Ants on a Pringle

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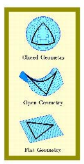
18.10.2011

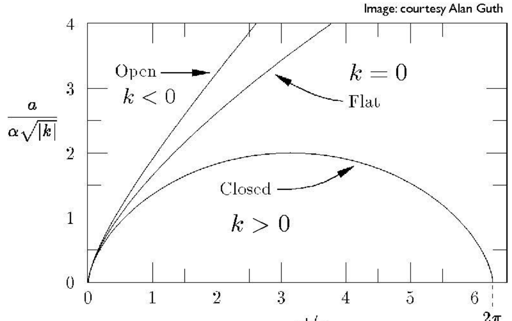
Introduction to non-Euclidean spaces

Recap



$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Image: courtesy Alan Guth



Note: matter dominated universes only!

k ?

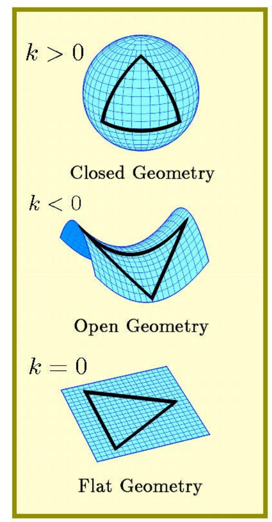
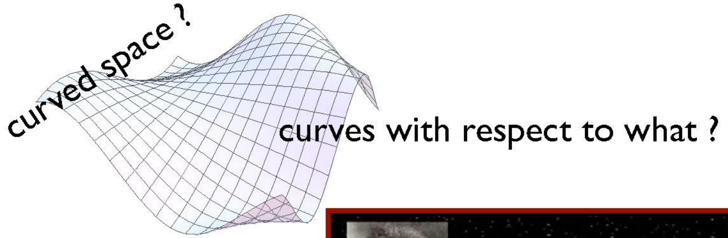
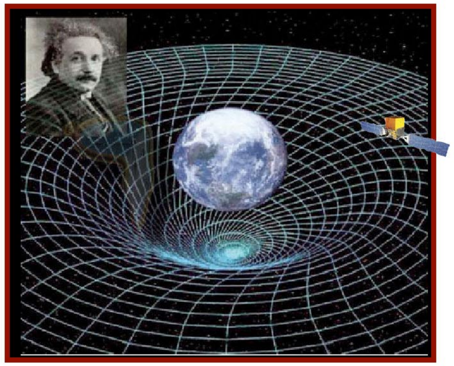


Image: courtesy Alan Guth



curved spacetime ?

non-Euclidean geometry



Note on image: I have added the "Dali" clock and Fermi satellite images to the original image created for the Gravity Probe B collaboration

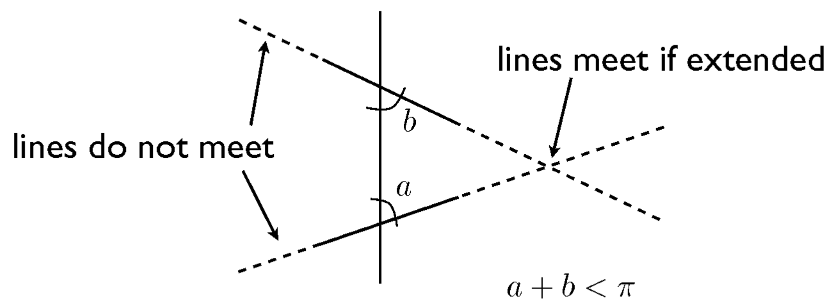
A bit of History ...

Euclid's *Postulates*

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given a straight line segment , a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.

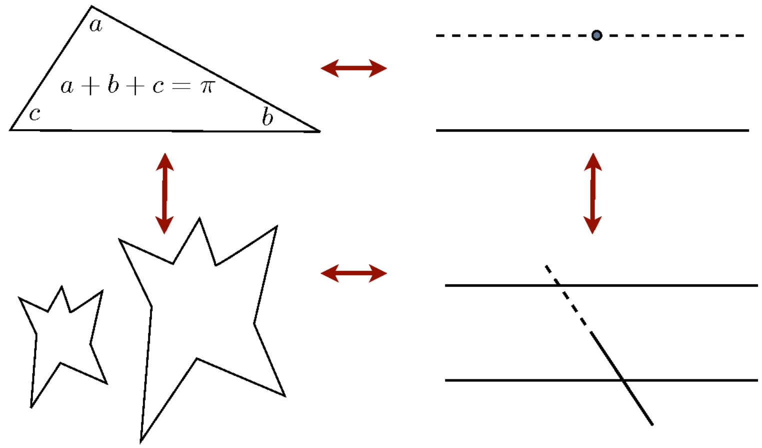
5. If a straight line falling on two straight lines makes the interior angles on the same side are less than the two right angles, the two straight lines if produced indefinitely meet on that line on which the angles are less than two right angles

5th Postulate



5. If a straight line falling on two straight lines makes the interior angles on the same side are less than the two right angles, the two straight lines if produced indefinitely meet on that line on which the angles are less than two right angles

Many tried, but ...



(1667-1733)

Saccheri

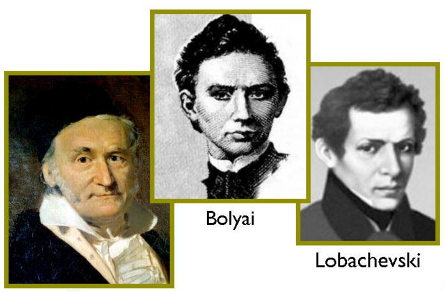
What if ...

~~5th Postulate~~

$\Downarrow$

inconsistent ?

~1750-1850



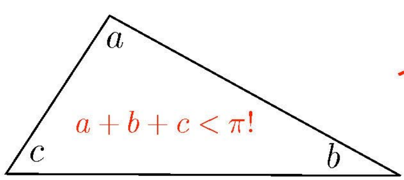
Gauss

Bolyai

Lobachevski

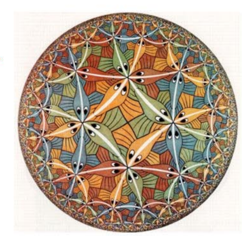
- infinite
- constant negative curvature

~~5th Postulate~~



GBL geometry with Klein

1. constant negative curvature
2. infinite
3. ~~5th postulate~~



(x_1, y_1)

(x_2, y_2)

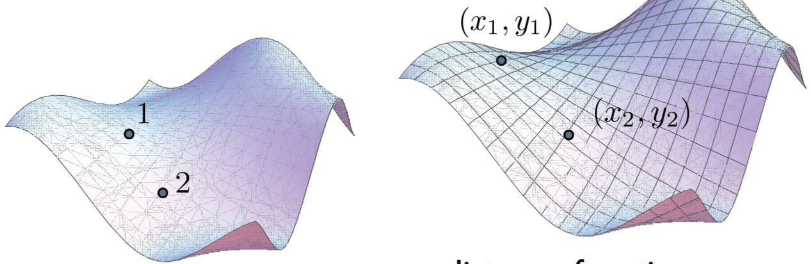
$x^2 + y^2 < 1$

$$d(1,2) = a \cosh^{-1}\left[\frac{1 - x_1x_2 - y_1y_2}{\sqrt{1 - x_1^2 - y_1^2}\sqrt{1 - x_2^2 - y_2^2}}\right]$$

Note: no global embedding in 3D Euclidean space possible

Geometry (after Klein)

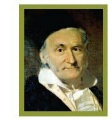
co-ordinates

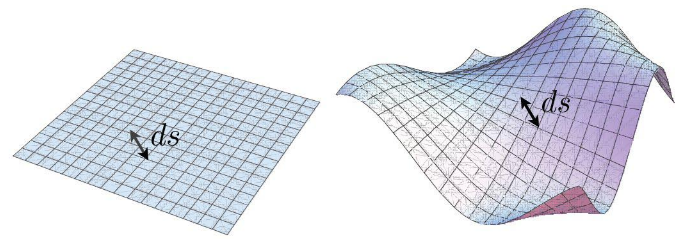


distance function

$d[(x_1, y_1), (x_2, y_2)]$

tiny distances

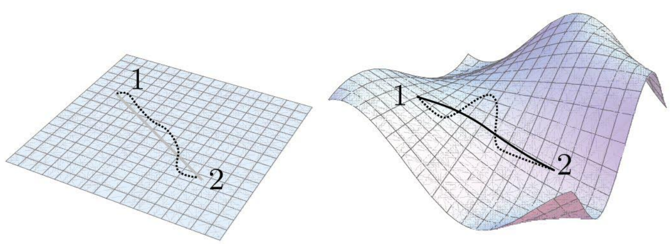




$ds^2 = dx^2 + dy^2$

$ds^2 = g_{xx}(x, y)dx^2 + 2g_{xy}(x, y)dxdy + g_{yy}(x, y)dy^2$

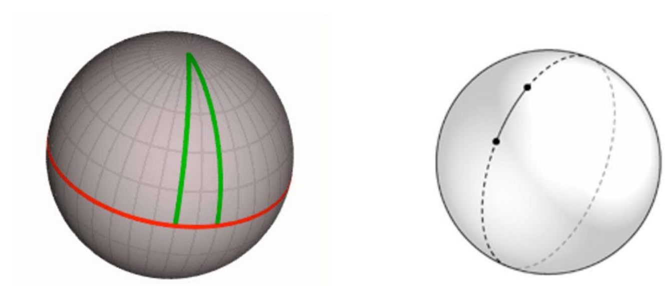
geodesics



a geodesic is a curve along which the distance between two given points is extremised.

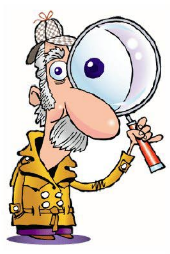
note: important!

sphere: geodesics



longitudes: yes
latitudes: no

quadratic form



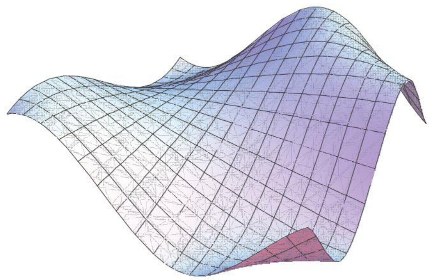
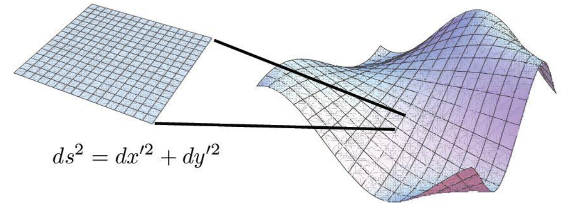


Image:www.easternct.edu/career/webresources.htm

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

locally Euclidean



$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

$$g_{xx}g_{yy} - g_{xy}^2 > 0$$

Intrinsic Geometry

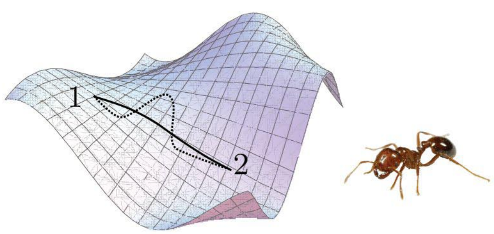
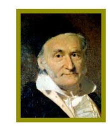
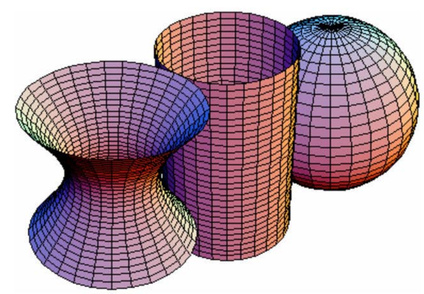


Image of ant from: http://www.termiteterry.com/

curved ?

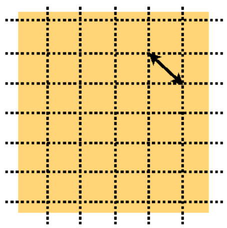


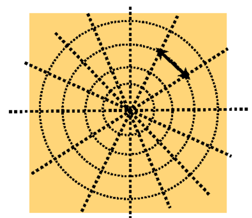


Above image:http://en.wikipedia.org/wiki/Gaussian_curvature

$$ds^2 = dx^2 + dy^2 \qquad ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

metric and co-ordinates



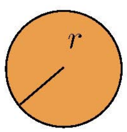


$$ds^2 = dx^2 + dy^2 \qquad ds^2 = dr^2 + r^2 d\theta^2$$

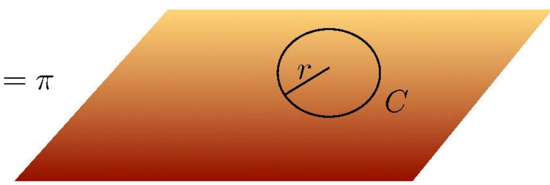
Curved ?

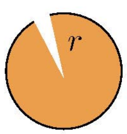
Image: www.earthshope.org/

curved ?

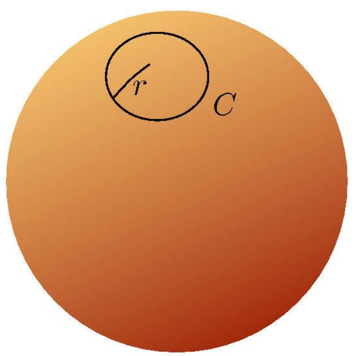


$$\frac{C}{2r} = \pi$$

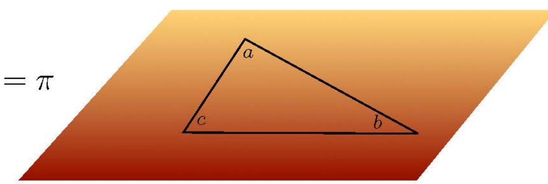




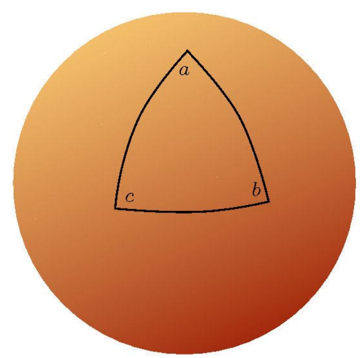
$$\frac{C}{2r} < \pi$$



$$a + b + c = \pi$$

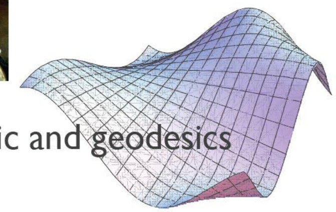


$$a + b + c > \pi$$



SUMMARY so far ...

intrinsic/extrinsic properties

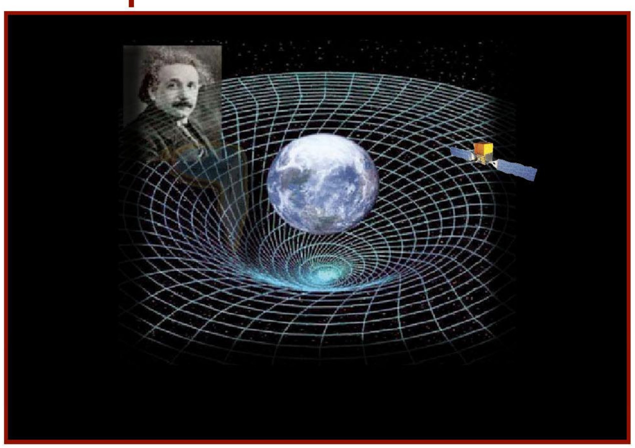


metric and geodesics

Riemannian
locally Euclidean
e.g. sphere, plane
curved or flat

pseudo-Riemannian
locally Lorentzian
e.g. spacetime
curved or flat

gravity: spacetime curvature



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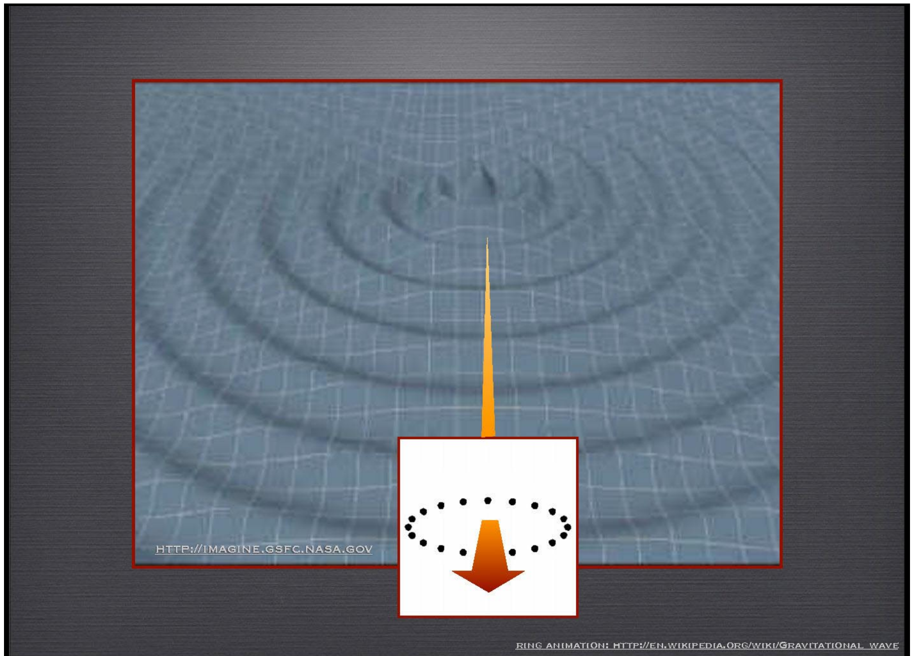

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References

- Alan's notes
- Rindler's book on Gravitation
- Personal notes (from class taught for EPGY)

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