Ants on a Pringle

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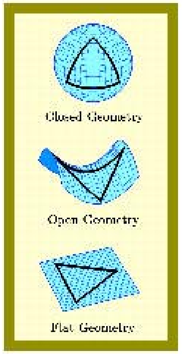
18.10.2011

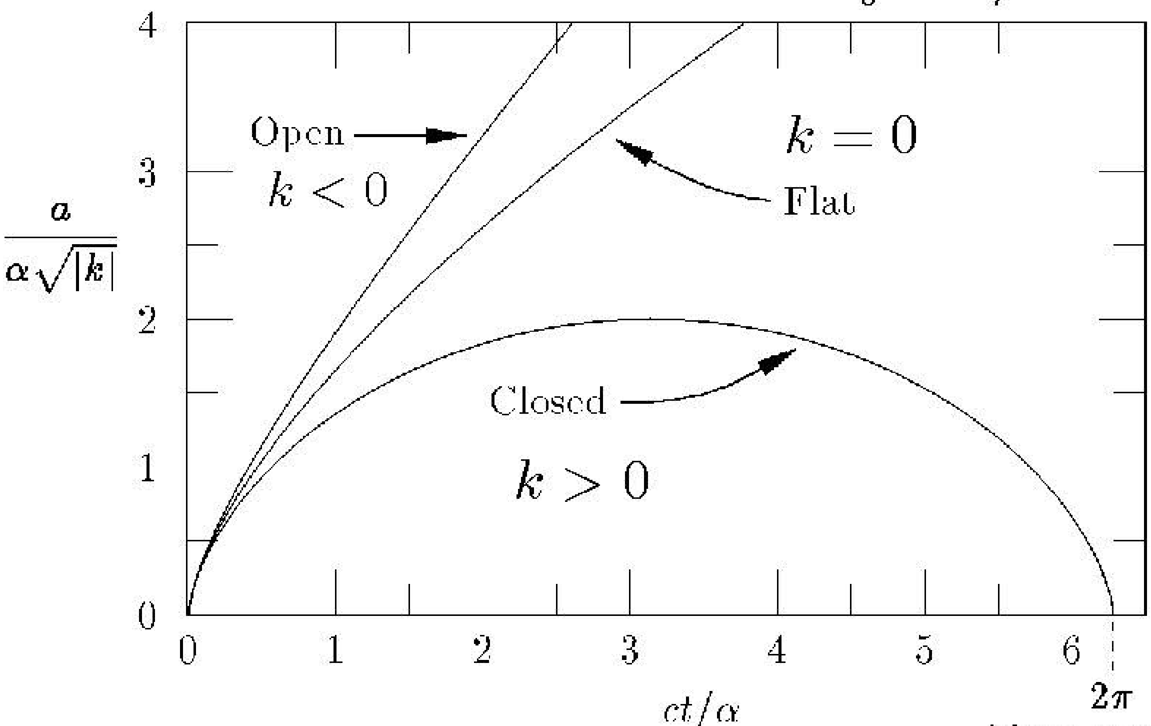
Introduction to non-Euclidean spaces



Recap

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

Image: courtesy Alan Guth



Note: matter dominated universes only!

k ?

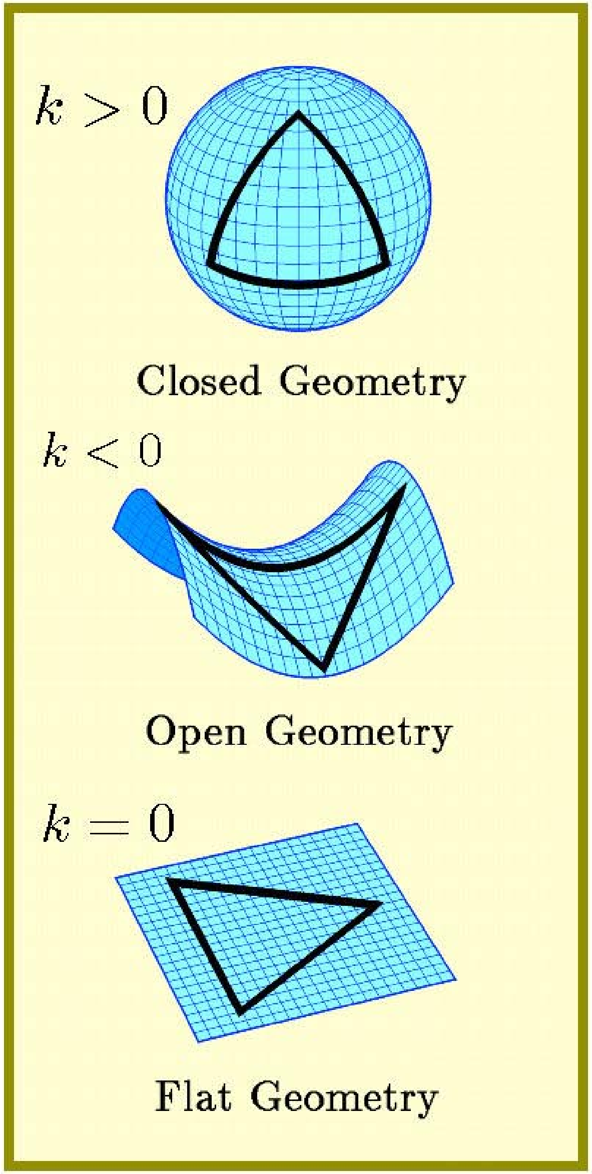
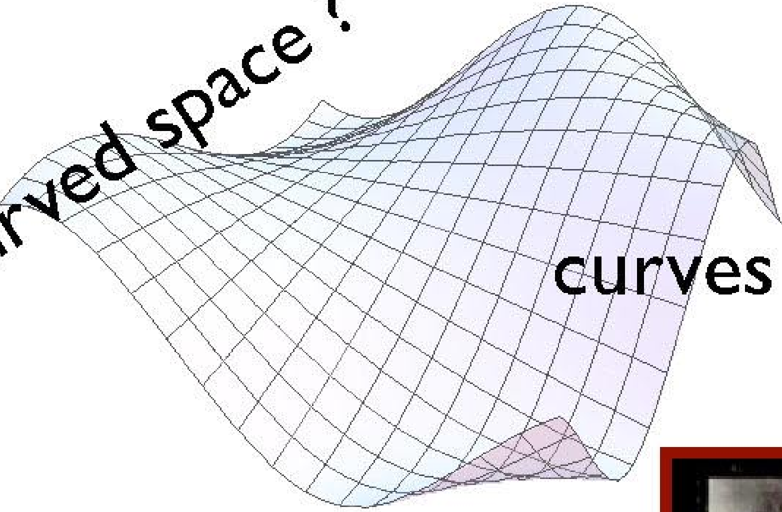


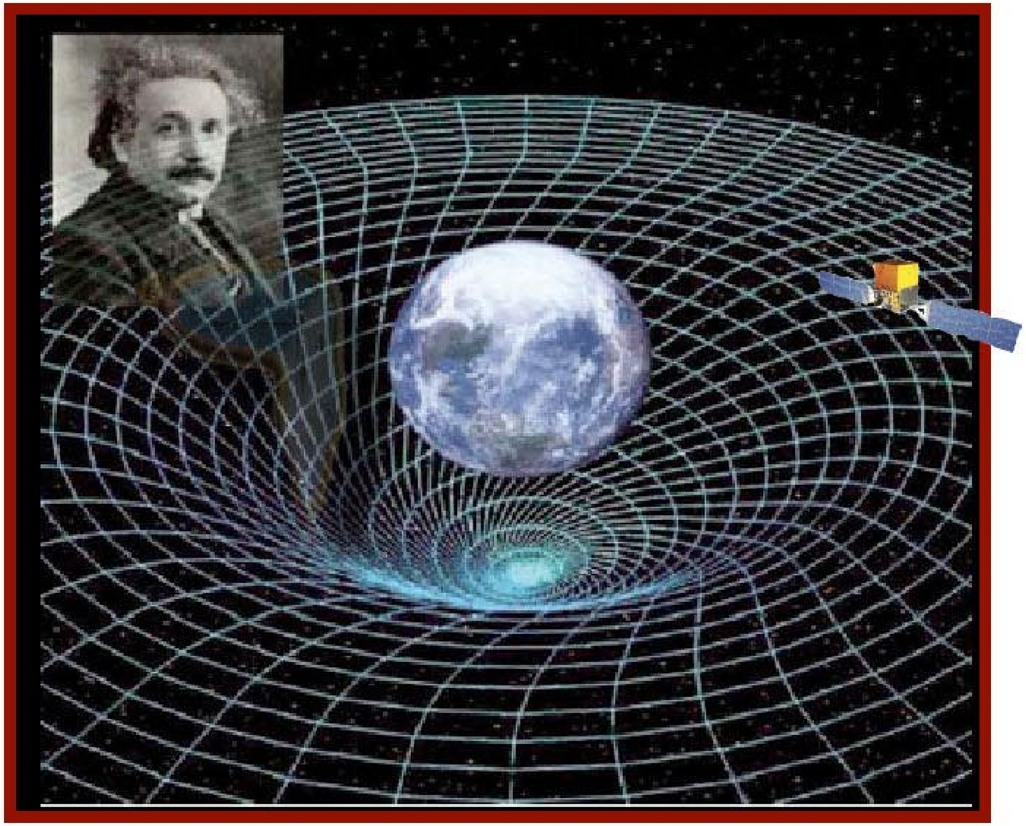
Image: courtesy Alan Guth

curved space ?



curves with respect to what ?

curved spacetime ?



non-Euclidean geometry

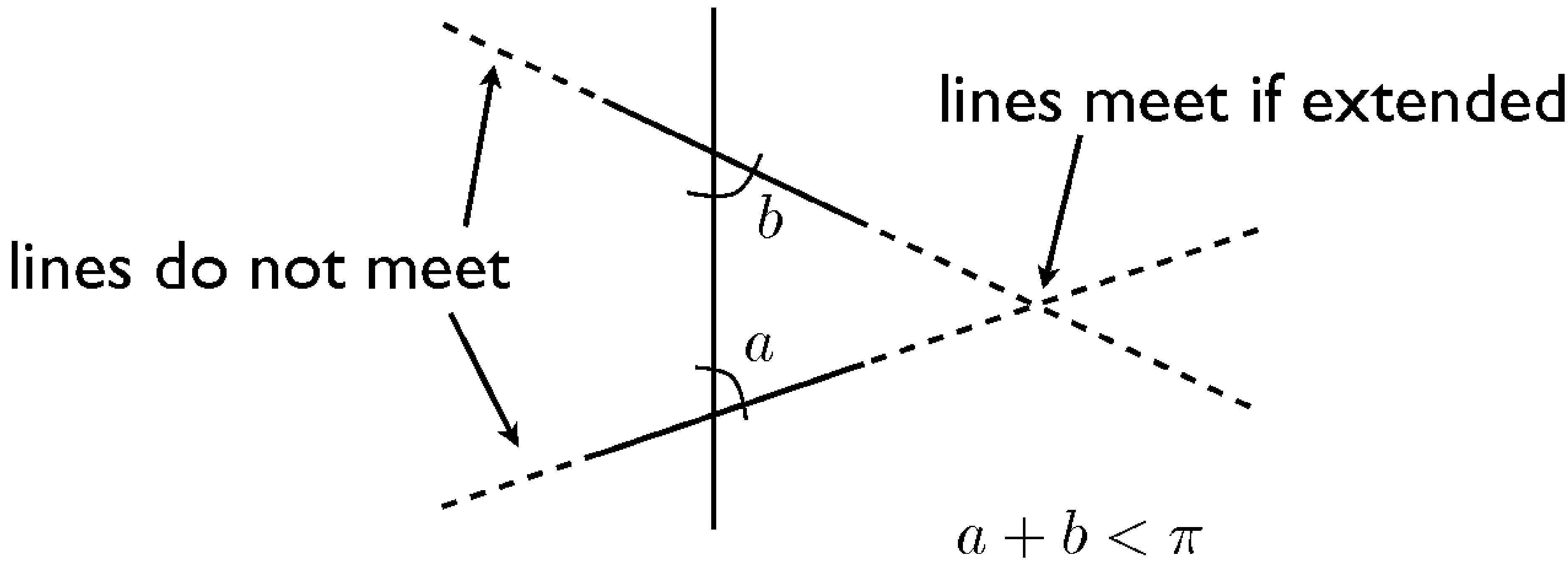
Note on image: I have added the "Dali" clock and Fermi satellite images to the original image created for the Gravity Probe B collaboration

A bit of History ...

Euclid's *Postulates*

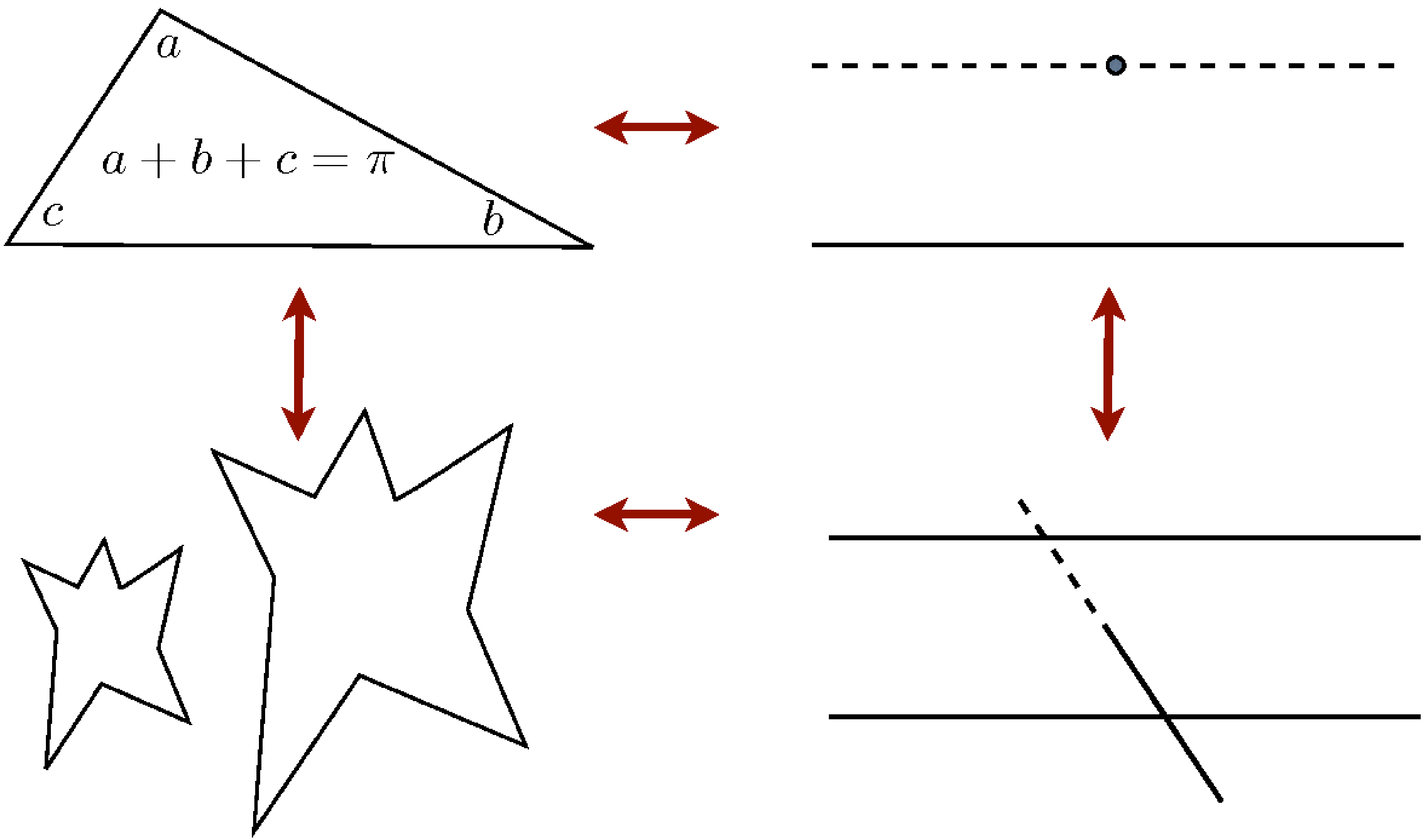
1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given a straight line segment , a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If a straight line falling on two straight lines makes the interior angles on the same side are less than the two right angles, the two straight lines if produced indefinitely meet on that line on which the angles are less than two right angles

5th Postulate



5. If a straight line falling on two straight lines makes the interior angles on the same side are less than the two right angles, the two straight lines if produced indefinitely meet on that line on which the angles are less than two right angles

Many tried, but ...



(1667-1733)

Saccheri

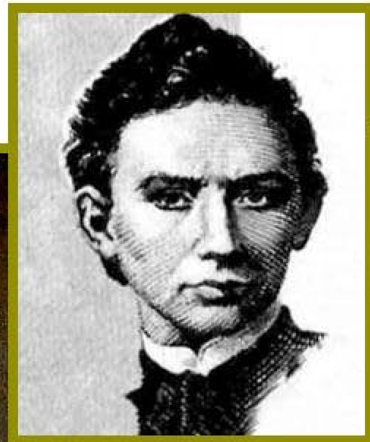
What if ...

~~5th Postulate~~

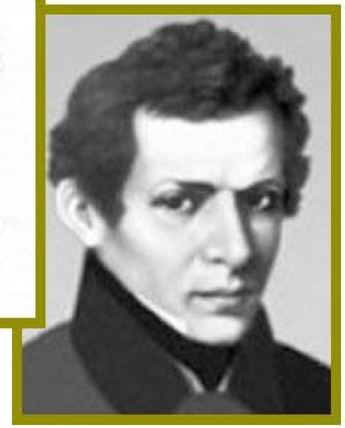
$$\Downarrow$$

inconsistent ?

~1750-1850



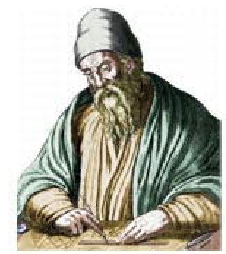
Bolyai



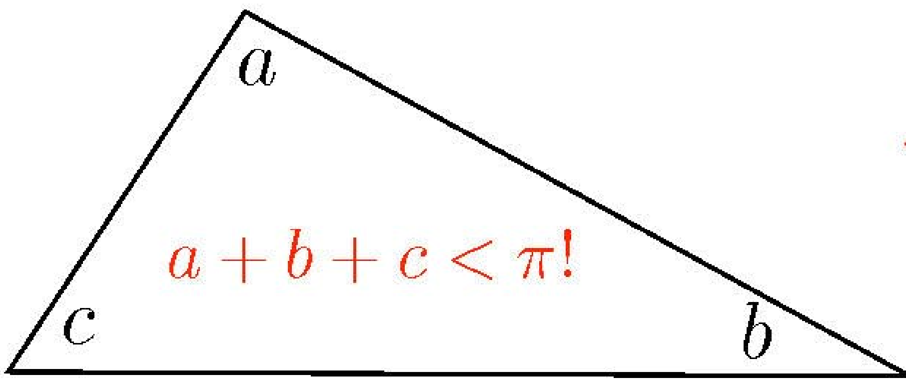
Lobachevski

Gauss

- infinite
- constant negative curvature

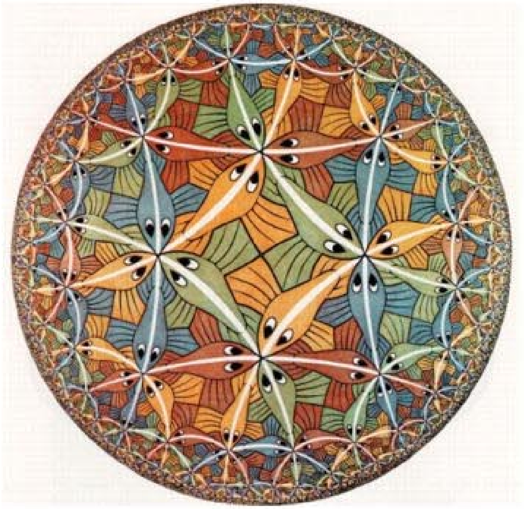


5th Postulate



GBL geometry with Klein

1. constant negative curvature
2. infinite
3. ~~5th postulate~~



(x_1, y_1)

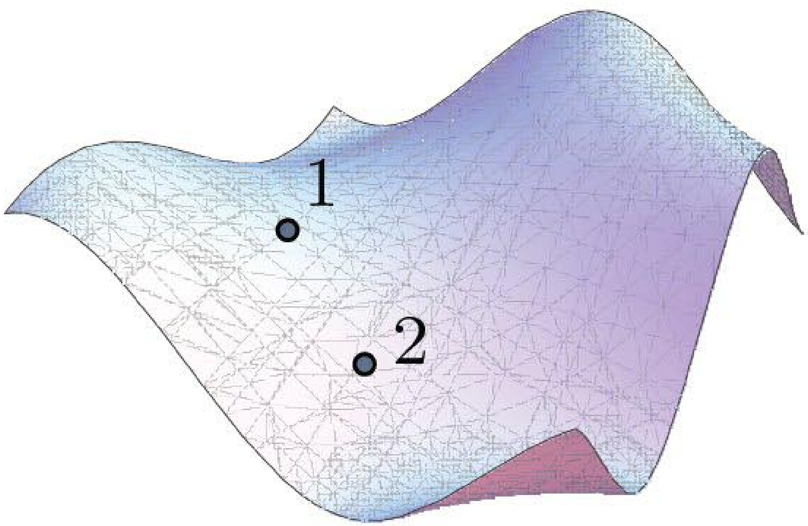
$$x^2 + y^2 < 1$$

$$d(1,2) = a \cosh^{-1} \left[\frac{1 - x_1 x_2 - y_1 y_2}{\sqrt{1 - x_1^2 - y_1^2}\sqrt{1 - x_2^2 - y_2^2}} \right]$$

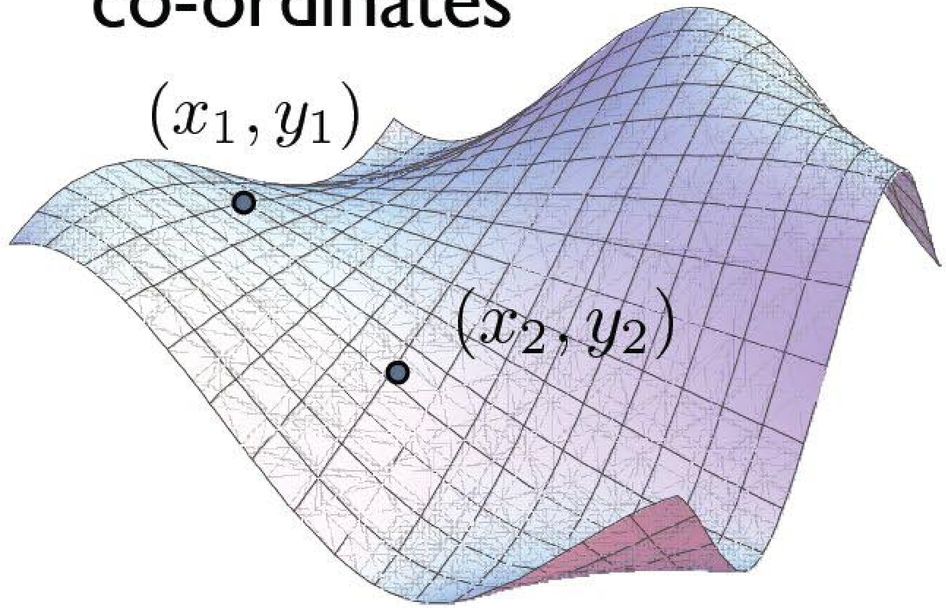
(x_2, y_2)

Note: no global embedding in 3D Euclidean space possible

Geometry (after Klein)



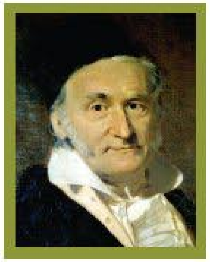
co-ordinates

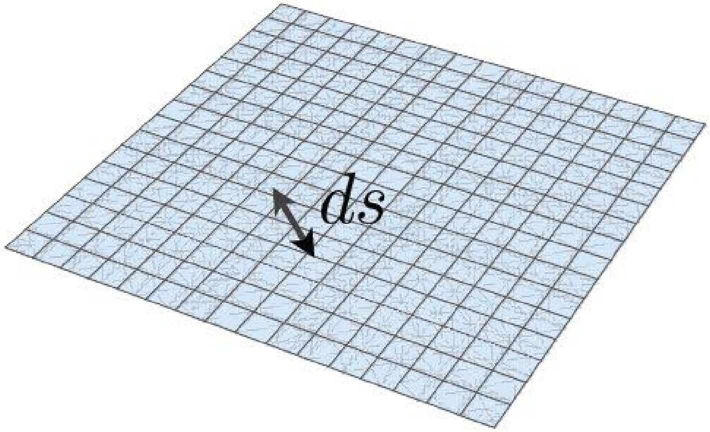


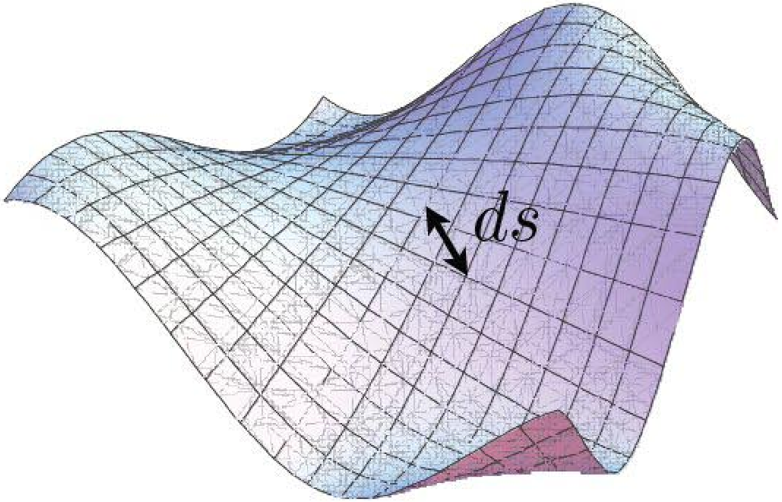
distance function

$$d[(x_1, y_1), (x_2, y_2)]$$

tiny distances



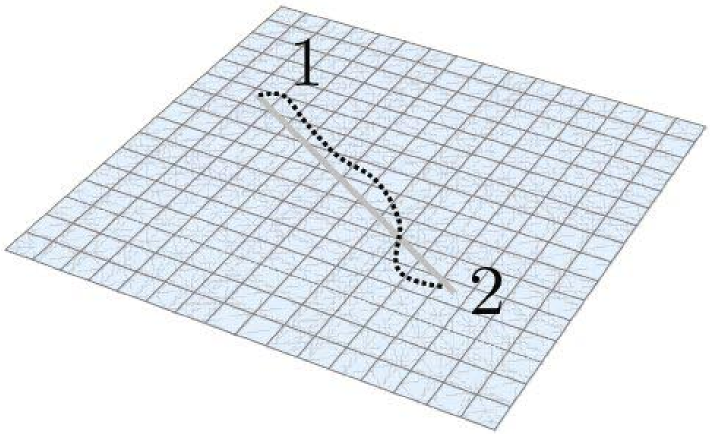


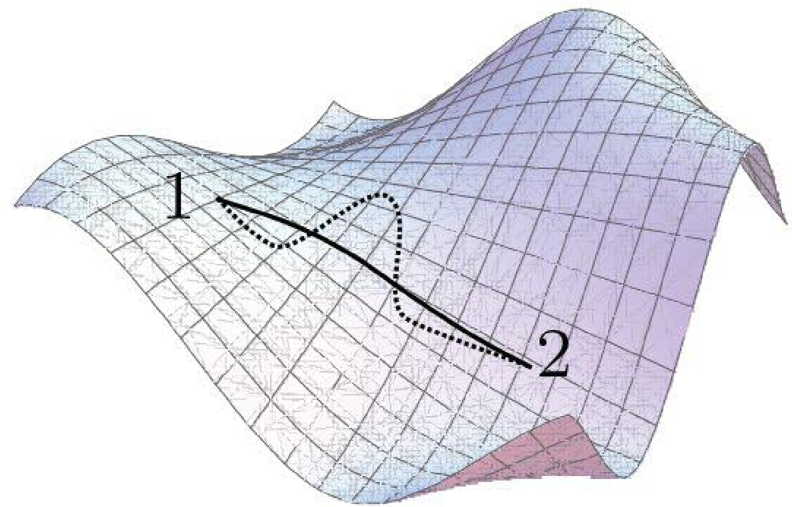


$$ds^2 = dx^2 + dy^2$$

$$ds^2 = g_{xx}(x, y)dx^2 + 2g_{xy}(x, y)dxdy + g_{yy}(x, y)dy^2$$

geodesics

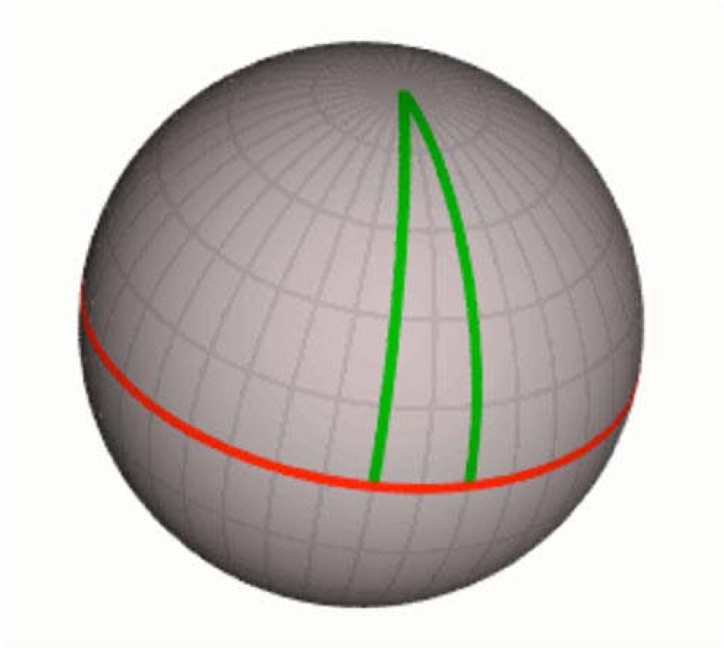


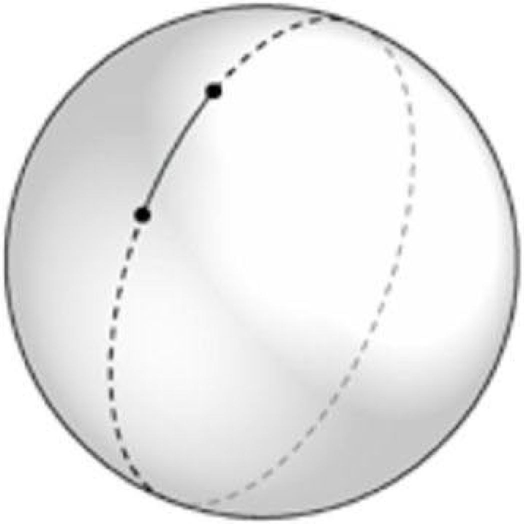


a geodesic is a curve along which the distance between two given points is extremised.

note: important!

sphere: geodesics





longitudes: yes
latitudes: no

quadratic form

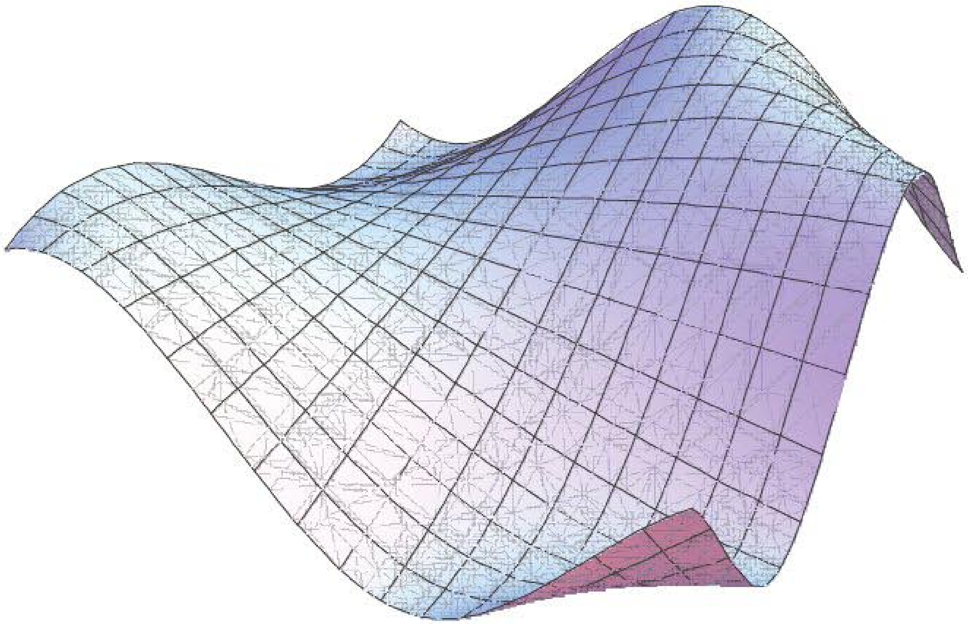
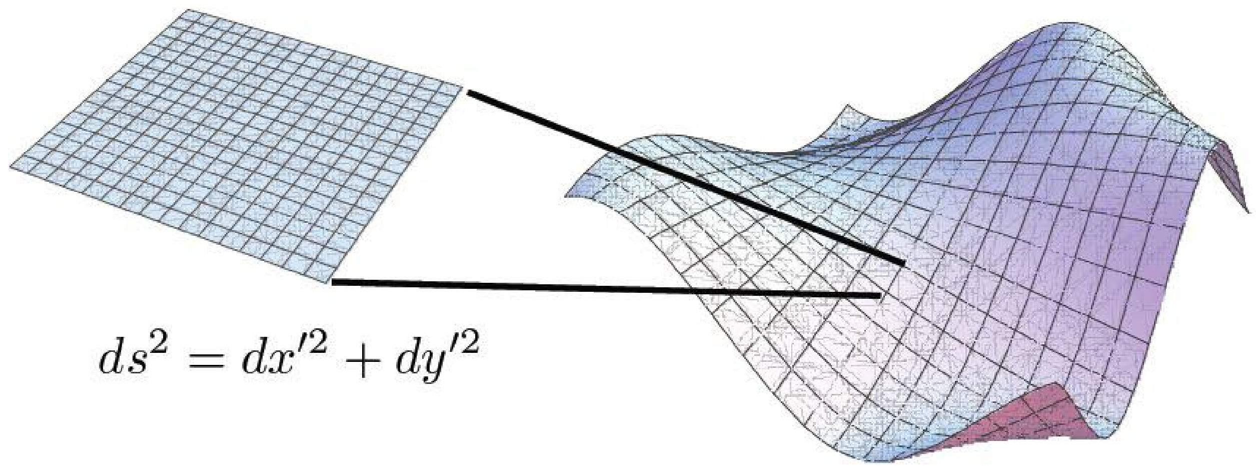


Image: www.easternct.edu/career/webresources.htm

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

locally Euclidean



$$ds^2 = dx'^2 + dy'^2$$

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

$$g_{xx}g_{yy} - g_{xy}^2 > 0$$

Intrinsic Geometry

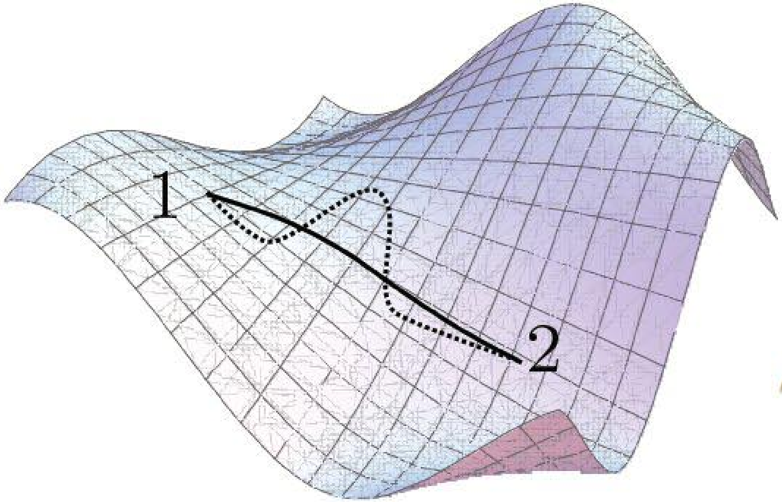
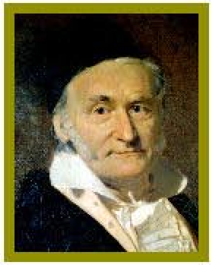
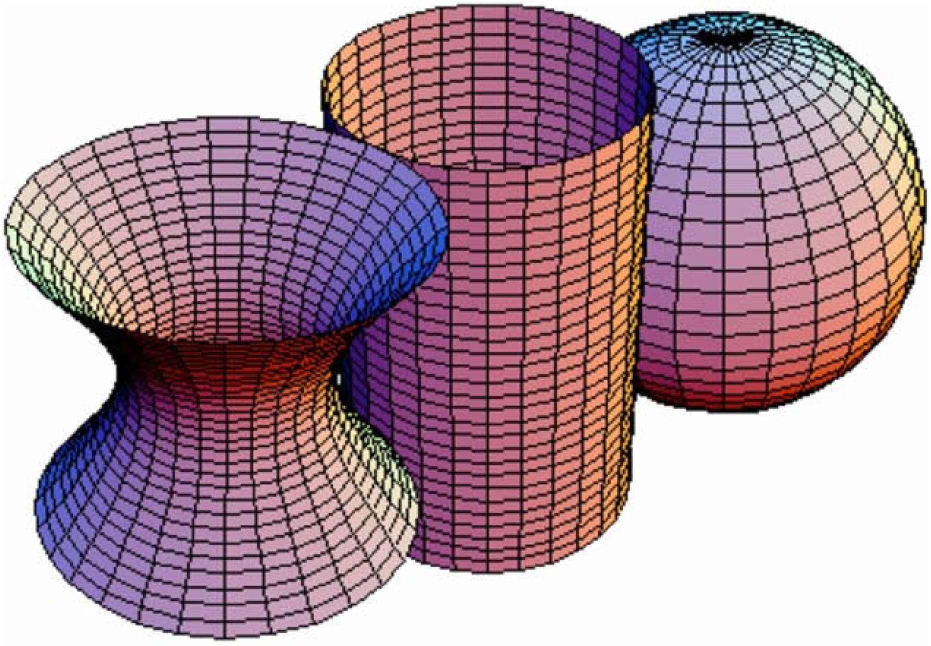


Image of ant from: http://www.termiteterry.com/

curved ?



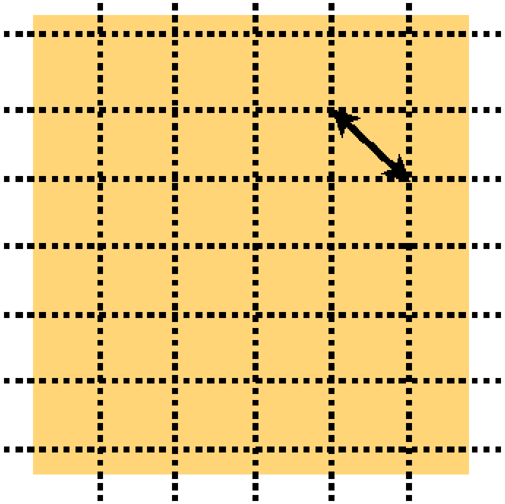


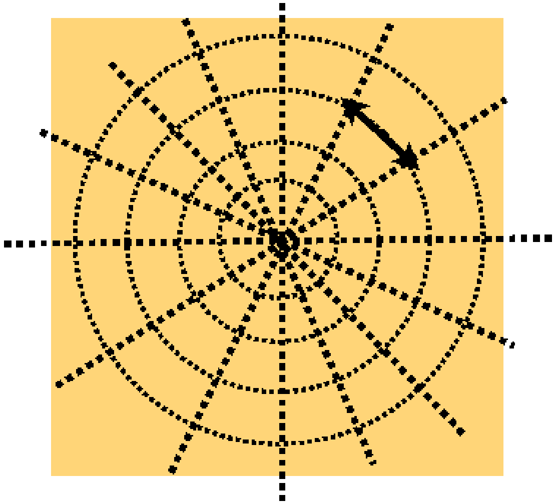
Above image:http://en.wikipedia.org/wiki/Gaussian_curvature

$$ds^2 = dx^2 + dy^2$$

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

metric and co-ordinates





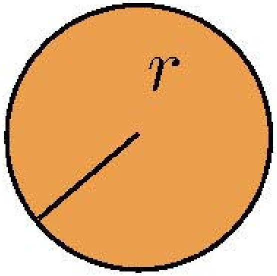
$$ds^2 = dx^2 + dy^2$$

$$ds^2 = dr^2 + r^2 d\theta^2$$

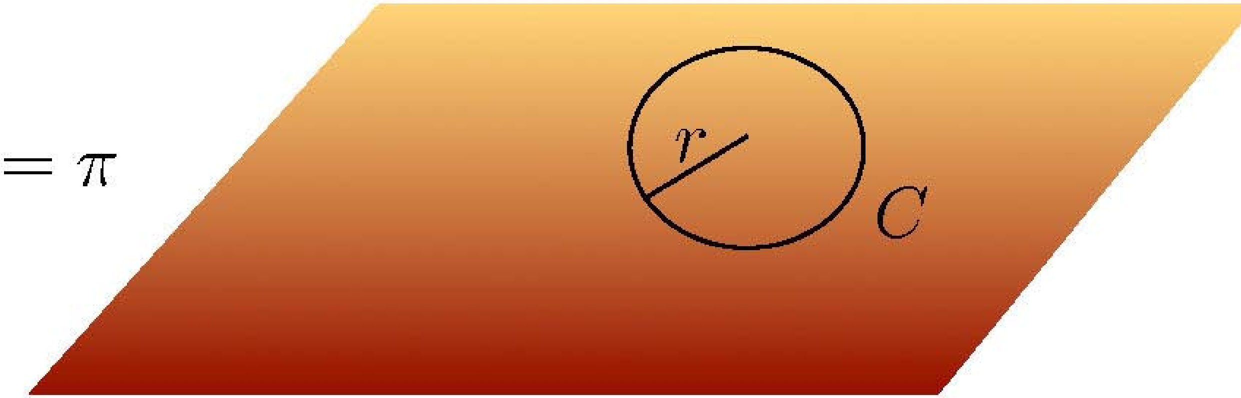
Curved ?

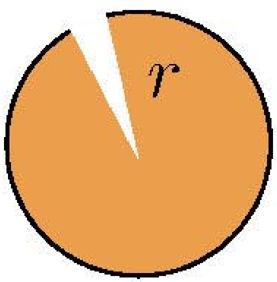
Image: www.earthshope.org/

curved ?

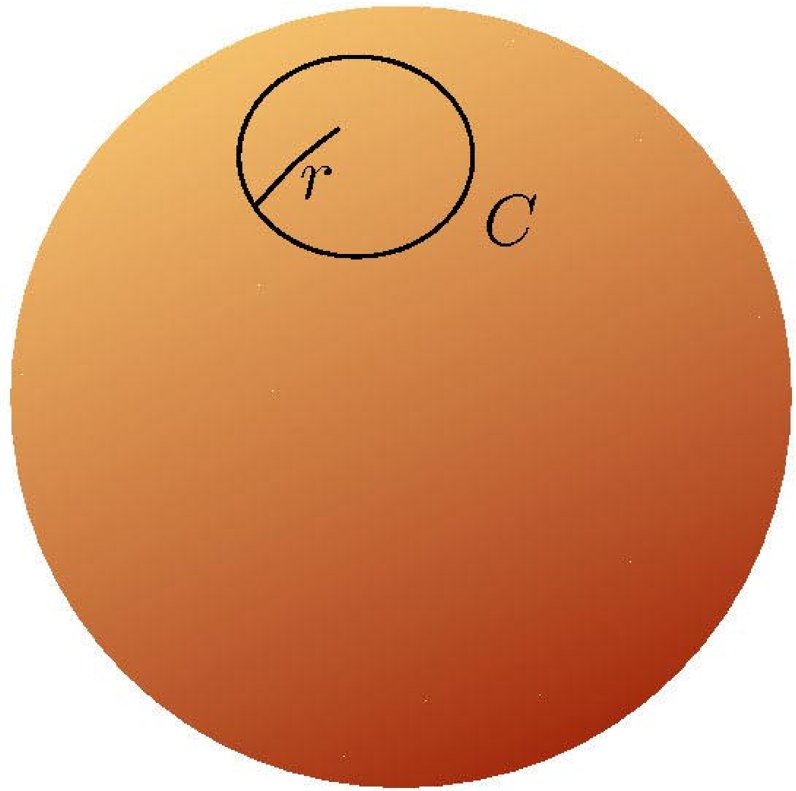


$$\frac{C}{2r} = \pi$$

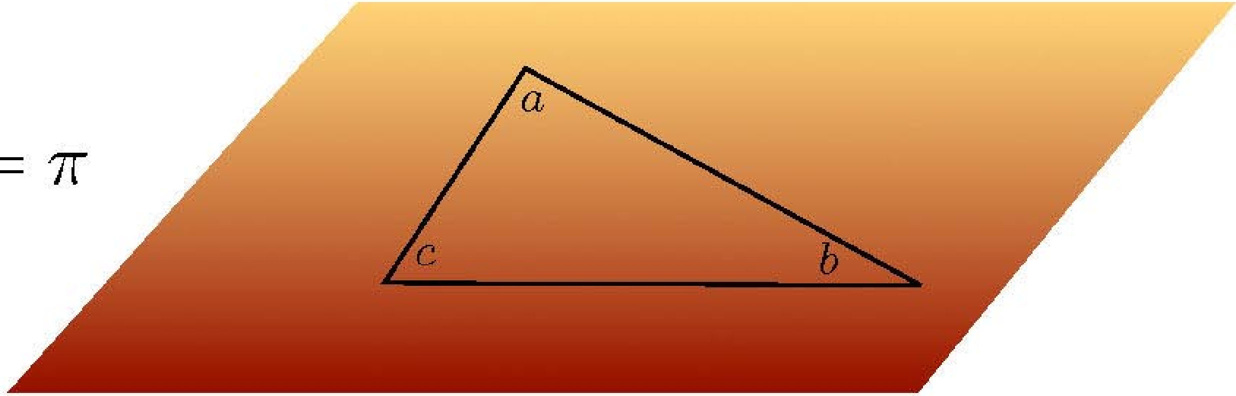




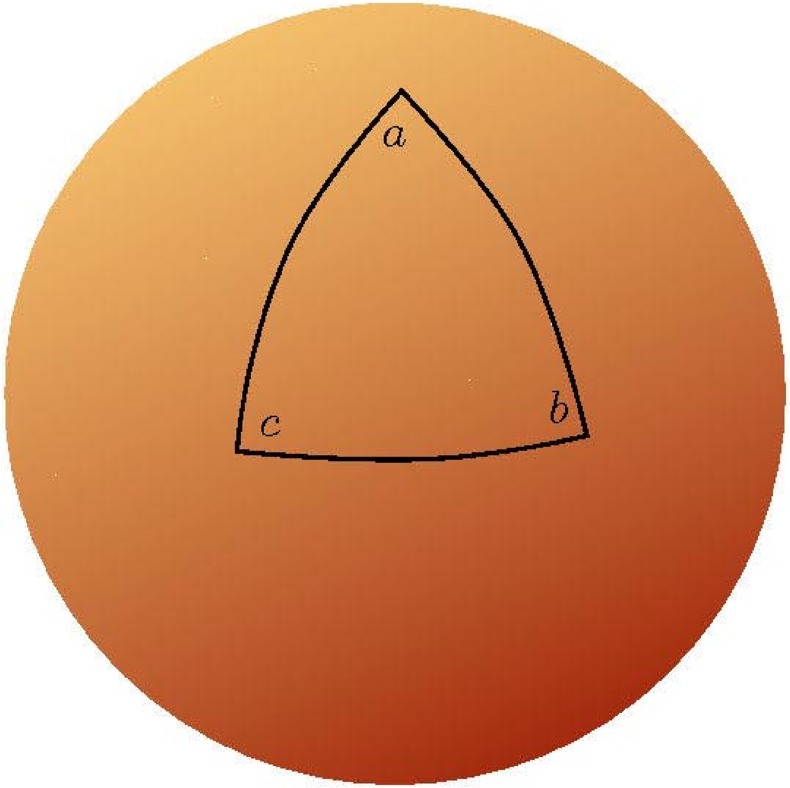
$$\frac{C}{2r} < \pi$$



$$a + b + c = \pi$$



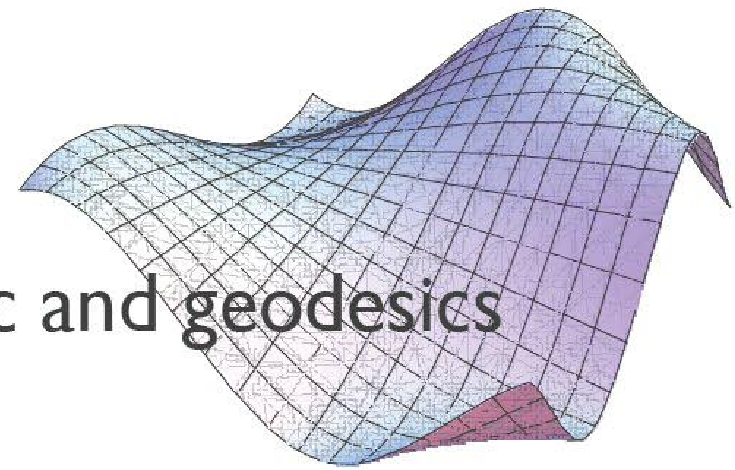
$$a + b + c > \pi$$




Pringles
SOUR CREAM & ONION
curved ?

SUMMARY so far ...

intrinsic/extrinsic properties

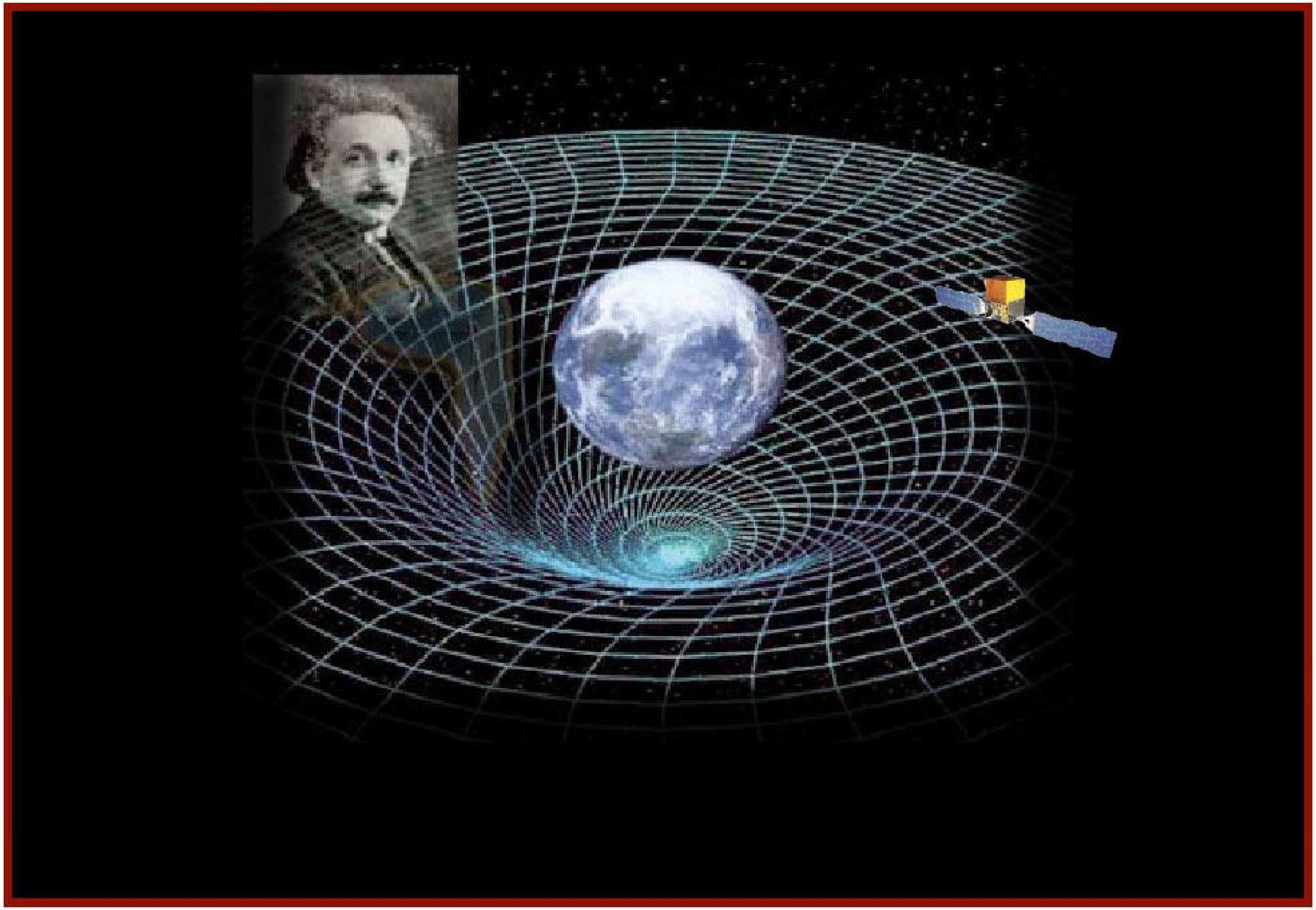


metric and geodesics

Riemannian
locally Euclidean
e.g. sphere, plane
curved or flat

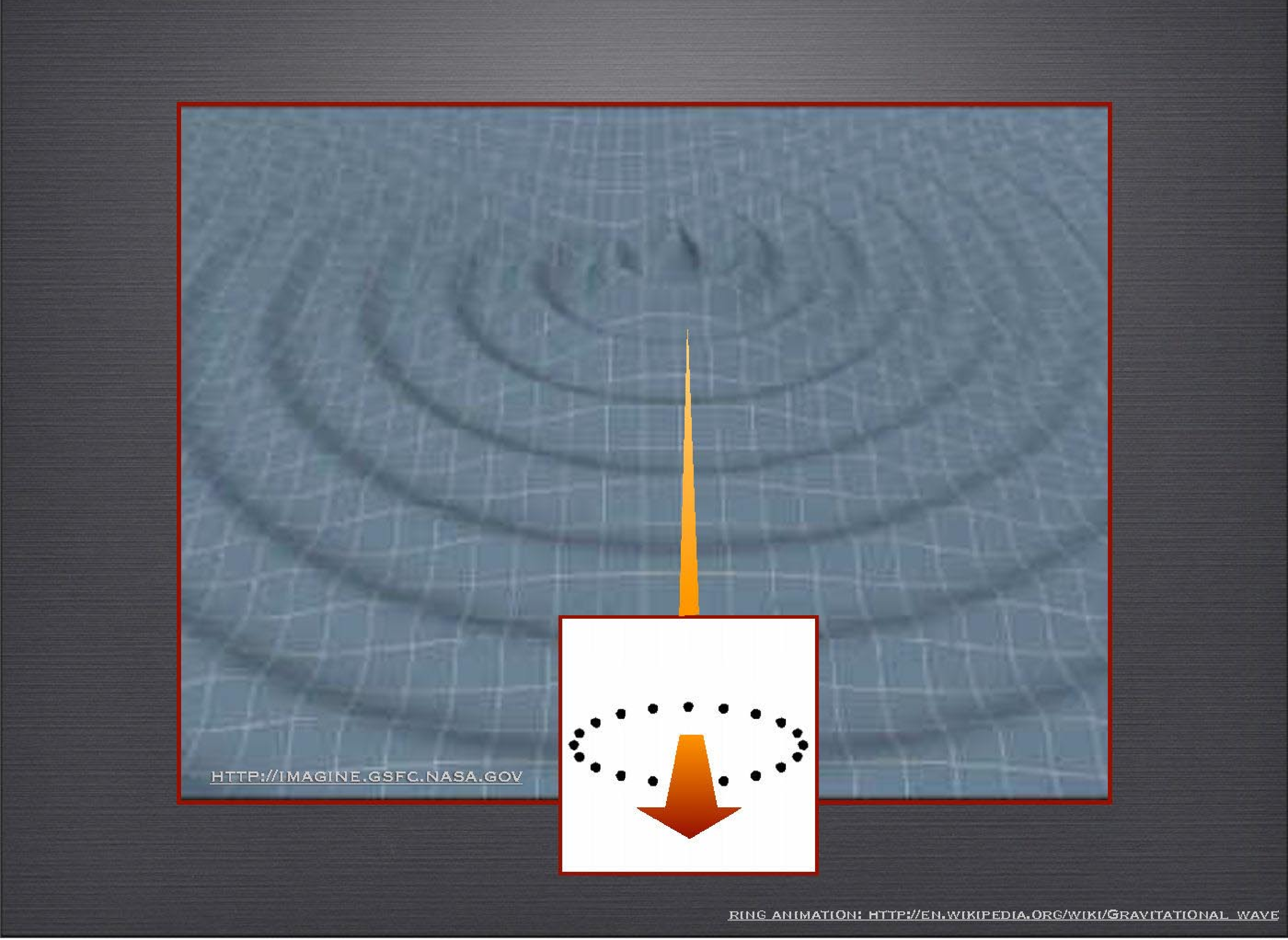
pseudo-Riemannian
locally Lorentzian
e.g. spacetime
curved or flat

gravity: spacetime curvature



Note on image: I have added the "Dali" clock and Fermi satellite animation to the original image created for the Gravity Probe B collaboration


HTTP://IMAGINE.GSFC.NASA.GOV


HTTP://IMAGINE.GSFC.NASA.GOV
RING ANIMATION: HTTP://EN.WIKIPEDIA.ORG/WIKI/GRAVITATIONAL_WAVE

References

- Alan's notes
- Rindler's book on Gravitation
- Personal notes (from class taught for EPGY)