Your Name

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 5, 2022
Prof. Alan Guth

QUIZ 1

Problem	Maximum	Score
1	20	
2	50	
3	30	
TOTAL	100	

PROBLEM 1: DID YOU DO THE READING? *(20 points)*

(a) *(5 points)* Why are there dark regions when we look toward the center of our galaxy using visible wavelength, or at another galaxy edge on?

(i) There are few stars in those areas.

(ii) The stars in those areas have a lower brightness than the stars in the halo.

(iii) Clouds of dust absorb the light of the stars located behind.

(iv) The gravitational field of the galaxy binds the path of photons away from the edge.

(b) *(5 points)* Using a radio antenna, Penzias and Wilson took about a year to determine that the signal they were receiving was truly of astronomical origin. What made it hard to establish that the signal they received was not just noise?

(i) It was hard to distinguish it from the electronic noise of the instruments.

(ii) It was hard to distinguish it from a microwave signal coming from stars or distant galaxies.

(iii) It was hard to distinguish it from radio communications on Earth.

(c) *(5 points)* The universe contains various types of particles; leptons, such as electrons and neutrinos, and baryons, such as neutrons and protons. Why do we refer to the matter of the universe solely as baryonic?

(i) There are much more baryons than leptons in the universe.

(ii) Baryons are heavier than leptons, so they dominate the mass density.

(iii) Leptons are a subcategory of baryons.

(iv) Leptons bind together to form baryons.

— Problem 1 continues on next page. —

(d) *(5 points)* State whether each of the following statements is true (T) or false (F):

(i) Before the 1840s it was a mystery why stars had visibly different colors, but in 1842 Johann Christian Doppler correctly explained this phenomenon as the consequence of the different velocities of stars. **T** or **F** .

(ii) Frauenhofer let light from the Sun pass through a slit and then through a glass prism. The resulting spectrum showed bright lines that could be identified with known elements. **T** or **F** .

(iii) In black-body radiation, the number of photons per unit volume is directly proportional to the cube of the temperature. **T** or **F** .

(iv) The model of the universe proposed by de Sitter in 1917 predicts a redshift proportional to the distance. **T** or **F** .

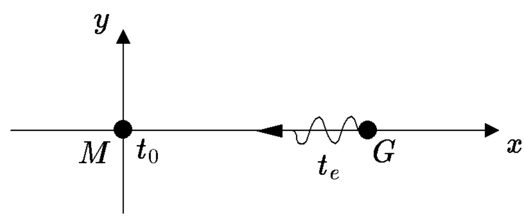
(v) The horizon distance bears its name because it is the edge of the universe; there are no stars or galaxies beyond it. **T** or **F** .

PROBLEM 2: INTERGALACTIC SIGNALING AND MARKSMANSHIP *(50 points)*

Consider a flat, matter-dominated universe, with a scale factor

$$a(t) = bt^{3/4} ,$$

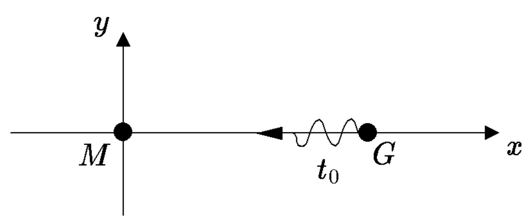
where b is a constant. Suppose that we live at cosmic time t_0 on galaxy M (maybe for Milky Way), located at the origin of a comoving coordinate system $(x, y, z) = (0, 0, 0)$. Galaxy G is located somewhere along the positive x axis, as shown in the diagram below:



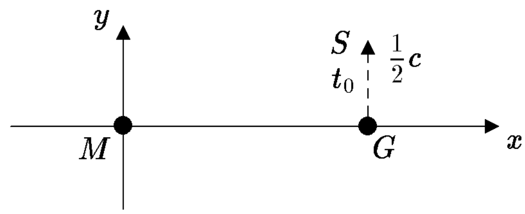
In all of the following questions, you can express your answer in terms of any or all of the given variables (i.e., any variable that has previously been assigned a symbol) and also any of the symbols that represent answers to the previous parts, whether or not you answered those parts.

(a) *(5 points)* What is the Hubble expansion rate H_0 that we would measure?

(b) *(7 points)* Suppose that we know that the light that we are receiving now (at time t_0) from G was emitted at time t_e. Given t_e and t_0, and the speed of light c, what is the physical distance $\ell_p(t_0)$ today between us and the galaxy G? What was the physical distance $\ell_p(t_e)$ at the time of emission t_e?



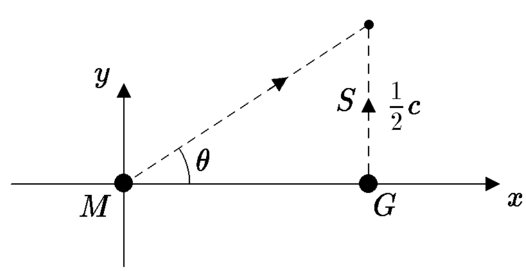
(c) *(8 points)* At the present cosmic time t_0, a light signal is emitted from G toward us, as shown in the diagram above. When we receive this light signal at some point in the future, what will be its redshift z_G?



(d) *(7 points)* Also at the present cosmic time t_0, a spaceship S is launched from the galaxy G in the y direction, as indicated in the diagram above. This is a science-fiction spaceship (just like one that you saw on the Quiz 1 Review Problems) which moves at speed $\frac{1}{2}c$ relative to the comoving observers. That is, it moves at speed $\frac{1}{2}c$ relative to the observers who are at rest in the comoving coordinate system. For simplicity, we will assume that the starship takes no time to reach its cruising velocity. Suppose, immediately after its launch, the spaceship emits a light signal directed toward us. When we eventually receive this signal (at M), what will be its redshift z_S? [*Hint:* it may help to imagine an observer at rest on G, who observes this light signal as it leaves the spaceship. You may assume that the redshift observed by this observer will be governed by special relativity. You can think of this observer as a relay station, since the signal will be unchanged if the observer absorbs it and retransmits it.]

(e) *(7 points)* Find the comoving coordinates $x_c(t)$ and $y_c(t)$ for the spaceship S at an arbitrary time $t > t_0$.

(f) *(7 points)* Find the physical distance $\ell_{p,S}(t)$ from S to M at an arbitrary time $t > t_0$.

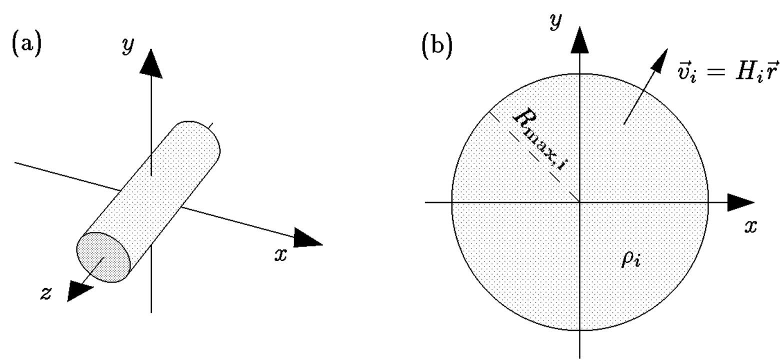


(g) *(9 points)* Now suppose that we, at the present time t_0, shoot a directed pulse of light at an angle θ with respect to the x axis, as shown in the diagram above. Our goal is to choose θ so that the light pulse intercepts the spaceship S. Write an equation that would determine the right value of θ to accomplish this interception. Your equation should involve only θ, given quantities, and answers to previous parts. You do *NOT* need to solve this equation (although you may find it easy to solve).

PROBLEM 3: A CYLINDRICAL UNIVERSE *(30 points)*

The following problem originated on Quiz 2 of 1994, where it counted 30 points. This year it was Problem 1 of Problem Set 3.

The lecture notes showed a construction of a Newtonian model of the universe that was based on a uniform, expanding, sphere of matter. In this problem we will construct a model of a cylindrical universe, one which is expanding in the x and y directions but which has no motion in the z direction. Instead of a sphere, we will describe an infinitely long cylinder of radius $R_{\rm max,i}$, with an axis coinciding with the z-axis of the coordinate system:



3D Perspective

2D Cross Section
(Different scale from (a))

We will use cylindrical coordinates, so

$$r = \sqrt{x^2 + y^2}$$

and

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \; ; \qquad \hat{r} = \frac{\vec{r}}{r} \; ,$$

where $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are the usual unit vectors along the x, y, and z axes. We will assume that at the initial time t_i, the initial density of the cylinder is ρ_i, and the initial velocity of a particle at position $\vec{r}$ is given by the Hubble relation

$$\vec{v}_i = H_i \vec{r} \; .$$

(a) *(5 points)* By using Gauss' law of gravity, it is possible to show that the gravitational acceleration at any point is given by

$$\vec{g} = -\frac{A\mu}{r}\hat{r} \; ,$$

where A is a constant and μ is the total mass per length contained within the radius r. Evaluate the constant A.

— *Problem 2 continues on next page.* —

(b) *(5 points)* As in the lecture notes, we let $r(r_i, t)$ denote the trajectory of a particle that starts at radius r_i at the initial time t_i. Find an expression for $\ddot{r}(r_i, t)$, expressing the result in terms of r, r_i, ρ_i, and any relevant constants. (Here an overdot denotes a time derivative.)

(c) *(7 points)* Defining

$$u(r_i, t) \equiv \frac{r(r_i, t)}{r_i} \ ,$$

show that $u(r_i, t)$ is in fact independent of r_i. This implies that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes. As before, we define the scale factor $a(t) \equiv u(r_i, t)$.

(d) *(5 points)* Express the mass density $\rho(t)$ in terms of the initial mass density ρ_i and the scale factor $a(t)$. Use this expression to obtain an expression for $\ddot{a}$ in terms of a, ρ, and any relevant constants.

(e) *(8 points)* Find an expression for a conserved quantity of the form

$$E = \frac{1}{2}\dot{a}^2 + V(a) \ .$$

What is $V(a)$? Will this universe expand forever, or will it collapse?