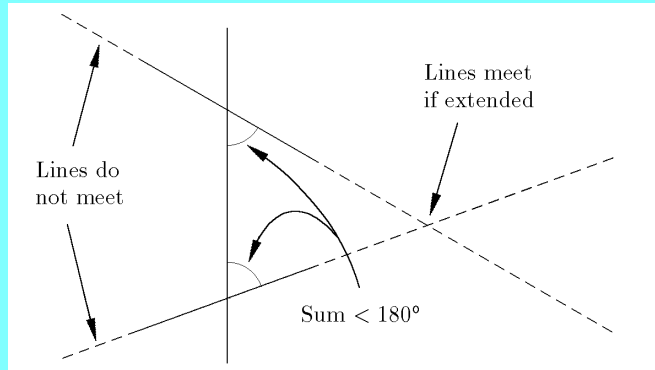


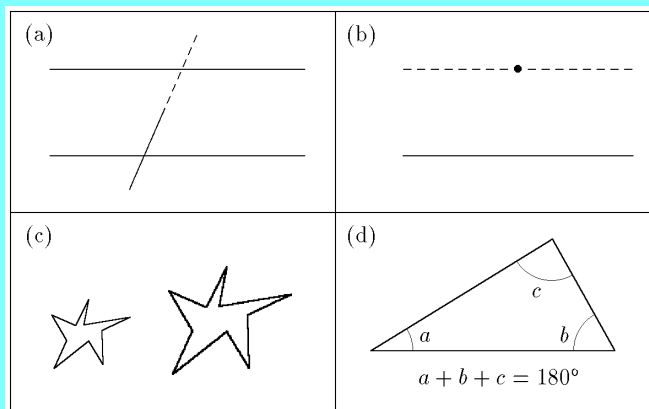
Euclid's Fifth Postulate



"If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines if produced indefinitely meet on that side on which the angles are less than two right angles." *[This statement is interpreted to imply that the two straight lines will never meet if extended on the opposite side.]*

-1-

Equivalent Statements:



- (a) "If a straight line intersects one of two parallels (i.e., lines which do not intersect however far they are extended), it will intersect the other also."
- (b) "There is one and only one line that passes through any given point and is parallel to a given line."
- (c) "Given any figure there exists a figure, similar to it, of any size."
- (d) "There is a triangle in which the sum of the three angles is equal to two right angles (i.e., 180°)."

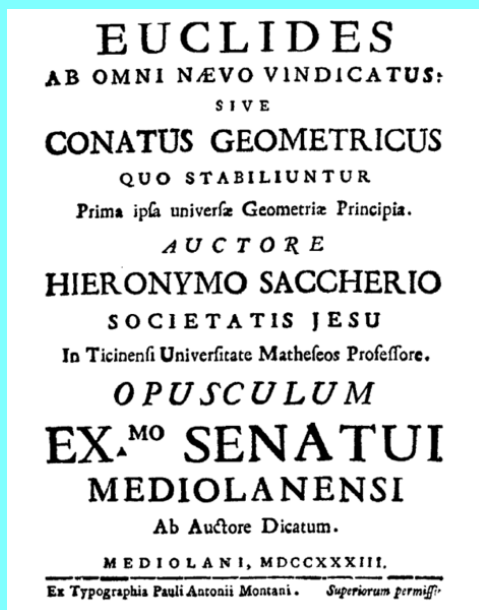
-2-

Giovanni Geralamo Saccheri, 1667-1733

- ★ Jesuit priest and mathematician.
- ★ In 1733, published a study of what geometry would be like if 5th postulate were false — hoped to find an inconsistency, but did not.
- ★ Discovered:
 - (a) If the sum of the angles of a triangle is more than 180° , then straight lines are finite.
 - (b) If the sum of the angles of a triangle is less than 180° , then triangles have a maximum finite area.
- ★ Conclusion: “the hypothesis of the acute angle is absolutely false; because it is repugnant to the nature of straight lines.”

-3-

Saccheri's Manuscript



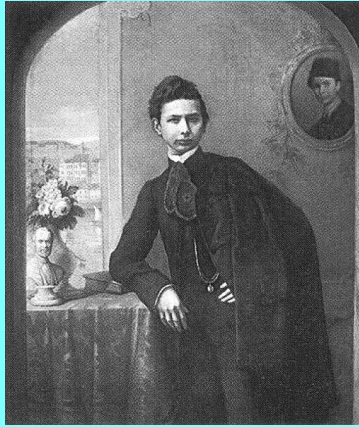
**Euclides ab omni
naevo vindicatus**
(Euclid Freed of Every Flaw)

-4-

Gauss-Bolyai-Lobachevski Geometry



Carl Friedrich Gauss
1777 – 1855
German Mathematician
and Physicist



Janos Bolyai
1802 – 1860
Austrian Army Officer



Nikolai Ivanovich Lobachevski
1793 – 1856
Russian Mathematician