8.286 Lecture 9
October 8, 2013

DYNAMICS OF HOMOGENEOUS EXPANSION, PART V (LAST!)

Summary of Lecture 8

Age of a Flat Matter-Dominated Universe:

$$a(t) \propto t^{2/3} \quad \Longrightarrow \quad t = \frac{2}{3} H^{-1}$$

For $H = 67.3 \pm 1.2$ km-s^{-1}-Mpc^{-1}, age $= 9.5 - 9.9$ billion years — but stars are older. Conclusion: our universe is nearly flat, but not matter-dominated.

The Big Bang Singularity: $a(0) = 0$, with infinite density, is a feature of our model, but not necessarily the real universe.

Horizon Distance: the present distance of the furthest particles from which light has had time to reach us.

$$a(t) \propto t^{2/3} \quad \Longrightarrow \quad \ell_{p,\text{horizon}} = 3ct = 2cH^{-1} \; .$$

Evolution of a Closed Universe:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} \; , \quad \rho(t) a^3(t) = \text{constant} \; .$$

New variables:

$$\tilde{a}(t) \equiv \frac{a(t)}{\sqrt{k}} \; , \qquad \tilde{t} \equiv ct \qquad \text{(both with units of distance)}$$

$$\left(\frac{d\tilde{a}}{d\tilde{t}}\right)^2 = \frac{2\alpha}{\tilde{a}} - 1 \text{ where } \quad \alpha = \frac{4\pi}{3} \frac{G\rho\tilde{a}^3}{c^2} = \text{constant} \; .$$

$$\left(\frac{d\tilde{a}}{d\tilde{t}}\right)^2 = \frac{2\alpha}{\tilde{a}} - 1 \quad \Longrightarrow \quad d\tilde{t} = \frac{\tilde{a}\, d\tilde{a}}{\sqrt{2\alpha\tilde{a} - \tilde{a}^2}} \; .$$

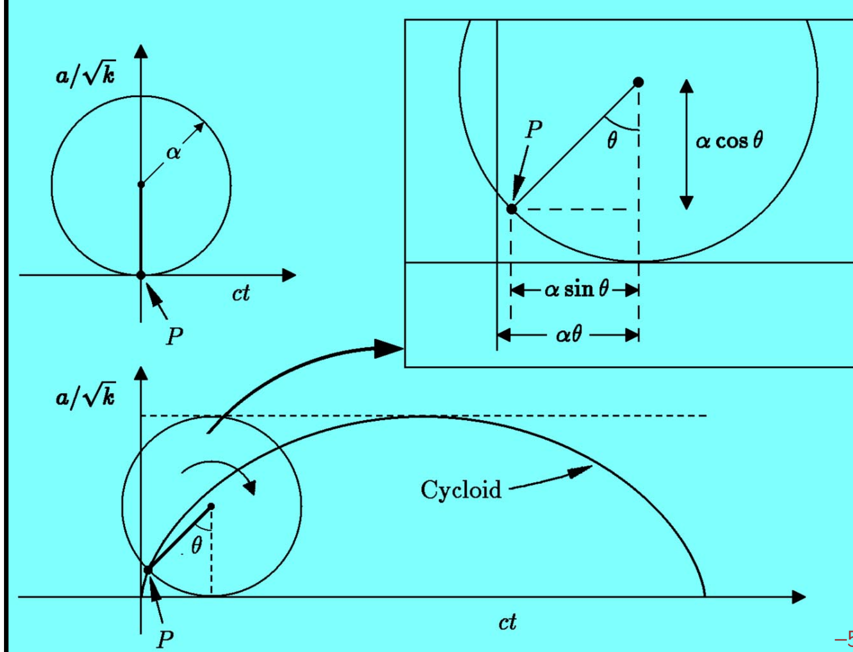
Then

$$\tilde{t}_f = \int_0^{\tilde{t}_f} d\tilde{t} = \int_0^{\tilde{a}_f} \frac{\tilde{a}\, d\tilde{a}}{\sqrt{2\alpha\tilde{a} - \tilde{a}^2}} \; ,$$

where $\tilde{t}_f$ is an arbitrary choice for a "final time" for the calculation, and $\tilde{a}_f$ is the value of $\tilde{a}$ at time $\tilde{t}_f$.

Evolution of a Closed Universe

$$ct = \alpha(\theta - \sin\theta) \ ,$$
$$\frac{a}{\sqrt{k}} = \alpha(1 - \cos\theta) \ .$$



Evolution of a Closed Universe

$$ct = \alpha(\theta - \sin\theta) \ ,$$
$$\frac{a}{\sqrt{k}} = \alpha(1 - \cos\theta) \ .$$

$$t = \frac{\Omega}{2|H|(\Omega - 1)^{3/2}} \left\{ \arcsin\left(\pm \frac{2\sqrt{\Omega - 1}}{\Omega} \right) \mp \frac{2\sqrt{\Omega - 1}}{\Omega} \right\} \ .$$

$$t = \frac{\Omega}{2|H|(\Omega - 1)^{3/2}} \left\{ \arcsin\left(\pm \frac{2\sqrt{\Omega - 1}}{\Omega} \right) \mp \frac{2\sqrt{\Omega - 1}}{\Omega} \right\} \ .$$

Quadrant	Phase	Ω	Sign Choice	$\sin^{-1}()$
1	Expanding	1 to 2	Upper	0 to $\frac{\pi}{2}$
2	Expanding	2 to ∞	Upper	$\frac{\pi}{2}$ to π
3	Contracting	∞ to 2	Lower	π to $\frac{3\pi}{2}$
4	Contracting	2 to 1	Lower	$\frac{3\pi}{2}$ to 2π