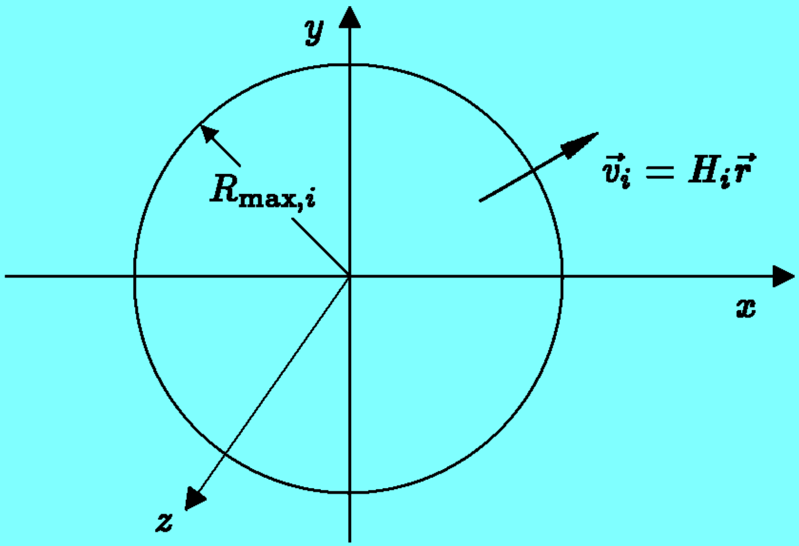
*8.286 Lecture 7*
*September 26, 2013*

# DYNAMICS OF HOMOGENEOUS EXPANSION, PART III

# Summary: Mathematical Model




$$t_i \equiv \text{time of initial picture}$$

$$R_{\max,i} \equiv \text{initial maximum radius}$$

$$\rho_i \equiv \text{initial mass density}$$

$$\vec{v}_i = H_i\vec{r}\ .$$

☆ Newtonian gravity of a shell:

Inside: $\vec{g} = 0$.

Outside: Same as point mass at center, with same $M$.

☆ Let $r(r_i, t) \equiv$ radius at $t$ of shell initially at $r_i$.

☆ Let $M(r_i) \equiv$ mass inside $r_i$-shell $= \frac{4\pi}{3} r_i^3 \rho_i$ at all times.

☆ $$\vec{g} = -\frac{GM(r_i)}{r^2}\hat{r} \quad \Longrightarrow \quad \ddot{r} = -\frac{4\pi}{3}\frac{G r_i^3 \rho_i}{r^2} \ , \text{ where } r \equiv r(r_i, t).$$

☆ Initial conditions: $r(r_i, t_i) = r_i \ , \quad \dot{r}(r_i, t_i) = H_i r_i \ .$

☆ Rescaling: Let $u(r_i, t) \equiv \dfrac{r(r_i, t)}{r_i} \equiv a(t)$ , where $r = a(t) r_i$ and

$$\boxed{\ddot{a} = -\frac{4\pi}{3}\frac{G\rho_i}{a^2} \ , \quad a(t_i) = 1 \ , \quad \dot{a}(t_i) = H_i \ ,} \quad \text{and}$$

$$\boxed{\ddot{a} = -\frac{4\pi}{3} G\rho(t) a \ .}$$

# Summary: A Conservation Law

$$\ddot{a} = -\frac{4\pi}{3}\frac{G\rho_i}{a^2} \quad \Longrightarrow \quad \dot{a}\left\{\ddot{a} + \frac{4\pi}{3}\frac{G\rho_i}{a^2}\right\} = 0 \quad \Longrightarrow \quad \frac{dE}{dt} = 0 \ ,$$

where

$$E = \frac{1}{2}\dot{a}^2 - \frac{4\pi}{3}\frac{G\rho_i}{a} \ .$$