MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — May 4, 1994

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QUIZ 3

USEFUL INFORMATION:

GEODESIC EQUATION:

$$\frac{d}{d\lambda}\left\{g_{ij}\frac{dx^j}{d\lambda}\right\} = \frac{1}{2}\left(\partial_i g_{k\ell}\right)\frac{dx^k}{d\lambda}\,\frac{dx^\ell}{d\lambda}$$

or:
$$\frac{d}{d\tau}\left\{g_{\mu\nu}\frac{dx^\nu}{d\tau}\right\} = \frac{1}{2}\left(\partial_\mu g_{\lambda\sigma}\right)\frac{dx^\lambda}{d\tau}\frac{dx^\sigma}{d\tau}$$

PHYSICAL CONSTANTS:

$$k = \text{Boltzmann's constant} = 1.381 \times 10^{-16}\,\text{erg}/{}^\circ\text{K}$$

$$= 8.617 \times 10^{-5}\,\text{eV}/{}^\circ\text{K}\ ,$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-27}\ \text{erg-sec}$$

$$= 6.582 \times 10^{-16}\ \text{eV-sec}\ ,$$

$$c = 2.998 \times 10^{10}\ \text{cm/sec}$$

$$1\ \text{eV} = 1.602 \times 10^{-12}\ \text{erg}\ .$$

COSMOLOGICAL EVOLUTION:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)R$$

EVOLUTION OF A FLAT ($\Omega \equiv \rho/\rho_c = 1$) UNIVERSE:

$$R(t) \propto t^{2/3} \qquad \text{(matter-dominated)}$$

$$R(t) \propto t^{1/2} \qquad \text{(radiation-dominated)}$$

COSMOLOGICAL REDSHIFT:

$$1+Z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

ROBERTSON-WALKER METRIC:

$$ds^2 = R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right\}$$

SCHWARZSCHILD METRIC:

$$ds^2 = -c^2 d\tau^2 = -\left(1-\frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1-\frac{2GM}{rc^2}\right)^{-1} dr^2$$
$$+\, r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 \ ,$$

BLACK-BODY RADIATION:

$$u = g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3}$$

$$n = g^*\frac{\zeta(3)}{\pi^2}\frac{(kT)^3}{(\hbar c)^3}$$

$$s = g\frac{2\pi^2}{45}\frac{k^4T^3}{(\hbar c)^3} \ ,$$

where

$$g \equiv \begin{cases} 1 \text{ per spin state for bosons (integer spin)} \\ 7/8 \text{ per spin state for fermions (half-integer spin)} \end{cases}$$

$$g^* \equiv \begin{cases} 1 \text{ per spin state for bosons} \\ 3/4 \text{ per spin state for fermions} \ , \end{cases}$$

and

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \ .$$

PARTICLE PROPERTIES:

While working on this exam you may refer to any of the tables in Lecture Notes 10.

PROBLEM 1: SHORT ANSWERS *(40 points)*

The following questions are each worth 5 points:

(a) The oldest rocks found on earth have been dated by radioactive elements, principally the decay of U^{238} to Pb^{206}. The age is estimated to be 2.1 billion years, 3.9 billion years, 6.3 billion years, or 9.7 billion years?

(b) When astronomers try to measure the distribution of radio galaxies in space, they find (A) that they appear to be uniformly distributed in space, (B) that there appear to be more nearby than far away, or (C) that there appear to be more far away than nearby?

(c) Which of the following hypothetical observations would be considered evidence for the existence of cosmic strings? Indicate as many as apply.

  (A) Double images of galaxies separated by a few arcseconds, especially if a line of such double images were observed.

  (B) A linelike discontinuity in the temperature of the cosmic background radiation.

  (C) Elongated galaxies with about 10^3 times more mass than ordinary galaxies.

  (D) A background of gravitational waves, detectable through jitter in the time intervals of pulsar signals.

(d) Most cosmologists believe that the entire visible universe, like the planet Earth, is composed of matter rather than antimatter. Which of the following are valid reasons for this belief? List as many as apply.

  (A) The polarization of the photons from distant galaxies confirms that they are composed of matter and not antimatter.

  (B) The fact that we receive photons and not antiphotons from distant galaxies implies that they are composed of matter and not antimatter.

  (C) The fact that gamma rays associated with matter-antimatter annihilation are not detected suggests that nearby clusters of galaxies must be entirely matter or else entirely antimatter. If both matter and antimatter have significant abundances in the visible universe, it is difficult to imagine how they became segregated into distinct regions.

  (D) The fact that we receive neutrinos and not antineutrinos from stars in distant galaxies implies that they are composed of matter rather than antimatter.

(e) If the matter/antimatter asymmetry of the observed universe evolved from an initial state of matter/antimatter symmetry, then the underlying particle physics must possess which of the following properties? List as many as apply. For each property that you list, give a one or two sentence explanation of why the property is needed for the matter/antimatter asymmetry to arise.

(A) The symmetry C, for charge conjugation, must be violated.

(B) The symmetry P, for parity or reflection symmetry, must be violated.

(C) The symmetry CP, for the combination of charge conjugation and parity, must be violated.

(D) The conservation of baryon number must be violated.

(E) The conservation of charge must be violated.

(f) Is the mass of a grand unified theory monopole expected to be about 1 GeV, 10^{10} GeV, 10^{16} GeV, or 10^{25} GeV?

(g) According to the unified electroweak theory, at the fundamental level an electron is identical to what other type of elementary particle? According to grand unified theories, it is identical to what two other types of particles?

(h) The Higgs fields that are introduced into the electroweak theory or grand unified theories to spontaneously break the internal symmetries are always (A) scalar fields, corresponding to spin 0 particles, (B) spinor fields, corresponding to spin $\frac{1}{2}$ particles, (C) vector fields, corresponding to spin 1 particles, or (D) tensor fields, corresponding to spin 2 particles?

PROBLEM 2: A REVISED THERMAL HISTORY OF THE UNIVERSE *(30 points)*

As promised there is one question verbatim from the homework or review problems— specifically, from Problem Set 6. Part (a) of this question calls for a numerical answer, but since you were not told to bring calculators, you need not carry out the arithmetic. Your answer should be expressed, however, in "calculator-ready" form— that is, it should be an expression involving pure numbers only (no units), with any necessary conversion factors included, and with the units of the final answer specified at the end. (For example, if you were asked how far a light pulse in vacuum travels in 5 minutes, you could express the answer as $2.998 \times 10^8 \times 5 \times 60$ *meters.) Similarly, in part (d) you may express your answer in terms of a trigonometric or exponential function, which you need not evaluate.*

Suppose a New Theory of the Weak Interactions (NTWI) was proposed, in which the weak interactions are somewhat weaker than in the standard model. This problem will deal with the cosmological consequences of such a theory.

The NTWI will predict that the neutrinos in the early universe will decouple at a higher temperature than in the standard model. Suppose that this decoupling takes place at $kT \approx 200$ MeV. This means that when the neutrinos cease to be thermally coupled to the rest of matter, the hot soup of particles would contain not only photons, neutrinos, and e^+-e^- pairs, but also μ^+, μ^-, π^+, π^-, and π^0 particles. (The muon, you will recall, is a particle that behaves almost identically to an electron, except that its rest energy is 106 MeV. The pions are the lightest of the mesons, with zero angular momentum and rest energies of 135 MeV and 140 MeV for the neutral and charged pions, respectively.)

(a) *(8 points)* According to the standard particle physics model, what is the mass density ρ of the universe when $kT \approx 200$ MeV? What is the value of ρ at this temperature, according to NTWI?

(b) *(8 points)* According to the standard model, the temperature today of the thermal neutrino background should be $(4/11)^{1/3} T_\gamma$, where $T_\gamma \approx 2.7°$ K is the temperature of the thermal photon background. What does the NTWI predict for the temperature of the thermal neutrino background?

(c) *(5 points)* According to the standard model, what is the ratio today of the number density of thermal neutrinos to the number density of thermal photons? What is this ratio according to NTWI?

(d) *(9 points)* Since the reactions that interchange protons and neutrons involve neutrinos, these reactions "freeze out" at roughly the same time as the neutrinos decouple. At later times the only reaction which effectively converts neutrons to protons is the free decay of the neutron. Despite the fact that neutron decay is a weak interaction, we will assume that it occurs with the usual 15 minute mean lifetime. Would the helium abundance predicted by the NTWI be higher or lower than the prediction of the standard model? To within 5 or 10%, what would the NTWI predict for the percent abundance (by weight) of helium in the universe?

PROBLEM 3: THE TOP QUARK *(30 points)*

On Tuesday, April 26, 1994, the CDF (Collider Detector at Fermilab) Collaboration, consisting of 440 physicists from 34 institutions (including MIT), released a paper titled "Evidence for Top Quark Production in $\bar{p}p$ Collisions at $\sqrt{s} = 1.8$ TeV." $\bar{p}p$ refers to the colliding beams of antiprotons and protons in the Fermilab accelerator, and $\sqrt{s}$ refers to the total energy: 1.8 tera-electron volts, or 1.8×10^{12} eV. They found a total of 12 events that looked like top quark production, but they estimate that 6 of these events were most likely due to "background"— that is, events that did not involve a top quark, but which looked similar. Nonetheless, the group estimates that there is only 1 chance in 400 that all 12 events could be attributed to background.

(a) *(8 points)* One method by which top quarks are believed to have been produced in this experiment is the reaction

$$u\,\bar{u} \longrightarrow t\,\bar{t}\ ,$$

where the u and $\bar{u}$ quarks are part of the initial proton and antiproton, respectively. (Note that $\bar{u}$ denotes the antiparticle of the u quark.) Draw a quark diagram for this process. Is it a strong, electromagnetic, weak, or gravitational process?

(b) *(7 points)* In the same $p\bar{p}$ collisions, $t\bar{t}$ pairs can also be produced from a different type of quark-antiquark pair. What type is that?

(c) *(8 points)* One of the principal decay modes for the t quark is

$$t \longrightarrow b\,e^{+}\,\nu_e\ .$$

Draw a quark diagram for this decay. Is it a strong, electromagnetic, weak, or gravitational process?

(d) *(7 points)* Another principal decay mode for the t quark is

$$t \longrightarrow b\,u\,\bar{d}\ .$$

Draw a quark diagram for this decay. Is it a strong, electromagnetic, weak, or gravitational process?

Have a good summer