MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — December 5, 2018
Prof. Alan Guth

QUIZ 3*

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Please answer all questions in this stapled booklet.

PROBLEM 1: DID YOU DO THE READING? *(20 points)*

(a) *(5 points)* Which one of the following statements about CMB is NOT correct?

(i) The dipole distortion is a simple Doppler shift, caused by the net motion of the observer relative to a frame of reference in which the CMB is isotropic.

(ii) After the dipole distortion of the CMB is subtracted away, the mean temperature averaging over the sky is <T> = 2.725K.

(iii) After the dipole distortion of the CMB is subtracted away, the temperature of the CMB varies by 0.3 microKelvin across the sky.

(iv) The photons of the CMB have mostly been traveling on straight lines since they were last scattered at $t \approx 370,000$ yr, at a location called the surface of last scattering.

(b) *(5 points)* The nonuniformities in the cosmic microwave background allow us to measure the ripples in the mass density of the universe at the time when the plasma combined to form neutral atoms, about 300,000 - 400,000 years after the big bang. These ripples are crucial for understanding what happened later, since they are the seeds which led to the complicated tapestry of galaxies, clusters of galaxies, and voids. Which of the following sentences describes how these ripples are created in the context of inflationary models:

(i) Magnetic monopoles can form randomly during the grand unified theory phase transition, resulting in nonuniformities in the mass density.

(ii) Cosmic strings, which are linelike topological defects, can form randomly during the grand unified theory phase transition, resulting in nonuniformities in the mass density.

(iii) They are generated by quantum fluctuations during inflation.

(iv) Since the early universe was very hot, there were large thermal fluctuations which ultimately evolved into the ripples in the mass density.

— Problem 1 continues on next page. —

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* This version includes corrections that were announced during the quiz.

(c) *(5 points)* In Chapter 8 of *The First Three Minutes*, Steven Weinberg describes the future of the universe (assuming, as was thought then to be the case, that the cosmological constant is zero). One possibility that he discusses is that the cosmic matter density could be greater than the critical density. Assuming that we live in such a universe, which of the following statements is NOT true?

(i) The universe is finite and its expansion will eventually cease, giving way to an accelerating contraction.

(ii) Three minutes after the temperature reaches a thousand million degrees (10^9 K), the laws of physics guarantee that the universe will crunch, and time will stop.

(iii) During at least the early part of the contracting phase, we will be able to observe both redshifts and blueshifts.

(iv) When the universe has recontracted to one-hundredth its present size, the radiation background will begin to dominate the sky, with a temperature of about 300 K.

(d) *(5 points)* Which of the following describes the Sachs-Wolfe effect?

(i) Photons from fluid which had a velocity toward us along the line of sight appear redder because of the Doppler effect.

(ii) Photons from fluid which had a velocity toward us along the line of sight appear bluer because of the Doppler effect.

(iii) Photons traveling toward us from the surface of last scattering appear redder because of absorption in the intergalactic medium.

(iv) Photons traveling toward us from the surface of last scattering appear bluer because of absorption in the intergalactic medium.

(v) Photons from overdense regions at the surface of last scattering appear redder because they must climb out of the gravitational potential well.

(vi) Photons from overdense regions at the surface of last scattering appear bluer because they must climb out of the gravitational potential well.

— End of Problem 1. —

PROBLEM 2: TIME EVOLUTION OF A UNIVERSE INCLUDING A HYPOTHETICAL KIND OF MATTER *(30 points)*

Suppose that a flat universe includes nonrelativistic matter, radiation, and also mysticium, where the mass density of mysticium behaves as

$$\rho_{\rm myst} \propto \frac{1}{a^5(t)}$$

as the universe expands. In this problem we will define

$$x(t) \equiv \frac{a(t)}{a(t_0)} ,$$

where t_0 is the present time. For the following questions, you need not evaluate any of the integrals that might arise, but they must be integrals of explicit functions with explicit limits of integration; remember that $a(t)$ is not given. You may express your answers in terms of the present value of the Hubble expansion rate, H_0, and the various contributions to the present value of Ω: $\Omega_{m,0}$, $\Omega_{\rm rad,0}$, and $\Omega_{\rm myst,0}$.

(a) *(7 points)* Write an expression for the Hubble expansion rate $H(x)$.

(b) *(7 points)* Write an expression for the current age of the universe.

(c) *(3 points)* Write an expression for the time $t(x)$ in terms of the value of x.

(d) *(3 points)* Write an expression for the total mass density $\rho(x)$ as a function of x.

(e) *(10 points)* Write an expression for present value of the physical horizon distance, $\ell_{p,\rm hor}(t_0)$.

PROBLEM 3: PROPERTIES OF BLACK-BODY RADIATION *(25 points)*

The following problem was Problem 6 of the Review Problems for Quiz 3.

In answering the following questions, remember that you can refer to the formulas on the formula sheets, circulated separately. Since you were not asked to bring calculators, you may leave your answers in the form of algebraic expressions, such as $\pi^{32}/\sqrt{5\zeta(3)}$.

(a) *(5 points)* For the black-body radiation (also called thermal radiation) of photons at temperature T, what is the average energy per photon?

(b) *(5 points)* For the same radiation, what is the average entropy per photon?

(c) *(5 points)* Now consider the black-body radiation of a massless boson which has spin zero, so there is only one spin state. Would the average energy per particle and entropy per particle be different from the answers you gave in parts (a) and (b)? If so, how would they change?

(d) *(5 points)* Now consider the black-body radiation of electron neutrinos at temperature T. These particles are fermions with spin 1/2, and we will assume that they are massless and have only one possible spin state. What is the average energy per particle for this case?

(e) *(5 points)* What is the average entropy per particle for the black-body radiation of neutrinos, as described in part (d)?

PROBLEM 4: THE CONSEQUENCES OF AN ALT-PHOTON *(25 points)*

Suppose that, in addition to the particles that are known to exist, there also existed an alt-photon, which has exactly the properties of a photon: it is massless, has two spin states (or polarization states), and has the same interactions with other particles that photons do. Like photons, it is its own antiparticle.

(a) *(5 points)* In thermal equilibrium at temperature T, what is the total energy density of alt-photons?

(b) *(5 points)* In thermal equilibrium at temperature T, what is the number density of alt-photons?

(c) *(10 points)* In this situation, what would be the temperature ratios T_ν/T_γ and $T_\nu/T_{\text{alt}\gamma}$ today?

(d) *(5 points)* Would the existence of this particle increase or decrease the abundance of helium, or would it have no effect?

Problem	**Maximum**	**Score**	**Initials**
1	20		
2	30		
3	25		
4	25		
TOTAL	100		