\ , \end{cases}$$

and

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202\ .$$

$$g_\gamma = g^*_\gamma = 2 \ ,$$

$$g_\nu = \underbrace{\frac{7}{8}}_{\substack{\text{Fermion}\\ \text{factor}}} \times \underbrace{3}_{\substack{\text{3 species}\\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\substack{\text{Particle/}\\ \text{antiparticle}}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4} \ ,$$

$$g^*_\nu = \underbrace{\frac{3}{4}}_{\substack{\text{Fermion}\\ \text{factor}}} \times \underbrace{3}_{\substack{\text{3 species}\\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\substack{\text{Particle/}\\ \text{antiparticle}}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2} \ ,$$

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\substack{\text{Fermion}\\ \text{factor}}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\substack{\text{Particle/}\\ \text{antiparticle}}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2} \ ,$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\substack{\text{Fermion}\\ \text{factor}}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\substack{\text{Particle/}\\ \text{antiparticle}}} \times \underbrace{2}_{\text{Spin states}} = 3 \ .$$

EVOLUTION OF A FLAT RADIATION-DOMINATED UNIVERSE:

$$\rho = \frac{3}{32\pi G t^2}$$

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}}$$

For $m_\mu = 106$ MeV $\gg kT \gg m_e = 0.511$ MeV, $g = 10.75$ and then

$$kT = \frac{0.860 \text{ MeV}}{\sqrt{t \text{ (in sec)}}} \left(\frac{10.75}{g}\right)^{1/4}$$

After the freeze-out of electron-positron pairs,

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \ .$$

COSMOLOGICAL CONSTANT:

$$u_{\text{vac}} = \rho_{\text{vac}} c^2 = \frac{\Lambda c^4}{8\pi G} \ ,$$

$$p_{\text{vac}} = -\rho_{\text{vac}} c^2 = -\frac{\Lambda c^4}{8\pi G} \ .$$

GENERALIZED COSMOLOGICAL EVOLUTION:

$$x \frac{dx}{dt} = H_0 \sqrt{\Omega_{m,0} x + \Omega_{\text{rad},0} + \Omega_{\text{vac},0} x^4 + \Omega_{k,0} x^2} \ ,$$

where

$$x \equiv \frac{a(t)}{a(t_0)} \equiv \frac{1}{1+z} \ ,$$

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0) H_0^2} = 1 - \Omega_{m,0} - \Omega_{\text{rad},0} - \Omega_{\text{vac},0} \ .$$

Age of universe:

$$\begin{aligned} t_0 &= \frac{1}{H_0} \int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0} x + \Omega_{\text{rad},0} + \Omega_{\text{vac},0} x^4 + \Omega_{k,0} x^2}} \\ &= \frac{1}{H_0} \int_0^\infty \frac{dz}{(1+z)\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\text{rad},0}(1+z)^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1+z)^2}} \ . \end{aligned}$$

Look-back time:

$$t_{\text{look-back}}(z) =$$

$$\frac{1}{H_0} \int_0^z \frac{dz'}{(1+z')\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{\text{rad},0}(1+z')^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1+z')^2}} \ .$$

PHYSICAL CONSTANTS:

$$G = 6.674 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} = 6.674 \times 10^{-8}\ \text{cm}^3 \cdot \text{g}^{-1} \cdot \text{s}^{-2}$$

$$\begin{aligned} k = \text{Boltzmann's constant} &= 1.381 \times 10^{-23}\,\text{joule/K} \\ &= 1.381 \times 10^{-16}\,\text{erg/K} \\ &= 8.617 \times 10^{-5}\,\text{eV/K} \end{aligned}$$

$$\begin{aligned} \hbar = \frac{h}{2\pi} &= 1.055 \times 10^{-34}\ \text{joule} \cdot \text{s} \\ &= 1.055 \times 10^{-27}\ \text{erg} \cdot \text{s} \\ &= 6.582 \times 10^{-16}\ \text{eV} \cdot \text{s} \end{aligned}$$

$$c = 2.998 \times 10^8\ \text{m/s}$$

$$= 2.998 \times 10^{10} \text{ cm/s}$$

$$\hbar c = 197.3 \text{ MeV-fm}, \quad 1 \text{ fm} = 10^{-15} \text{ m}$$

$$1 \text{ yr} = 3.156 \times 10^{7} \text{ s}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ joule} = 1.602 \times 10^{-12} \text{ erg}$$

$$\begin{aligned} 1 \text{ GeV} = 10^9 \text{ eV} &= 1.783 \times 10^{-27} \text{ kg (where } c \equiv 1) \\ &= 1.783 \times 10^{-24} \text{ g} . \end{aligned}$$

Planck Units: The Planck length ℓ_P, the Planck time t_P, the Planck mass m_P, and the Planck energy E_p are given by

$$\begin{aligned} \ell_P = \sqrt{\frac{G\hbar}{c^3}} &= 1.616 \times 10^{-35} \text{ m} , \\ &= 1.616 \times 10^{-33} \text{ cm} , \\ t_P = \sqrt{\frac{\hbar G}{c^5}} &= 5.391 \times 10^{-44} \text{ s} , \\ m_P = \sqrt{\frac{\hbar c}{G}} &= 2.177 \times 10^{-8} \text{ kg} , \\ &= 2.177 \times 10^{-5} \text{ g} , \\ E_P = \sqrt{\frac{\hbar c^5}{G}} &= 1.221 \times 10^{19} \text{ GeV} . \end{aligned}$$

CHEMICAL EQUILIBRIUM:

(This topic was not included in the course in 2013, but the formulas are nonetheless included here for logical completeness. They will not be relevant to Quiz 3. They are relevant to Problem 13 in these Review Problems, which is also not relevant to Quiz 3. Please enjoy looking at these items, or enjoy ignoring them!)

Ideal Gas of Classical Nonrelativistic Particles:

$$n_i = g_i \frac{(2\pi m_i kT)^{3/2}}{(2\pi\hbar)^3} \, e^{(\mu_i - m_i c^2)/kT} .$$

$$\begin{aligned} \text{where } n_i &= \text{number density of particle} \\ g_i &= \text{number of spin states of particle} \\ m_i &= \text{mass of particle} \\ \mu_i &= \text{chemical potential} \end{aligned}$$

For any reaction, the sum of the μ_i on the left-hand side of the reaction equation must equal the sum of the μ_i on the right-hand side. Formula assumes gas is nonrelativistic ($kT \ll m_i c^2$) and dilute ($n_i \ll (2\pi m_i kT)^{3/2}/(2\pi\hbar)^3$).