MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe May 18, 1994
Prof. Alan Guth

QUIZ 3 SOLUTIONS

PROBLEM 1: SHORT ANSWERS

a) This subject is discussed in **The Big Bang**, by Joseph Silk, on p. 70. The answer is 3.9 billion years.

b) This subject is also discussed in the book by Joseph Silk, on p. 81. The answer is (C): there appear to be more radio galaxies far away than nearby. It is believed that this is an evolutionary effect. When we look far away we are seeing into the past, since light travels at a finite velocity. Apparently radio galaxies were more intense in the past, so we see a larger fraction of them.

c) This subject is also discussed in Joseph Silk's book, on pp. 126–27. Statement (A) is true. A cosmic string can cause a double image, with one image visible on each side of the string. If the string crosses the line of sight of several distant galaxies, then these double images will form a line. Statement (B) is also true: a moving cosmic string actually causes a temperature differential between the radiation passing on either side of the string. Statement (C) is false: the galaxies that would be formed by cosmic string perturbations would look like normal galaxies, so it is conceivable that all the galaxies that we observe were in fact formed by cosmic strings. Statement (D) is true: cosmic strings actually dissipate most of their energy in gravitational radiation. Today these waves would distort the space through which the pulsar's radio waves are traveling, resulting in a nonuniformity in the time intervals between signals.

d) This issue is discussed in *"The Cosmic Asymmetry Between Matter and Antimatter"*, by Frank Wilczek (**Scientific American**, December 1980). Statement (A) is false: polarization can provide evidence of magnetic fields, but it does not distinguish between matter and antimatter. Statement (B) is also false: a photon is its own antiparticle, so there is no such thing as an antiphoton. Statement (C) is true. It may not sound like a completely compelling reason, but it is the best that we have. Statement (D) is false. In principle, an anti-star can be distinguished from a star by the fact that it

emits mainly antineutrinos rather than neutrinos. Unfortunately, however, we can detect neither neutrinos nor antineutrinos from any star other than our sun.

e) This issue is also discussed in the **Scientific American** article by Frank Wilczek. Statement (A) is true. If C (charge conjugation symmetry) were not violated, then the symmetry would guarantee equal probabilities for the production of matter and antimatter. While the randomness of the production process would lead to a random excess of one type over the other, the laws of statistics imply that this imbalance would be far smaller than the observed excess of matter over antimatter. Statement (B) is false: parity symmetry relates a proton with spin along its momentum to a proton with spin opposite its momentum, but it says nothing about the relation between protons and antiprotons. Statement (C) is true. If CP (charge conjugation times parity symmetry) were not also violated, then this symmetry would guarantee equal probabilities for the production of matter and antimatter. That is, for every right-handed proton that is produced, there would be an equal probability that a left-handed antiproton was produced. (Here "right-handed" and "left-handed" refer to the spin of the proton, which is either along the momentum or opposite it. Many accounts place more emphasis on CP violation then on C violation, but they are both essential. CP tends to be given more press coverage, apparently for historical reasons. C violation was known much earlier, so for many years CP symmetry seemed to be the obstacle to baryogenesis. So when CP violation was finally discovered, it was natural to think of it as the key to baryogenesis.) Statement (D) is true: if baryon number were exactly conserved, then matter and antimatter would always have to be produced in equal amounts, so the excess of matter could not be explained. Statement (E) is false: charge is believed to be exactly conserved, but this poses no obstacle to baryogenesis.

f) 10^{16} GeV, as discussed in Lecture Notes 11.

g) In the electroweak theory, the electron is fundamentally the same as the neutrino. In grand unified theories, the electron is fundamentally the same as both the neutrino and the quark. (Many students mentioned the symmetry between the electron, the muon, and the tau, which is a part of the symmetry between generations of fermions. This symmetry remains a mystery, not explained by either the electroweak theory or grand unified theories.)

h) Choice (A): the Higgs fields are scalars. This is essential for the consistency of the theory, since the Higgs fields have a nonzero value in the vacuum. If they were not scalars, then this nonzero value would be measured differently by observers who were rotated with respect to each other, so the rotational invariance of the vacuum would be violated. (The electron, by the way, is a spin-$\frac{1}{2}$ particle, the photon is an example of a spin 1 particle, and the graviton has spin 2.)

PROBLEM 2: A REVISED THERMAL HISTORY OF THE UNIVERSE

(a) In the standard model, the black-body radiation at $kT \approx 200$ MeV contains the following contributions:

$$\begin{array}{ll} \text{Photons:} & g = 2 \\ e^+e^-: & g = 4 \times \frac{7}{8} = 3\frac{1}{2} \\ \nu_e, \nu_\mu, \nu_\tau: & g = 6 \times \frac{7}{8} = 5\frac{1}{4} \\ \mu^+\mu^-: & g = 4 \times \frac{7}{8} = 3\frac{1}{2} \\ \pi^+\pi^-\pi^0 & g = 3 \end{array}$$

$$g_{\rm TOT} = 17\frac{1}{4}$$

$$\rho = \frac{u}{c^2} = g_{\rm TOT}\frac{\pi^2}{30}\frac{(kT)^4}{\hbar^3 c^5}$$

$$= \left(17\frac{1}{4}\right)\frac{\pi^2}{30}\frac{\left[200 \times 10^6\,\text{eV} \times \frac{1.602\times 10^{-12}\,\text{erg}}{\text{eV}}\right]^4}{\left(1.055 \times 10^{-27}\,\text{erg-sec}\right)^3\left(2.998 \times 10^{10}\,\text{cm/sec}\right)^5}$$

Units:

$$\frac{\text{erg}^4}{\text{erg}^3\text{-sec}^3\text{-cm}^5\text{-sec}^{-5}} = \frac{\text{erg-sec}^2}{\text{cm}^5}$$

$$= \frac{\left(\text{gm-cm}^2\text{-sec}^{-2}\right)\text{sec}^2}{\text{cm}^5} = \text{gm/cm}^3$$

$$\rho = \left(17\frac{1}{4}\right)\frac{\pi^2}{30}\frac{\left[200 \times 10^6 \times 1.602 \times 10^{-12}\right]^4}{\left(1.055 \times 10^{-27}\right)^3\left(2.998 \times 10^{10}\right)^5}\frac{\text{gm}}{\text{cm}^3}$$

$$= \boxed{2.10 \times 10^{15}\ \text{gm/cm}^3\ .}$$

Note: In the past many students have left out the conversion factor 1.602×10^{-12} erg/eV, and instead used $\hbar = 6.582 \times 10^{-16}$ eV-sec. Few of these students worked out the units of the answer. It turns out to be eV-sec^2/cm^5, which is a most peculiar set of units to measure a mass density.

In the NTWI, the answer is the same. The neutrinos are just ceasing to interact with the rest of matter, but this does *not* cause them to disappear.

(b) As long as the universe is in thermal equilibrium, entropy is conserved. The entropy in a given volume of the comoving coordinate system is

$$R^3(t)\,s\,V_{\rm coord}\quad ,$$

where s is the entropy density and $R^3 V_{\rm coord}$ is the physical volume. So

$$R^3(t)\,s$$

is conserved. After the neutrinos decouple,

$$R^3 s_\nu \quad \text{and} \quad R^3 s_{\rm other}$$

are separately conserved ($s_{\rm other}$ = entropy of everything except ν).

Note that

$$s = gAT^3\quad ,$$

where A is a constant. Before the disappearance of the e, μ, and π particles from the thermal equilibrium radiation,

$$s_\nu = \left(5\frac{1}{4}\right)AT^3$$

$$s_{\rm other} = 12AT^3\quad .$$

So

$$\frac{s_\nu}{s_{\text{other}}} = \frac{5\frac{1}{4}}{12} \quad .$$

If $R^3 s_\nu$ and $R^3 s_{\text{other}}$ are conserved, then so is s_ν / s_{other}. By today, the entropy previously shared amoung the various particles still in equilibrium after neutrino decoupling has been transfered to the photons so that

$$s_{\text{other}} = s_{\text{photons}} = 2AT_\gamma^3 \quad .$$

The entropy in neutrinos, being separately conserved, is still

$$s_\nu = \left(5\frac{1}{4}\right) AT_\nu^3 \quad .$$

Since s_ν / s_{other} is constant we know that

$$\frac{\left(5\frac{1}{4}\right) T_\nu^3}{2T_\gamma^2} = \frac{s_\nu}{s_{\text{other}}} = \frac{\left(5\frac{1}{4}\right)}{12}$$

$$\Longrightarrow \quad \boxed{T_\nu = \left(\frac{1}{6}\right)^{1/3} T_\gamma} \quad .$$

(c) One can write

$$n = g^* BT^3 \quad ,$$

where B is a constant. Here $g_\gamma^* = 2$, and $g_\nu^* = 6 \times \frac{3}{4} = 4\frac{1}{4}$. In the standard model, one has today

$$\frac{n_\nu}{n_\gamma} = \frac{g_\nu^* T_\nu^3}{g_\gamma^* T_\gamma^3} = \frac{\left(4\frac{1}{2}\right)}{2} \frac{4}{11} = \boxed{\frac{9}{11}\ .}$$

In the NTWI,

$$\frac{n_\nu}{n_\gamma} = \frac{\left(4\frac{1}{2}\right)}{2} \frac{1}{6} = \boxed{\frac{3}{8}\ .}$$

(d) At $kT = 200$ MeV, the thermal equilibrium ratio of neutrons to protons is given by

$$\frac{n_{\text{n}}}{n_{\text{p}}} = e^{-1.29\,\text{MeV}/200\,\text{MeV}} \approx 1 \quad .$$

In the standard theory this ratio would decrease rapidly as kT fell below the p-n mass difference of 1.29 MeV, but in the NTWI the neutrons can disappear only through the slow process of free neutron decay. Thus, the NTWI predicts a higher neutron abundance than the standard model. At about $3\frac{3}{4}$ minutes all of the neutrons will become bound into He, exactly as in the standard model — this is a strong interaction, and will happen in the same way in either theory. The higher neutron abundance of the NTWI implies a

$$\boxed{\text{higher predicted He abundance.}}$$

To estimate the He abundance, note that in the absence of neutron decay, the neutron-proton ratio would be frozen at about one, and at $3\frac{3}{4}$ minutes essentially *all* the nucleons in the universe would collect into He. However, the neutron abundance is reduced by free decay by a factor

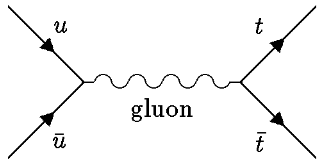
$$e^{-3\frac{3}{4}\,\text{mins}/15\,\text{mins}} \approx 0.78 \quad ,$$

so the He production is reduced by this factor. Thus,

$$\boxed{\text{Predicted He abundance by weight} \approx 80\%\ .}$$

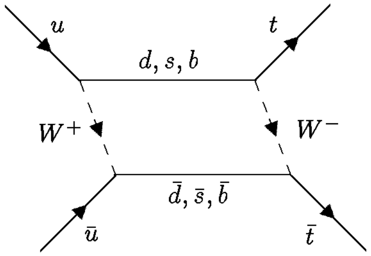
PROBLEM 3: THE TOP QUARK

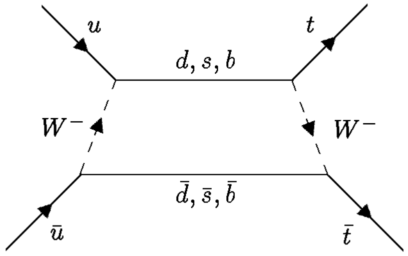
a) The primary mechanism for $u\,\bar{u} \longrightarrow t\,\bar{t}$ is illustrated by the diagram:



Since the process involves only gluons as intermediate particles, it is a strong interaction.

A number of students drew weak interaction diagrams, such as





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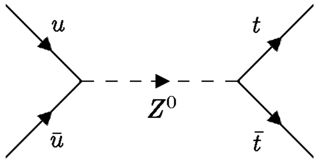
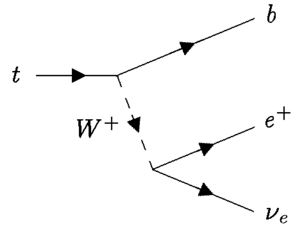


Here the label "d, s, b" indicates a single particle which might be a d, s, ***OR*** b; more explicitly, I could have drawn three diagrams and labeled one with a d, one with an s, and one with a b. These diagrams are valid, but the presence of the intermediate vector bosons W^+, W^-, and Z^0 implies that they are weak interactions. These processes, therefore, are much rarer than the dominant strong interaction that was sketched above. (More precisely, in the language of quantum theory, these processes make only a negligible contribution to the probability amplitude for the process $u\,\bar u \longrightarrow t\,\bar t$.) I gave partial credit for diagrams of this sort, but only the strong interaction diagram received full credit.

b) The proton contains u and d quarks, while the antiproton contains only $\bar u$ and $\bar d$ quarks. Thus, the combination $d\,\bar d$ can produce $t\,\bar t$ pairs in the same way that $u\,\bar u$ pairs do. (Some students gave $c\,\bar c$ as an answer to this

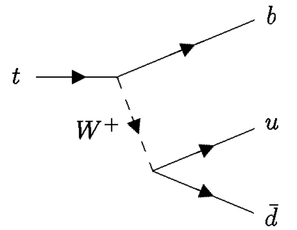
question. It is true that $c\,\bar c$ can produce a $t\,\bar t$ pair by an analogous process, but the c and $\bar c$ quarks are not contained in the protons or antiprotons.)

c) The diagram for $t \longrightarrow b\,e^+\,\nu_e$ is:

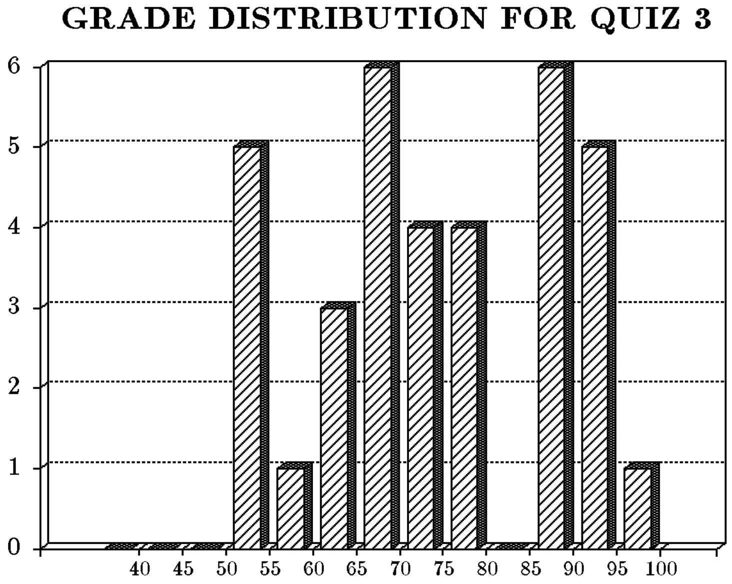


It is a weak interaction, due to the presence of the W^+ in the intermediate state. Note that the emission or absorption of a gluon never changes the identity of a quark, so there is no way that this reaction can happen by gluons alone.

d) The diagram for $t \longrightarrow b\,u\,\bar d$ is given by:



As in (c), this is a weak interaction process.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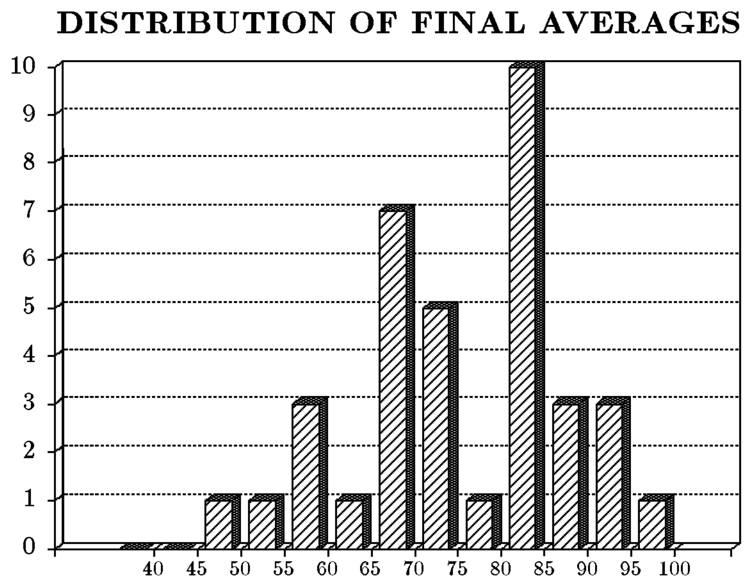
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