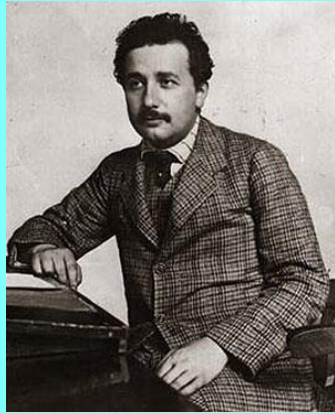
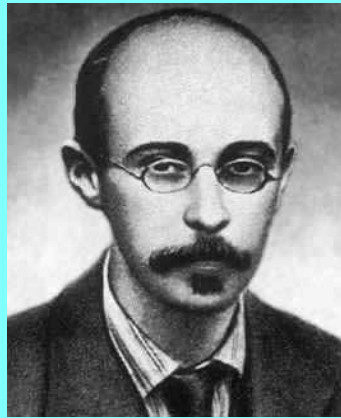


## Albert Einstein and the Friedmann Equations



Albert Einstein



Alexander A. Friedmann

## Publication of the Friedmann Equations

### On the Curvature of Space

A. Friedmann  
Petersburg  
Received June 29, 1922  
*Zeitschrift für Physik*



## Einstein's Reaction

REMARK ON THE WORK OF A. FRIEDMANN (FRIEDMANN 1922)  
“ON THE CURVATURE OF SPACE”

A. Einstein, Berlin

Received September 18, 1922

*Zeitschrift für Physik*

The work cited contains a result concerning a non-stationary world which seems suspect to me. Indeed, those solutions do not appear compatible with the field equations (A). From the field equations it follows necessarily that the divergence of the matter tensor  $T_{ik}$  vanishes. This along with the anzatzes (C) and (D) leads to the condition

$$\partial\rho/\partial x_4 = 0$$

which together with (8) implies that the world-radius R is constant in time. The significance of the work therefore is to demonstrate this constancy.

REFERENCES: Friedmann, A. 1922, *Zs. f. Physik.*, 10, 377.

Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

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## Sequence of Events

June 29, 1922: Friedmann's paper received at *Zeitschrift für Physik*.

September 18, 1922: Einstein's refutation received at *Zeitschrift für Physik*.

December 6, 1922: Friedmann learns about Einstein's objection from his friend, Yuri A. Krutkov, who is visiting in Berlin. Friedmann writes a detailed letter to Einstein. Einstein is traveling and does not read it.

May, 1923: Einstein meets Krutkov in Leiden, both attending the farewell lecture by Lorentz, who was retiring.

Krutkov's letters to his sister: "On Monday, May 7, 1923, I was reading, together with Einstein, Friedmann's article in the *Zeitschrift für Physik*." May 18: "I defeated Einstein in the argument about Friedmann. Petrograd's honor is saved!"\*

May 31, 1923: Einstein's retraction of his refutation is received at *Zeitschrift für Physik*.

\* Quoted in *Alexander A. Friedmann: the Man who Made the Universe Expand*, by E.A. Tropp, V. Ya. Frenkel, & A.D. Chernin.

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## Einstein's Retraction

A NOTE ON THE WORK OF A. FRIEDMANN  
"ON THE CURVATURE OF SPACE"

A. Einstein, Berlin  
Received May 31, 1923  
*Zeitschrift für Physik*

I have in an earlier note (Einstein 1922) criticized the cited work (Friedmann 1922). My objection rested however — as Mr. Krutkov in person and a letter from Mr. Friedmann convinced me — on a calculational error. I am convinced that Mr. Friedmann's results are both correct and clarifying. They show that in addition to the static solutions to the field equations there are time varying solutions with a spatially symmetric structure.

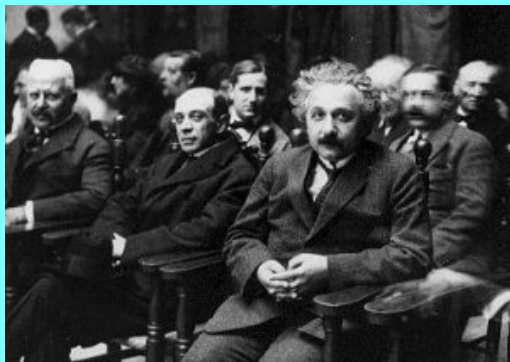
### REFERENCES:

Einstein, A. 1922, *Zs. f. Phys.*, 11, 326.  
Friedmann, A. 1922, *Ebenda*, 10, 377.

Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

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## Einstein and Krutkov



Albert Einstein  
Barcelona, 1923



Yuri A. Krutkov.

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Einstein's Draft

Notiz zu der Arbeit von A. Friedmann  
 „Über die Krümmung des Raumes“<sup>x</sup>

Ich habe in einer früheren Notiz<sup>x</sup> an  
 der genannten Arbeit<sup>\*\*</sup> Kritik geübt.  
 Mein Einwand bezog sich aber - wie  
 ich nicht auf Abweichung von Raum  
 an Grund einer ~~anderen~~ <sup>unvollständigen</sup>  
 Kritik ~~überzeugt~~ <sup>überzeugt</sup> habe - auf einen  
 Rechenfehler. Ich halte Herrn Friedmanns  
 Resultate für richtig und interessant und lehne  
 es nicht ab, dass die Feldgleichungen  
 dynamisch neben den statischen <sup>dynamische</sup>  
 (d. h. mit der Zeitkoordinate <sup>zeitlich-symmetrische</sup>  
~~unveränderliche~~)  
<sup>für die Krümmung</sup>  
 Lösungen <sup>zulassen</sup>, deren physikalische  
 Bedeutung ~~keine~~ <sup>ganz</sup> geschrieben sein  
 dürfte.  
 A. Einstein..

<sup>x</sup> Ztschr. für Physik 1922 11.B. § 326  
<sup>\*\*</sup> Ztschr. für Physik 1922 10.B. § 322.

“a physical significance can hardly be ascribed to them.”

Einstein's draft of 1923 in which he withdrew his earlier objection to Friedmann's dynamic solutions to the field equations. The last bit of the last sentence was: "a physical significance can hardly be ascribed to them". He crossed this out before sending the note to print.

\* From *The Invented Universe*, by Pierre Kerszberg