MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — December 2, 2020
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QUIZ 3

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

(a) *(5 points)* In what sense is the Big Bang theory incomplete?

(i) It doesn't explain the origin of our universe. It only explains what happened after the "big bang."

(ii) It doesn't explain how matter clumped into galaxies.

(iii) It is incompatible with our observation of a homogeneous cosmic microwave background (CMB).

(iv) It always leads to a Big Crunch at the end of time, which is incompatible with the accelerated expansion we observe today.

(v) It isn't incomplete in any way; it is a fully self-contained theory that explains why the universe is as flat and homogeneous as we observe it.

(b) *(5 points)* The cosmic microwave background is nearly isotropic, up to some small fluctuations. Is our observation of these fluctuations from Earth affected by the motion of our galaxy?

(i) No, because our galaxy isn't located at any special point in space, so the universe we observe must be statistically homogeneous in every direction. That is to say, after averaging over small patches of the sky, every direction looks exactly the same.

(ii) No, because we can always consider the reference frame where our galaxy is at rest, and the fact that there is local thermal equilibrium in our universe means that we will observe a homogeneous pattern of radiation once we go to the locally equilibrated frame.

(iii) Yes. The rapid motion of our galaxy, because of its massive kinetic energy, leads to a strong gravitational field that distorts the CMB in such a way that most CMB photons we observe had their trajectory bent by more than 90 degrees.

(iv) Yes. Galaxies generally have "peculiar" velocities (small departures from a completely uniform Hubble expansion). This means that the CMB we observe is not perfectly isotropic because we are moving relative to the frame of reference in which the CMB is isotropic, leading to what is known as the "dipole distortion" of the CMB.

(v) No, because the peculiar velocity of the Milky way is too small to produce any significant distortion in our observations of the CMB.

— Problem 1 continues on next page. —

(c) *(5 points)* Which of the following sequences of events is correctly ordered from earliest to latest?

(i) Radiation-matter equality, Inflation, Nucleosynthesis

(ii) Last scattering surface, Recombination, Formation of galaxies

(iii) Radiation-matter equality, Recombination, Photon decoupling

(iv) Nucleosynthesis, Recombination, Inflation

(v) Inflation, Formation of galaxies, Last scattering surface

(d) *(5 points)* What problematic aspects of the conventional Big Bang theory does the inflationary theory explain? Consider the following possibilities:

(A) The flatness problem

(B) The horizon problem

(C) The monopole problem

Which one of the following combinations is the best answer to the question?

(i) Only (A)

(ii) Only (B)

(iii) Only (C)

(iv) (A) and (B)

(v) (A), (B), and (C)

(e) *(5 points)* Ryden gives the equation of motion for the inflaton field as

$$\ddot{\phi} + 3H(t)\,\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi} \ .$$

Ryden explains that the inflaton field normally reaches terminal velocity. Explain in a sentence or two what this means.

PROBLEM 2: A NEW THEORY OF THE WEAK INTERACTIONS *(40 points)*

This problem was Problem 11 of the Review Problems for Quiz 3 (2020).

Suppose a New Theory of the Weak Interactions (NTWI) was proposed, which differs from the standard theory in two ways. First, the NTWI predicts that the weak interactions are somewhat weaker than in the standard model. In addition, the theory implies the existence of new spin-$\frac{1}{2}$ particles (fermions) called the R^+ and R^-, with a rest energy of 50 MeV (where 1 MeV $= 10^6$ eV). This problem will deal with the cosmological consequences of such a theory.

The NTWI will predict that the neutrinos in the early universe will decouple at a higher temperature than in the standard model. Suppose that this decoupling takes place at $kT \approx 200$ MeV. This means that when the neutrinos cease to be thermally coupled to the rest of matter, the hot soup of particles would contain not only photons, neutrinos, and e^+-e^- pairs, but also μ^+, μ^-, π^+, π^-, and π^0 particles, along with the R^+-R^- pairs. (The muon is a particle which behaves almost identically to an electron, except that its rest energy is 106 MeV. The pions are the lightest of the mesons, with zero angular momentum and rest energies of 135 MeV and 140 MeV for the neutral and charged pions, respectively. The π^+ and π^- are antiparticles of each other, and the π^0 is its own antiparticle. Zero angular momentum implies a single spin state.) You may assume that the universe is flat.

(a) *(10 points)* According to the standard particle physics model, what is the mass density ρ of the universe when $kT \approx 200$ MeV? What is the value of ρ at this temperature, according to NTWI? Use either g/cm^3 or kg/m^3. (Calculators should NOT be used, but you can save time by not carrying out the arithmetic. If you do this, you should give the answer in "calculator-ready" form, by which I mean an expression involving pure numbers (no units), with any necessary conversion factors included, and with the units of the answer specified at the end. For example, if asked how far light travels in 5 minutes, you could answer $2.998 \times 10^8 \times 5 \times 60$ m.)

(b) *(10 points)* According to the standard model, the temperature today of the thermal neutrino background should be $(4/11)^{1/3} T_\gamma$, where T_γ is the temperature of the thermal photon background. What does the NTWI predict for the temperature of the thermal neutrino background?

— *Problem 2 continues on next page.* —

(c) *(10 points)* According to the standard model, what is the ratio today of the number density of thermal neutrinos to the number density of thermal photons? What is this ratio according to NTWI?

(d) *(10 points)* Since the reactions which interchange protons and neutrons involve neutrinos, these reactions "freeze out" at roughly the same time as the neutrinos decouple. At later times the only reaction which effectively converts neutrons to protons is the free decay of the neutron. Despite the fact that neutron decay is a weak interaction, we will assume that it occurs with the usual 886-second mean lifetime. Would the helium abundance predicted by the NTWI be higher or lower than the prediction of the standard model? To within 5 or 10%, what would the NTWI predict for the percent abundance (by weight) of helium in the universe? (As in part (a), you can leave the answer in calculator-ready form.)

Useful information: The proton and neutron rest energies are given by $m_p c^2 = 938.27$ MeV and $m_n c^2 = 939.57$ MeV, with $(m_n - m_p)c^2 = 1.29$ MeV. The mean lifetime for the neutron decay, $n \to p + e^- + \bar{\nu}_e$, is given by $\tau_n = 886$ s. In the standard model of cosmology, the deuterium bottleneck breaks at $t_{\rm DB} \approx 200$ s.

PROBLEM 3: A MESSAGE FROM A DISTANT GALAXY *(35 points)*

Our universe, at the present time, is well-described as a flat universe with a Hubble expansion rate H_0. Nonrelativistic matter comprises a fraction $\Omega_{m,0}$ of the critical density, radiation comprises a fraction $\Omega_{r,0}$, and vacuum energy comprises a fraction $\Omega_{v,0}$. For the following questions, you may leave your answers in terms of integrals that you do not evaluate, but you should be sure to specify the limits of integration. The answer to each part can be expressed in terms of the given variables, and/or the variables that represent the answers to any previous part.

(a) *(5 points)* Suppose that a galaxy G is observed at a redshift z_G. What was the cosmic time of emission t_e of the light that we are now receiving from G? ("Cosmic time" is the time variable of the Robertson-Walker metric.)

(b) *(10 points)* What is the present value of the physical distance $\ell_{p,0}$ to the galaxy G?

(c) *(10 points)* Suppose that a civilization in galaxy G sends us a light signal, originating at the present cosmic time. The light signal will reach us at a time called t_r. Write an equation that would determine the value of $a_r \equiv a(t_r)$, the scale factor at the time that we receive the signal. You will be happy to know that you are not expected to solve this equation. NOTE ADDED: besides the variables previously mentioned, your answer might also depend on $a(t_0)$.

(d) *(10 points)* At what cosmic time t_r would we receive the signal from the galaxy G?

Problem	**Maximum**	**Score**	**Initials**
1	25		
2	40		
3	35		
TOTAL	100		