MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — September 30, 2020
Prof. Alan Guth

QUIZ 1

PROBLEM 1: DID YOU DO THE READING? *(30 points)*

(a) *(5 points)* In what way do spectral lines help us understand the properties of nearby stars? Consider the following options:

(A) We can determine their chemical composition.

(B) We can determine their speed relative to us.

(C) We can determine their acceleration relative to us.

(D) We can determine their mass.

(E) We can determine their volume.

Which one of the following combinations is correct:

(i) (A) and (B)

(ii) (A) and (C)

(iii) (B) and (C)

(iv) (D) and (E)

(v) (A), (B), (C), (D), and (E)

(b) *(5 points)* What is the cosmological principle?

(i) We live in the center of the universe.

(ii) The universe is homogeneous and isotropic on large distance scales.

(iii) The universe is isotropic.

(iv) All galaxies are evenly spaced throughout the universe.

(v) Newtonian gravity applies on large distance scales.

(c) *(5 points)* Roughly, what is the size of the Milky Way? And, at what length scale does the cosmological principle become valid?

(i) 10^7 light years and 100 Mpc

(ii) 10^5 light years and 1 Mpc

(iii) 10^5 light years and 100 Mpc

(iv) 10^3 light years and 1 Mpc

(v) 10^3 light years and 100 Mpc

— Problem 1 continues on next page. —

(d) *(5 points)* Which one of the following statements is a valid description of the history of what is now known as the "Cosmic Microwave Background"?

(i) The CMB radiation initially had a temperature less than 2.7 degrees Kelvin, but it increased gradually to its current value as it absorbed the energy of the light from stars.

(ii) Plasma instabilities in the early universe generated large fluctuations in electromagnetic radiation, which we now see as a "background" to astronomical observations.

(iii) The universe was originally filled with microwave radiation, and it persists until today.

(iv) It is a primordial kind of radiation with a blackbody spectrum of 2.7 degrees Kelvin at the time it was generated.

(v) At some point in the history of our universe, photons, which were once in thermal equilibrium with other particles, stopped interacting because electrons and atomic nuclei combined into atoms. The photons then free-streamed throughout the universe, from then until now.

(e) *(5 points)* Why is the night sky not uniformly bright due to the universe being homogeneously populated with stars (or other bright objects)? Consider the following options:

(A) Space is not transparent.

(B) The universe is not infinitely large.

(C) The universe is not infinitely old.

(D) The absolute brightness of stars depends on their distance away from us.

(E) The cosmological redshift makes stars look dimmer and dimmer as they are further away from us.

Which one of the following combinations is correct?

(i) (A) and (C)

(ii) (B) and (D)

(iii) (C) and (E)

(iv) (A), (C), and (E)

(v) (B), (C), and (D)

— *Problem 1 continues on next page.* —

(f) *(5 points)* Which of the following statements best describes Einstein's equivalence principle?

(i) Physical laws do not depend on the place of the universe where we make measurements.

(ii) General relativity is equivalent to special relativity when only massless particles are considered.

(iii) An observer on the surface of the Earth experiences the same external forces as a free-falling observer.

(iv) An observer on the surface of the Earth, without looking through a window, cannot tell whether they are actually on Earth or if they are on a rocket ship accelerating at 9.8 $\mathrm{m/s^2}$ in empty space.

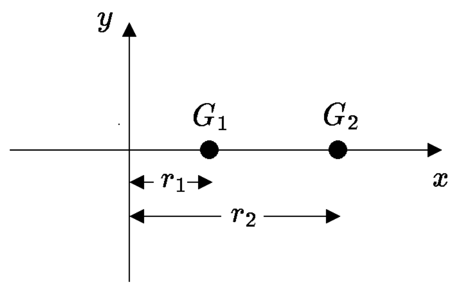
(v) Any calculation done in the framework of General Relativity must be equivalent to another in Newtonian gravity in the appropriate limit.

PROBLEM 2: INTERGALACTIC SIGNALING AND TRAVEL *(30 points)*

Consider a flat, matter-dominated universe, with a scale factor

$$a(t) = bt^{2/3} ,$$

where b is a constant. Suppose that galaxy G_1 is located at comoving coordinates $(x, y, z) = (r_1, 0, 0)$ and galaxy G_2 is located at $(x, y, z) = (r_2, 0, 0)$, as shown in the following diagram:



(a) *(10 points)* At cosmic time $t = t_1$, a light signal is sent from G_1, in the direction of G_2. At what time t_2 does the light signal reach G_2? Your answer should be expressed in terms of some or all of the given variables t_1, b, r_1, r_2, and the speed of light c.

(b) *(6 points)* When the light signal arrives at G_2, what is its redshift z? Your answer can be expressed in terms of any or all of the variables mentioned in part (a), and also t_2, the answer to part (a).

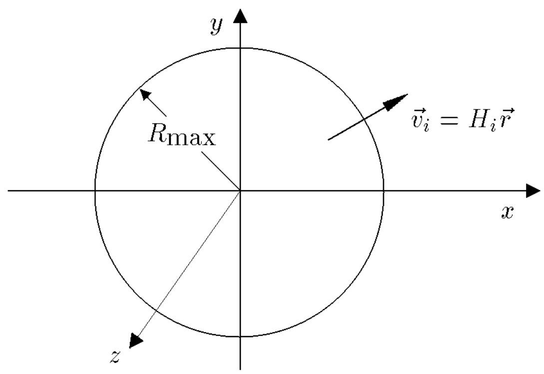
(c) *(7 points)* Now suppose that at time t_1, the galaxy G_1 also launches a starship towards G_2. This is a science-fiction starship, which moves at speed $\frac{1}{2}c$ relative to the comoving observers. That is, it moves at speed $\frac{1}{2}c$ relative to the observers who are at rest in the comoving coordinate system. For simplicity, we will assume that the starship takes no time to reach its cruising velocity, and no time to decelerate at the end of its journey. At what time t_3 does the starship arrive at G_2? Your answer may depend on any of the allowed variables in part (b).

(d) *(7 points)* We have not discussed how the clocks on such a starship would behave, but general relativity implies that, as the starship passes any comoving observer, the comoving observer would measure the clocks on the starship to be time-dilated exactly as they would be in special relativity, for the same relative velocity. Given this fact, what time will the clocks on the starship read when the ship arrives at G_2?

PROBLEM 3: A POSSIBLE MODIFICATION OF NEWTON'S LAW OF GRAVITY *(30 points)*

The following problem was Problem 4 on Problem Set 3.

In Lecture Notes 3 we developed a Newtonian model of cosmology, by considering a uniform sphere of mass, centered at the origin, with initial mass density ρ_i and an initial pattern of velocities corresponding to Hubble expansion: $\vec{v}_i = H_i\vec{r}$:



We denoted the radius at time t of a particle which started at radius r_i by the function $r(r_i, t)$. Assuming Newton's law of gravity, we concluded that each particle would experience an acceleration given by

$$\vec{g} = -\frac{GM(r_i)}{r^2(r_i,t)}\,\hat{r}\ ,$$

where $M(r_i)$ denotes the total mass contained initially in the region $r < r_i$, given by

$$M(r_i) = \frac{4\pi}{3} r_i^3 \rho_i\ .$$

Suppose that the law of gravity is modified to contain a new, repulsive term, producing an acceleration which grows as the nth power of the distance, with a strength that is independent of the mass. That is, suppose $\vec{g}$ is given by

$$\vec{g} = -\frac{GM(r_i)}{r^2(r_i,t)}\,\hat{r} + \gamma r^n(r_i,t)\,\hat{r}\ ,$$

where γ is a constant. The function $r(r_i, t)$ then obeys the differential equation

$$\ddot{r} = -\frac{GM(r_i)}{r^2(r_i,t)} + \gamma r^n(r_i,t)\ .$$

— *Problem 3 continues on next page.* —

(a) *(7 points)* As done in the lecture notes, we define

$$u(r_i, t) \equiv r(r_i, t)/r_i \ .$$

Write the differential equation obeyed by u. *(Hint: be sure that u is the only time-dependent quantity in your equation; r, ρ, etc. must be rewritten in terms of u, ρ_i, etc.)*

(b) *(6 points)* For what value of the power n is the differential equation found in part (a) independent of r_i?

(c) *(7 points)* Write the initial conditions for u which, when combined with the differential equation found in (a), uniquely determine the function u.

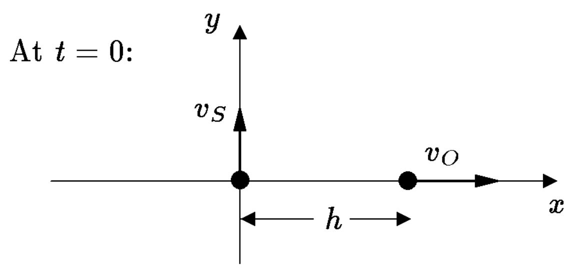
(d) *(10 points)* If all is going well, then you have learned that for a certain value of n, the function $u(r_i, t)$ will in fact not depend on r_i, so we can define

$$a(t) \equiv u(r_i, t) \ .$$

Show, for this value of n, that the differential equation for a can be integrated once to obtain an equation related to the conservation of energy. The desired equation should include terms depending on a and $\dot{a}$, but not $\ddot{a}$ or any higher derivatives.

PROBLEM 4: THE NONRELATIVISTIC TRANSVERSE DOPPLER SHIFT *(10 points)*

At time $t = 0$, a source of sound waves is located at the origin of a coordinate system, and an observer is located at a distance h along the positive x axis, as shown in the diagram below. The source is moving at constant speed v_S along the positive y axis, and the observer is moving at constant speed v_O along the positive x axis. Denote the speed of sound by u.



(a) *(6 points)* At $t = 0$, the sound source emits a wave crest. At what time t_1 does it arrive at the observer?

(b) *(4 points)* Under the usual assumption that the wavelength of the wave is very short compared to any other distance, what is the Doppler shift of the sound wave, as received by the observer?

Problem	**Maximum**	**Score**	**Initials**
1	30		
2	30		
3	30		
4	10		
TOTAL	100		