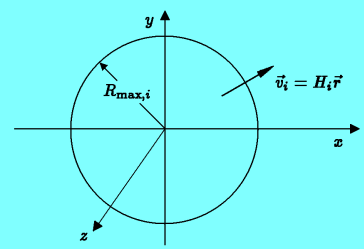
8.286 Lecture 7
September 26, 2013

DYNAMICS OF HOMOGENEOUS EXPANSION, PART III

Summary: Mathematical Model



$$t_i \equiv \text{time of initial picture}$$
$$R_{\max,i} \equiv \text{initial maximum radius}$$
$$\rho_i \equiv \text{initial mass density}$$
$$\vec{v}_i = H_i \vec{r}\ .$$

- Newtonian gravity of a shell:
  - Inside: $\vec{g} = 0$.
  - Outside: Same as point mass at center, with same M.
- Let $r(r_i, t) \equiv$ radius at t of shell initially at r_i.
- Let $M(r_i) \equiv$ mass inside r_i-shell $= \frac{4\pi}{3} r_i^3 \rho_i$ at all times.
- $\vec{g} = -\dfrac{GM(r_i)}{r^2}\hat{r} \quad \Longrightarrow \quad \ddot{r} = -\dfrac{4\pi}{3}\dfrac{Gr_i^3\rho_i}{r^2}$, where $r \equiv r(r_i, t)$.
- Initial conditions: $r(r_i, t_i) = r_i\ , \quad \dot{r}(r_i, t_i) = H_i r_i\ .$
- Rescaling: Let $u(r_i, t) \equiv \dfrac{r(r_i, t)}{r_i} \equiv a(t)$, where $r = a(t) r_i$ and

$$\ddot{a} = -\frac{4\pi}{3}\frac{G\rho_i}{a^2}\ , \quad a(t_i) = 1\ , \quad \dot{a}(t_i) = H_i\ ,$$

and

$$\ddot{a} = -\frac{4\pi}{3} G\rho(t) a\ .$$

Summary: A Conservation Law

$$\ddot{a} = -\frac{4\pi}{3}\frac{G\rho_i}{a^2} \quad \Longrightarrow \quad \dot{a}\left\{\ddot{a} + \frac{4\pi}{3}\frac{G\rho_i}{a^2}\right\} = 0 \quad \Longrightarrow \quad \frac{dE}{dt} = 0\ ,$$

where

$$E = \frac{1}{2}\dot{a}^2 - \frac{4\pi}{3}\frac{G\rho_i}{a}\ .$$