8.286 Lecture 6
September 24, 2013

DYNAMICS OF HOMOGENEOUS EXPANSION

Review: Can a Uniform Infinite Distribution of Mass Be Stable?

Gauss's Law of Gravity:

$$\vec{g} = -\frac{GM}{r^2}\hat{r} \quad \Longrightarrow \quad \oint \vec{g} \cdot d\vec{a} = -4\pi G M_{\text{enclosed}}$$

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Poisson's Equation:

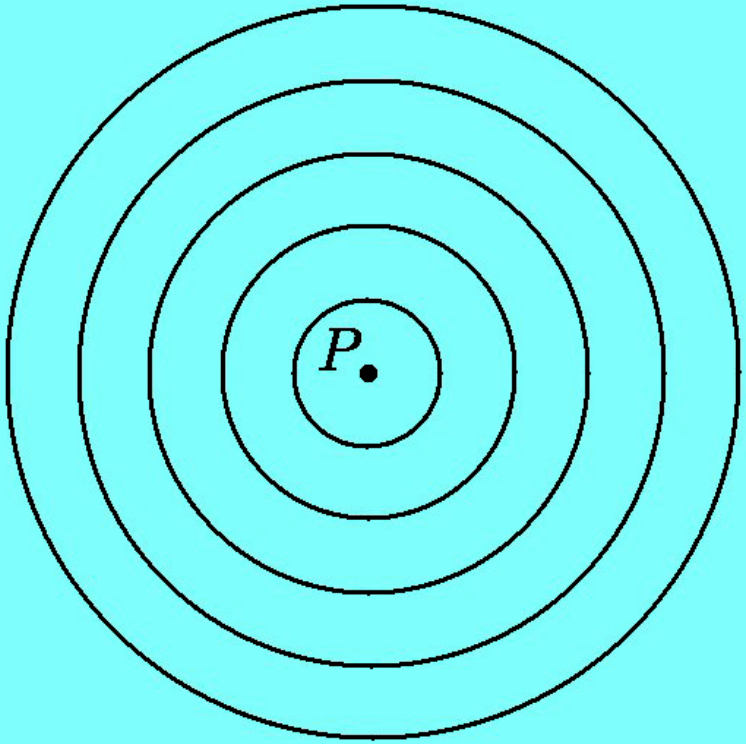
$$\nabla^2\phi = 4\pi G\rho \ , \quad \text{where } \vec{g} = -\vec{\nabla}\phi \ .$$

where ρ is the mass density, ∇^2 is the Laplacian:

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \ ,$$

and $\vec{\nabla}$ is the gradient:

$$\vec{\nabla} \equiv \hat{\imath}\,\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \ .$$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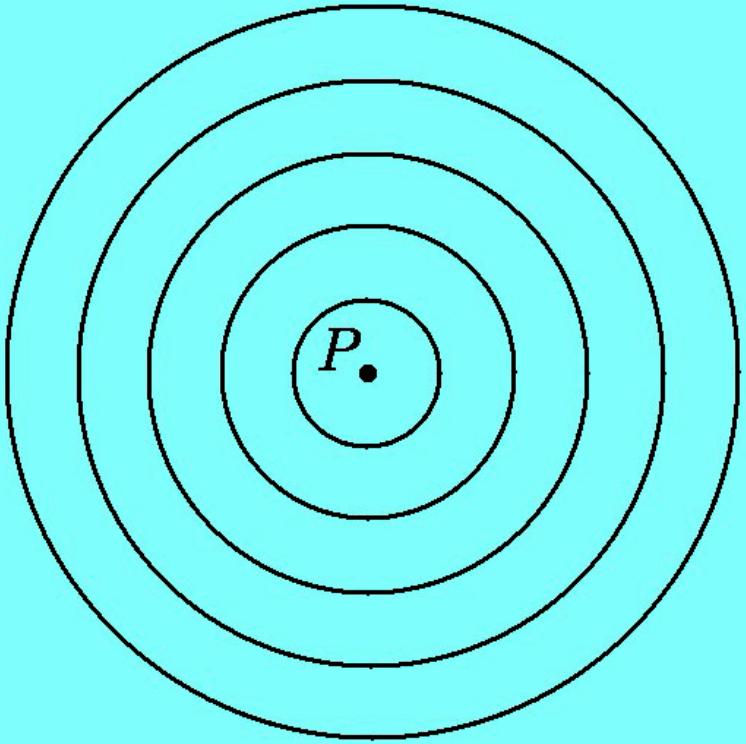

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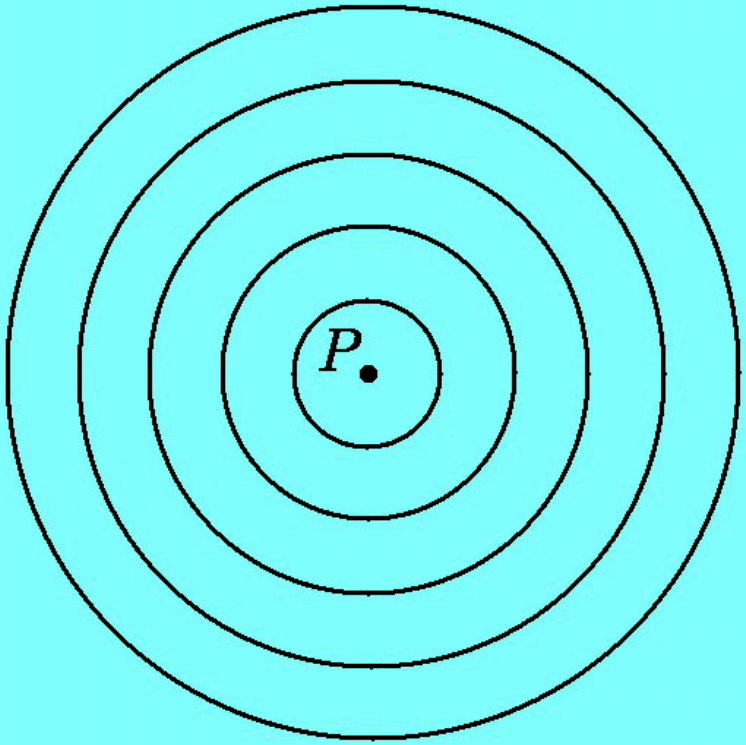

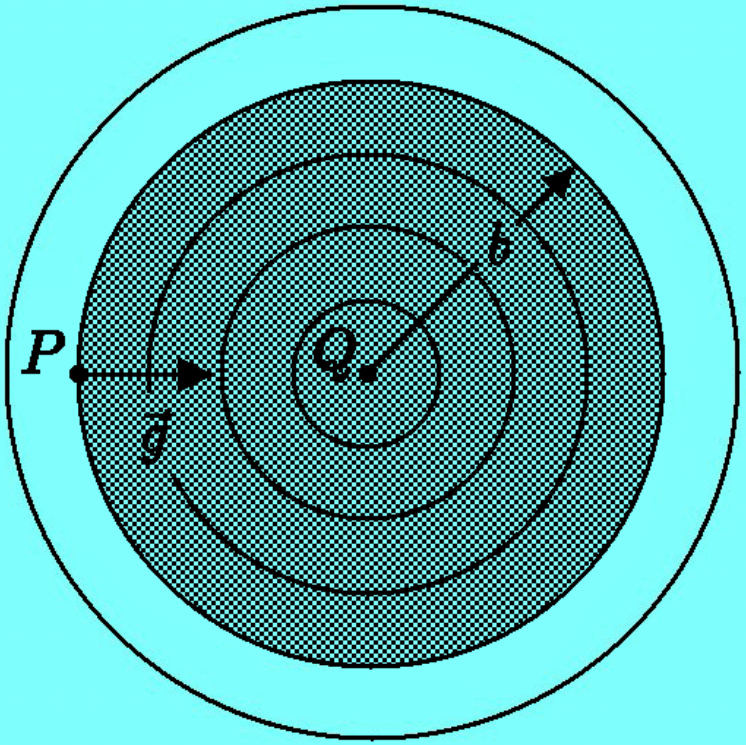

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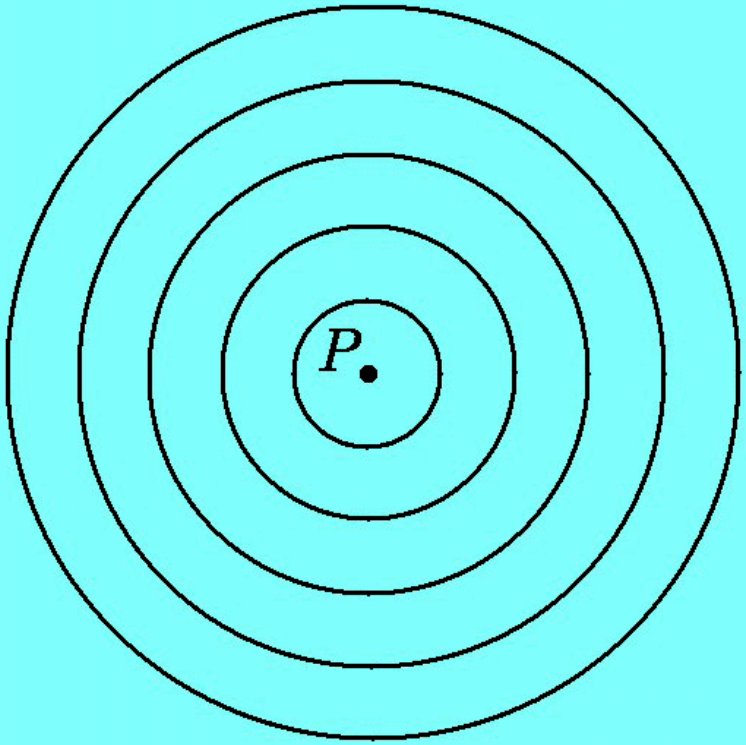

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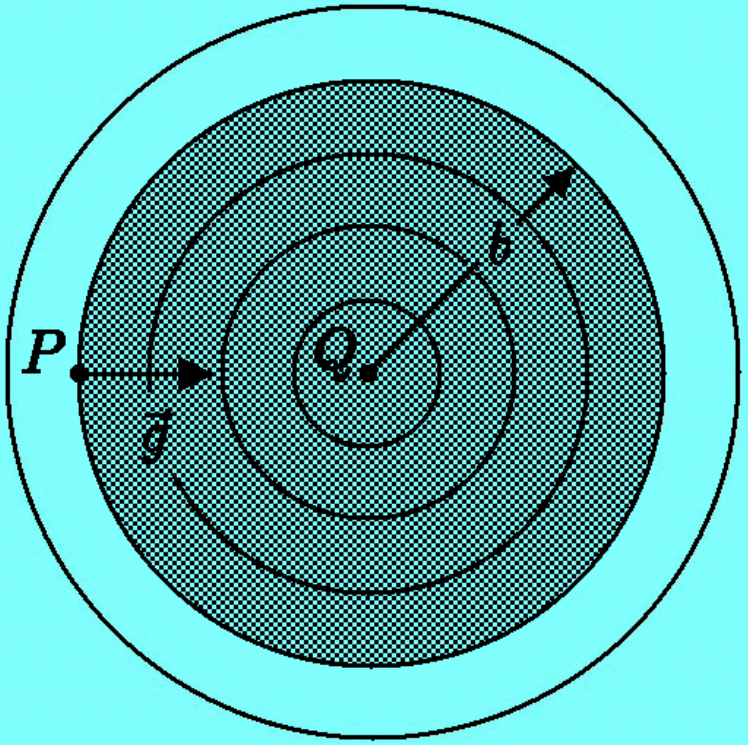

$$\vec{g} = \frac{GM}{b^2}\hat{e}_{QP}$$

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- In the absence of an inertial frame, all accelerations, like velocities, are relative.
- When all accelerations are relative, any observer can consider herself to be non-accelerating. She would then see all other objects accelerating radially toward herself. Like the velocities of Hubble expansion, this picture looks like it has a unique center, but really it is homogenous.

Mathematical Model