MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 3, 2018
Prof. Alan Guth

QUIZ 1

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Please answer all questions in this stapled booklet.

PROBLEM 1: DID YOU DO THE READING? *(30 points)*

(a) *(5 points)* After telescopes became available, more and more extended objects in the sky, called nebulae, were discovered, but those were thought as members of our galaxy. Who is the person who first proposed that some of the nebulae are galaxies like our own located outside our galaxy?

(i) Isaac Newton

(ii) Immanuel Kant

(iii) Edwin Hubble

(iv) Albert Einstein

(b) *(5 points)* Before 1923, questions of the nature of the spiral and elliptical nebulae could not be settled without some reliable method of determining how far away they are. In 1923, Edwin Hubble was for the first time able to resolve the Andromeda Nebula (galaxy) into separate stars and estimated the distance to the Andromeda Nebula. What observational quantity did he measure to estimate the distance?

(i) the radial velocity of individual stars in the Adromeda Nebula

(ii) the radial velocity of the Andromeda Nebula itself

(iii) the periods of variation of a class of stars in the Andromeda Nebula

(iv) the parallax of bright stars in the Adromeda Nebula

— Problem 1 continues on next page. —

(c) *(5 points)* In 1917, a year after the completion of Einstein's general theory of relativity, _________ looked specifically for a solution that would be homogeneous, isotropic, and static, and thus was forced to mutilate the equations by introducing a term, the so-called cosmological constant. In the same year, another solution of the modified theory was found by the Dutch astronomer _________. Although this solution appeared to be static, it had the remarkable property of predicting a redshift proportional to the distance. In 1922, the general homogeneous and isotropic solution of the original Einstein equations was found by the Russian mathematician _________, which provides a mathematical background for the most modern cosmological theories. Which is the right answer to fill in the blanks in turn?

(i) Friedmann — Einstein — de Sitter

(ii) Friedmann — de Sitter — Einstein

(iii) Einstein — Friedmann — de Sitter

(iv) Einstein — de Sitter — Friedmann

(v) de Sitter — Einstein — Friedmann

(vi) de Sitter — Friedmann — Einstein

(d) *(5 points)* After radio noises with the equivalent temperature of about 3.5° K were detected, Penzias, Wilson, Dicke, Peebles, Roll, and Wilkinson decided to publish a pair of companion letters in the Astrophysical Journal, in which Penzias and Wilson would announce their observations, and Dicke, Peebles, Roll, and Wilkinson would explain the cosmological interpretation. What is the title of the paper written by Penzias and Wilson?

(i) "A Measurement of Excess Antenna Temperature at 4,080 Mc/s"

(ii) "Cosmic Black-Body Radiation"

(iii) "Origin of the Microwave Radio Background"

(iv) "Three Degrees Above Zero: Bell Labs in the Information Age"

— Problem 1 continues on next page. —

(e) *(5 points)* The universe contains different types of particles. Which of the following statements is NOT true?

(i) A baryon is defined as a particle made of three quarks.

(ii) Electrons and neutrinos are leptons.

(iii) There are three types of neutrinos and they all have zero charge.

(iv) The component of the universe made of ions, atoms, and molecules is generally referred to as baryonic matter, since only the baryons (protons and neutrons) contribute significantly to the mass density.

(v) About three-fourths of the baryonic matter in the universe is currently in the form of helium.

(f) *(5 points)* If one averages over sufficiently large scales, the universe appears to be homogeneous and isotropic. How large must the averaging scale be before this homogeneity and isotropy set in?*

(i) 1000 Mpc. (1 Mpc $= 10^6$ pc, 1 pc $= 3.086 \times 10^{16}$ m $= 3.262$ light-year).

(ii) 100 Mpc.

(iii) 1 Mpc.

(iv) 100 kpc (1 kpc = 1000 pc).

(v) 1 AU (1 AU $= 1.496 \times 10^{11}$ m).

— *End of Problem 1.* —

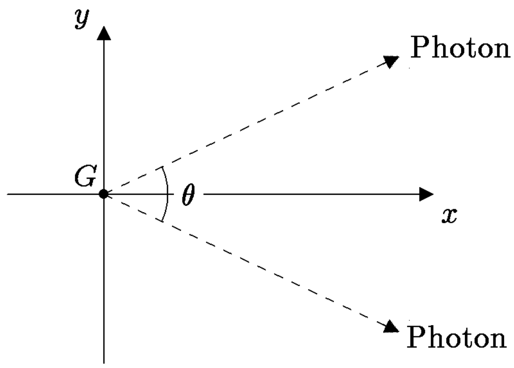
* This question was a replacement, listed on a separate sheet when the quiz was administered.

PROBLEM 2: LIGHT RAYS TRAVELING THROUGH A MATTER-DOMINATED FLAT UNIVERSE *(40 points)*

Consider a flat, matter-dominated universe, with a scale factor given by

$$a(t) = bt^{2/3} ,$$

where b is a constant. Now consider a galaxy G in this universe which at time t_1 emits two photons, with an angular separation θ between their paths, as shown in the diagram:

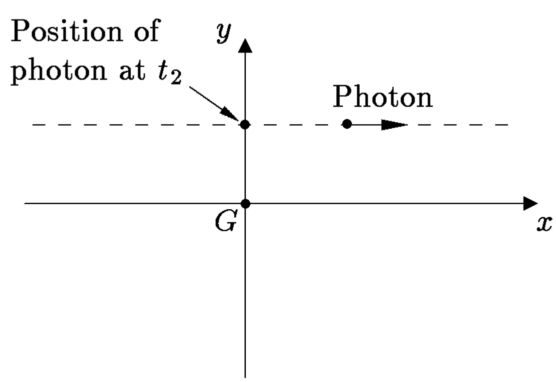


(a) *(10 points)* At cosmic time t (for $t > t_1$), what is the physical distance $\ell_{1,\text{phys}}(t)$ of each of these photons from the galaxy G?

(b) *(5 points)* If the frequency of the photons was ν_1 when they were emitted, what is their frequency $\nu(t)$ at cosmic time t (for $t > t_1$)? $\nu(t)$ should be the frequency as it would be measured by a comoving observer, i.e. an observer at rest with respect to the matter at the same location.

(c) *(10 points)* What is the physical distance $\ell_{2,\text{phys}}(t)$ between the two photons at time t (for $t > t_1$)?

Now consider a different situation, but in the same universe. This time we consider a photon that travels past the galaxy G, traveling in the x direction, in the x-y plane, as shown in the diagram below. We are told that the photon crosses the y axis at time t_2, and at that time the photon is a physical distance h from the galaxy.



(d) *(10 points)* What is the physical distance $\ell_{3,\mathrm{phys}}(t)$ between the photon and the galaxy G at arbitrary time t, which might be earlier or later than t_2?

(e) *(5 points)* At time t_2, what is the recessional speed $d\ell_{3,\mathrm{phys}}(t)/dt$ of the photon from the galaxy. *Hint:* if you are clever, this can be done with very little calculation.

PROBLEM 3: THE STEADY-STATE UNIVERSE THEORY *(30 points)*

The following problem was Problem 2, Quiz 1, 2000. It was also Problem 2 of the Quiz 1 Review Problems, 2018.

Until the discovery of the cosmic microwave background, the steady state theory was considered a viable model of the universe. As the name suggests, this theory is based on the hypothesis that the large-scale properties of the universe do not change with time. The expansion of the universe was an established fact when the steady-state theory was invented, but the steady-state theory reconciles the expansion with a steady-state density of matter by proposing that new matter is created as the universe expands, so that the matter density does not fall. Like the conventional theory, the steady-state theory describes a homogeneous, isotropic, expanding universe, so the same comoving coordinate formulation can be used.

a) *(15 points)* The steady-state theory proposes that the Hubble constant, like other cosmological parameters, does not change with time, so $H(t) = H_0$. Find the most general form for the scale factor function $a(t)$ which is consistent with this hypothesis.

b) *(15 points)* Suppose that the mass density of the universe is ρ_0, which of course does not change with time. In terms of the general form for $a(t)$ that you found in part (a), calculate the rate at which new matter must be created for ρ_0 to remain constant as the universe expands. Your answer should have the units of mass per unit volume per unit time. *[If you failed to answer part (a), you will still receive full credit here if you correctly answer the question for an arbitrary scale factor function $a(t)$.]*

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QUIZ 1 FORMULA SHEET

DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{observed}})}{a(t_{\text{emitted}})}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \,, \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:
Trailing clock reads later by an amount $\beta \ell_0 / c$.

KINEMATICS OF A HOMOGENEOUSLY EXPANDING UNIVERSE:

Hubble's Law: $v = Hr$,
where v = recession velocity of a distant object, H = Hubble expansion rate, and r = distance to the distant object.

Present Value of Hubble Expansion Rate (Planck 2018):

$$H_0 = 67.66 \pm 0.42 \text{ km-s}^{-1}\text{-Mpc}^{-1}$$

Scale Factor: $\ell_p(t) = a(t)\ell_c$,
where $\ell_p(t)$ is the physical distance between any two objects, $a(t)$ is the scale factor, and ℓ_c is the coordinate distance between the objects, also called the comoving distance.

Hubble Expansion Rate: $H(t) = \dfrac{1}{a(t)}\dfrac{\mathrm{d}a(t)}{\mathrm{d}t}$.

Light Rays in Comoving Coordinates: Light rays travel in straight lines with speed $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{c}{a(t)}$.

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} , \quad \ddot{a} = -\frac{4\pi}{3}G\rho a ,$$

$$\rho(t) = \frac{a^3(t_i)}{a^3(t)}\,\rho(t_i)$$

$$\Omega \equiv \rho/\rho_c , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} .$$

Flat ($k = 0$): $a(t) \propto t^{2/3}$, $\Omega = 1$