Your Name

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — December 7, 2022
Prof. Alan Guth

QUIZ 3

Please answer all questions in this stapled booklet.
Ask if you need extra pages, but we expect that you will not.
Closed book, no calculators, no internet.
Formula Sheet will be handed out separately.
We intend to scan the quizzes, so please write inside the margins,
and if you use a pencil, write darkly. Thanks!

Problem	**Maximum**	**Score**
1	20	
2	20	
3	30	
4	30	
TOTAL	100	

PROBLEM 1: DID YOU DO THE READING? *(20 points)*

(a) *(2 points)* At the time of recombination, the temperature is

(i) above the proton and electron masses

(ii) below the proton mass but above the electron mass

(iii) below the proton and electron masses

(b) *(2 points)* At the time of recombination, the universe is

(i) matter-dominated

(ii) radiation-dominated

(iii) vacuum-energy dominated

(c) *(2 points)* The reaction that produces the most deuterium in the early universe is

(i) proton-proton fusion ($p + p \to D+$ maybe other elements)

(ii) proton-neutron fusion ($p + n \to D+$ maybe other elements)

(iii) neutron-neutron fusion ($n + n \to D+$ maybe other elements)

(d) *(2 points)* For the microcanonical ensemble (describing an isolated system in thermal equilibrium), the probability that the system is in a state i with energy E_i is proportional to

(i) $\Omega_{\text{bath}}(E_{\text{tot}} - E_i)$, where E_{tot} is the energy of the heat bath and the small system and Ω_{bath} is the density of states of the bath

(ii) $\exp(-E_i/kT)$

(iii) $1/M$ if $E < E_i < E + \delta E$, where M is the total number of states in this energy range

(e) *(2 points)* In the early universe, two important reactions occurred: deuterium production and hydrogen production from proton-electron recombination. How do the binding energies of deuterium and hydrogen compare?

(i) the binding energy of deuterium is larger

(ii) the binding energies of deuterium and hydrogen are (almost) exactly the same

(iii) the binding energy of hydrogen is larger

— *Problem 1 continues on the next page.* —

(f) *(2 points)* The universe can be approximated as

(i) homogeneous and isotropic at all scales

(ii) homogeneous and isotropic on small scales ($\lesssim$ 100 Mpc)

(iii) homogeneous and isotropic on large scales ($\gtrsim$ 100 Mpc)

(iv) inhomogeneous and anisotropic at all scales

(g) *(2 points)* In order to analyze the ionization and recombination of hydrogen as a function of temperature in the early universe, we studied the ratio of the number density of hydrogen atoms n_H to the product of the number densities of the free elements that react to create them: protons, with number density n_p, and electrons, with number density n_e. For 2 points credit, you can EITHER state the name that is given to the equation that describes this ratio, or you can write the equation. (If you try to write the equation, be sure to notice the relevant formulas on the formula sheet.)

(h) *(2 points)* Gravity tends to make small density perturbations grow rapidly with time. What keeps small density perturbations from creating instabilities in the matter density and leading to collapse? (For example, what stops the air in this room from undergoing gravitational collapse.)

(i) *(2 points)* Once the photons are decoupled, the photons and baryons form two separate gases, instead of a single photon-baryon flui d. The speed of sound in the baryonic gas is [larger than / equal to / smaller than] the speed of sound in the photon gas.
Circle one

(j) *(2 points)* Baryon acoustic oscillations are due to the interaction of baryons with

(i) photons

(ii) dark matter

(iii) neutrinos

— *End of Problem 1.* —

PROBLEM 2: SHORT ANSWER QUESTIONS: GRAND UNIFIED THEORIES, MAGNETIC MONOPOLES, AND INFLATION *(20 points)*

(a) *(2 points)* A quark is specified by its flavor [u(p), d(own), c(harmed), s(trange), t(op), b(ottom)], its spin [up or down, along any chosen z axis], whether it is a quark or antiquark, and its color. **T** or **F** .

(b) *(2 points)* Any isolated system of quarks must be a color singlet. A single red quark is an example of a color singlet, since it has only one color. **T** or **F** .

(c) *(3 points)* Write the definition of the group SU(3). Assume that the reader does not know the meaning of "unitary" or "special", but that they do know the standard mathematical notation of matrix multiplication AB, matrix inversion A^{-1}, matrix transpose A^T, matrix adjoint $A^\dagger$, etc.

— *Problem 2 continues on the next page.* —

In electromagnetism, the electric and magnetic fields $\vec{E}$ and $\vec{B}$ can be written in terms of the scalar and vector potentials ϕ and $\vec{A}$ as

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \ , \quad \vec{B} = \vec{\nabla} \times \vec{A} \ .$$

For any function $\Lambda(\vec{x}, t)$, one can define a gauge transformation on ϕ and $\vec{A}$ by

$$\phi'(\vec{x}, t) = \phi(\vec{x}, t) - \frac{\partial \Lambda(\vec{x}, t)}{\partial t} \ , \quad \vec{A}'(\vec{x}, t) = \vec{A}(\vec{x}, t) + \vec{\nabla}\Lambda(\vec{x}, t) \ .$$

(d) *(2 points)* If $\vec{E}'(\vec{x}, t)$ and $\vec{B}'(\vec{x}, t)$ are the new electric and magnetic fields, calculated from $\phi'(\vec{x}, t)$ and $\vec{A}'(\vec{x}, t)$, how are they related to the original $\vec{E}(\vec{x}, t)$ and $\vec{B}(\vec{x}, t)$? [If you know the answer, you can write it without any derivation.]

$$\vec{E}'(\vec{x}, t) = \vec{E}(\vec{x}, t) \ + \ \underline{\hspace{3cm}}$$

$$\vec{B}'(\vec{x}, t) = \vec{B}(\vec{x}, t) \ + \ \underline{\hspace{3cm}}$$

(e) *(3 points)* Consider the line integral

$$W = \oint_P \vec{A}(\vec{x}, t) \cdot d\vec{x} \ ,$$

where P is a closed loop in (three-dimensional) space. If $\vec{A}(\vec{x}, t)$ is replaced by its gauge transform $\vec{A}'(\vec{x}, t)$, how is the resulting W' related to the original W? [Here you should show a short derivation.]

— *Problem 2 continues on the next page.* —

(f) *(2 points)* The SU(5) grand unified theory relies on the fact that the gauge group of the standard model, SU(3)×SU(2)×U(1), is a subgroup of SU(5). Is SU(3)×SU(2)×U(1) also a subgroup of SU(6)? **Yes** or **No** .

(g) *(2 points)* Inflation explains the homogeneity of the universe by modifying the nature of the initial singularity, so the universe is created in a homogeneous state. **T** or **F** .

(h) *(2 points)* Inflation proposes that the early universe went through a period of nearly exponential expansion driven by the repulsive gravity caused by a material with a negative pressure. **T** or **F** .

(i) *(2 points)* In inflationary models, the enormous expansion tends to smooth out any nonuniformities that may have been present before inflation. Nonetheless, there remain faint ripples in the cosmic microwave background, because there was not enough inflation to smooth the universe completely. **T** or **F** .

— End of Problem 2 —

PROBLEM 3: THE FREEZE-OUT OF A FICTITIOUS PARTICLE X *(30 points)*

The following problem was Problem 3 of Quiz 3, 2016, and Problem 10 of the Review Problems for Quiz 3, 2022.

Suppose that, in addition to the particles that are known to exist, there also existed a family of three spin-1 particles, X^+, X^-, and X^0, all with masses 0.511 MeV/c^2, exactly the same as the electron. The X^- is the antiparticle of the X^+, and the X^0 is its own antiparticle. Since the X's are spin-1 particles with nonzero mass, each particle has three spin states.

The X's do not interact with neutrinos any more strongly than the electrons and positrons do, so when the X's freeze out, all of their energy and entropy are given to the photons, just like the electron-positron pairs.

(a) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c^2, what is the total energy density of the X^+, X^-, and X^0 particles?

(b) *(5 points)* In thermal equilibrium when $kT \gg 0.511$ MeV/c^2, what is the total number density of the X^+, X^-, and X^0 particles?

(c) *(15 points)* The X particles and the electron-positron pairs freeze out of the thermal equilibrium radiation at the same time, as kT decreases from values that are large compared to 0.511 MeV/c^2 to values that are small compared to it. If the X's, electron-positron pairs, photons, and neutrinos were all in thermal equilibrium before this freeze-out, what will be the ratio T_ν/T_γ, the ratio of the neutrino temperature to the photon temperature, after the freeze-out?

(d) *(5 points)* If the mass of the X's was, for example, 0.100 MeV/c^2, so that the electron-positron pairs froze out first, and then the X's froze out, would the final ratio T_ν/T_γ be higher, lower, or the same as the answer to part (c)? Explain your answer in a sentence or two.

— *End of Problem 3* —

PROBLEM 4: SIZE OF THE HORIZON ON THE SURFACE OF LAST SCATTERING *(30 points)*

In lecture we discussed an approximate calculation of the horizon problem as it relates to the cosmic microwave background, crudely estimating that the radius of the surface of last scattering was about 23 times larger than the horizon distance at the time of last scattering (which is also called the time of decoupling). Here you are asked to write the equations that determine this number accurately, taking into account all the known contributions to the energy density of the universe.

You should assume that the universe is flat, and that you are given the present values of the contributions to Ω of the known components: $\Omega_{\text{vac},0}$ for vacuum energy, $\Omega_{m,0}$ for nonrelativistic matter (dark matter and baryons), and $\Omega_{\text{rad},0}$ for radiation, with

$$\Omega_{\text{vac},0} + \Omega_{m,0} + \Omega_{\text{rad},0} = 1 \ . \tag{P4.1}$$

You are also given the present value H_0 of the Hubble expansion rate, the redshift z_{ls} of the surface of last scattering, and of course the speed of light c. Following Ryden's conventions, you should take $a(t_0) \equiv 1$.

(a) *(15 points)* What is the coordinate radius $\ell_{\text{ls},c}$ of the surface of last scattering? Your answer should take the form of a definite integral, with explicit limits of integration. (Note that the function $a(t)$ is not given, so it should not appear in your final answer.)

(b) *(10 points)* What is the coordinate size $h_{\text{ls},c}$ of the horizon distance at the time of last scattering? Your answer should again take the form of an integral, as in part (a).

(c) *(5 points)* If two points on the surface of last scattering were separated from each other by a horizon distance at the time of last scattering, what will be the angular separation between these two points as seen on Earth today? You may use small angle approximations. Your answer can include any of the given variables, and also the answers $\ell_{\text{ls},c}$ and $h_{\text{ls},c}$ to the previous two parts, whether or not you answered those parts.