MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — May 28, 1996
Prof. Alan Guth

QUIZ 4 SOLUTIONS

PROBLEM 1: DID YOU DO THE READING?

(a) According to general relativity, the only properties of a black hole that can be measured by an outside observer are its mass, its angular momentum, and its electric charge. Thus, if a baryon falls into a black hole, all evidence of its baryon number is lost, and therefore its baryon number has not been conserved. While a completely successful quantum theory of general relativity does not yet exist, one expects that a quantum description of "virtual black holes" on subatomic length scales would lead to processes such as the decay of the proton. *Grading comment: the mention of virtual black holes was not required for full credit.*

(b) A redshift survey is the tabulation of the redshifts of galaxies in some region of the sky. Assuming the validity of Hubble's law, one can use the redshifts as estimates of distances, which can be combined with the measured angular positions on the sky to construct a three-dimensional map of the galaxies. A void is a volume of space with few or no visible galaxies. Voids are very common, and frequently have diameters of about 300 million light-years.

PROBLEM 2: BIG BANG NUCLEOSYNTHESIS

(a) It would go up. With a higher density of baryons, deuterium would become stable at a higher temperature. Nucleosynthesis would then happen earlier, so fewer neutrons would have time to decay. Since essentially all the available neutrons are bound into helium at the time of nucleosynthesis, this means that more helium would be produced.

(b) It would go down. Deuterium is produced in the big bang as an intermediate substance which has not completed its reactions (i.e., deuterium is the "carbon monoxide" of the early universe). If there were a higher density of baryons, the reactions would be more efficient, and there would be fewer incomplete reactions.

(c) It would go up. Fewer neutrons would decay, and thus a larger number would be available for nucleosynthesis.

(d) It would go up. An increased binding energy for deuterium would make it stable at a higher temperature. The "deuterium bottleneck" would therefore break sooner, and nucleosynthesis would take place earlier. Fewer neutrons would decay before nucleosynthesis, so more would be available.

(e) It would go up. Real electrons and positrons contribute $7/2$ to g, since there are two spin states each for electrons and positrons, for a total of four spin states, and a factor of $7/8$ because spin-$\frac{1}{2}$ particles are fermions. If electrons and positrons had spin 1, their contribution to g would go up to 6, since there would be three spin states for each particle, and no factor of $7/8$ associated with fermions. A larger g, as with an extra species of neutrinos, means that the universe would cool faster. The temperature at which the deuterium bottleneck breaks would be reached faster, so fewer neutrons would have time to decay, and more would be available for nucleosynthesis.

A Subtlety: One student pointed out that if electrons and positrons had spin 1, then the proton-neutron conversion reactions, such as

$$e^- + p \longleftrightarrow \nu_e + n \ ,$$

or

$$e^+ + n \longleftrightarrow \overline{\nu_e} + p \ ,$$

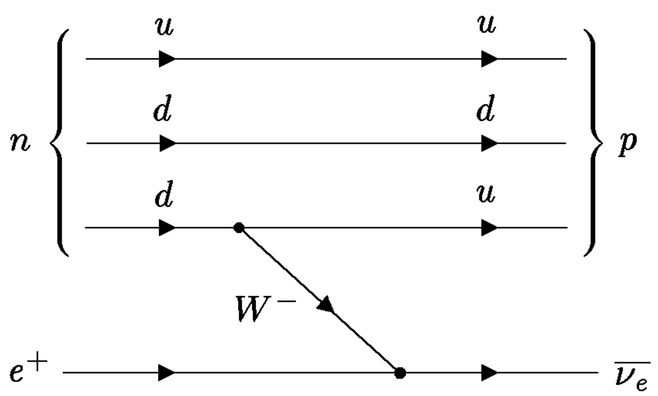
would not be possible. The combination of the two particles on the left would have a half-integral angular momentum, while the particles on the right would have an integral total angular momentum. This is not the issue that I intended to ask about, so if I ever ask this question again I will change the spin of the neutrinos so that the reactions above remain possible. If the above reactions were prevented, then the neutron-proton ratio would freeze out at a much higher temperature, resulting in more neutrons. So this effect, like the effect discussed in the previous paragraph, would lead to more helium production.

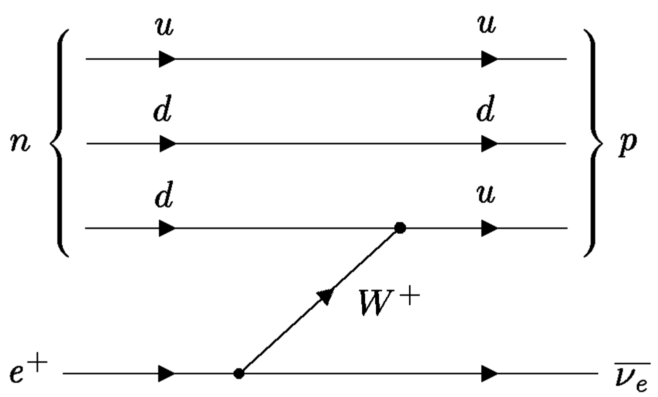
PROBLEM 3: QUARK DIAGRAM FOR $e^+ + n \rightarrow p +$ NEUTRINO

(a) The particle must be an antineutrino, since conservation of lepton number requires the X particle to have the same lepton number as the positron. According to the standard conventions, electrons and neutrinos have lepton number $+1$, while positrons and antineutrinos have lepton number -1. (Some people thought that spin was the relevant conserved quantity. Remember, however, that neutrinos and antineutrinos have the same magnitude of spin. The spin of an antineutrino is opposite in direction to that of a neutrino moving in the same direction, but since the spin of the initial particles could point in any direction, the direction of the neutrino spin cannot restrict the possible reactions.)

(b) The X particle must be an electron antineutrino, since electron number, muon number, and tau number are each separately conserved. (The sum of these three numbers is called lepton number.) The positron has electron number -1, and the electron antineutrino is the only type of neutrino which has the same electron number. (Note, however, that electron number, muon number, tau number, and lepton number are conserved only in the same sense that baryon number is — experimentally these conservation laws are never seen to be violated, but for theoretical reasons the conservation laws are not believed to be exact.)

(c) There are two choices for the simplest diagram of this reaction:





There are other more complicated diagrams that are also possible, and which were also given full credit.

PROBLEM 4: THE FLATNESS PROBLEM IN A UNIVERSE WITH $p = \frac{1}{2}u$

(a) The necessary ingredients are the evolution equation

$$H^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

and the relation for the critical density

$$H^2 = \frac{8\pi}{3} G\rho_c \ .$$

Combining the two equations gives

$$\frac{8\pi}{3} G\rho_c = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2} \ ,$$

or

$$\rho_c = \rho - \frac{3kc^2}{8\pi G R^2} \ .$$

Then

$$\frac{\rho - \rho_c}{\rho} = \frac{3kc^2}{8\pi G R^2 \rho} \ ,$$

and

$$\frac{\rho - \rho_c}{\rho} = \frac{\rho/\rho_c - 1}{\rho/\rho_c} = \frac{\Omega - 1}{\Omega} \ .$$

Finally,

$$\boxed{\frac{\Omega - 1}{\Omega} = \frac{3kc^2}{8\pi G R^2 \rho} \ .}$$

(b) Using the fact that $R(t) \propto t^{4/9}$, it follows that

$$H(t) = \frac{\dot{R}}{R} = \frac{4}{9\,t} \ .$$

For a nearly flat universe

$$H^2 \approx \frac{8\pi}{3} G\rho \ ,$$

from which it follows that in this case

$$\rho \propto \frac{1}{t^2} \ .$$

Then, using the answer to the previous part,

$$\begin{aligned} \frac{\Omega - 1}{\Omega} &\propto \frac{\text{const}}{R^2 \rho} \\ &\propto \frac{\text{const}}{(t^{4/9})^2 (1/t^2)} \\ &\propto t^{10/9} \ , \end{aligned}$$

so

$$\boxed{\gamma = \frac{10}{9} \ .}$$