

Summary of Lecture 15: Relativistic Four-Momentum

$$p^0 = \frac{E}{c}$$
,  $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$ .

 $p^{\mu}$  transforms from one inertial frame to another in the same way as  $x^{\mu} = (ct, \vec{x})$ .

$$\vec{p} = \gamma m_0 \vec{v}$$
,  $E = \gamma m_0 c^2 = \sqrt{(m_0 c^2)^2 + |\vec{p}|^2 c^2}$ ,

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \ .$$

Lorentz-invariant square of  $p^{\mu}$ :

$$p^2 \equiv |\vec{p}|^2 - (p^0)^2 = |\vec{p}|^2 - \frac{E^2}{c^2} = -(m_0 c)^2$$
.

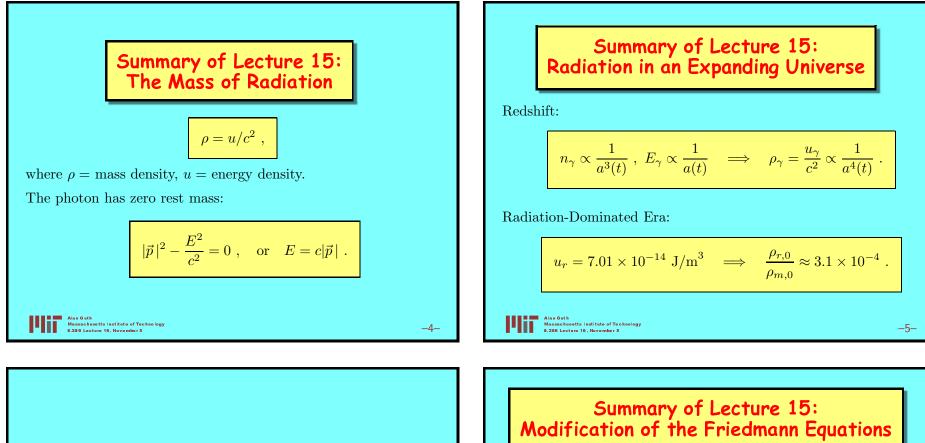
The Hydrogen Atom:

$$m_H = m_p + m_e - \Delta E/c^2 ,$$

where  $m_H$ ,  $m_p$ , and  $m_e$  are the masses of the hydrogen atom, the proton, and the electron, and  $\Delta E = 13.6$  eV is the binding energy.

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$$\frac{\rho_r(t)}{\rho_m(t)} = \frac{a(t_0)}{a(t)} \times 3.1 \times 10^{-4} ,$$

$$\frac{\rho_r(t_{\rm eq})}{\rho_m(t_{\rm eq})} \equiv 1 \quad \Longrightarrow \quad \frac{a(t_0)}{a(t_{\rm eq})} = \frac{1}{3.1 \times 10^{-4}} \approx 3200 \ . \label{eq:rho}$$

Assuming that  $a(t) \propto t^{2/3}$  from  $t_{\rm eq}$  until  $t_0$  (crude approximation), find  $t_{\rm eq} = (3.1 \times 10^{-4})^{3/2} t_0 \approx 75,000$  yr, for  $t_0 = 13.8$  Gyr.

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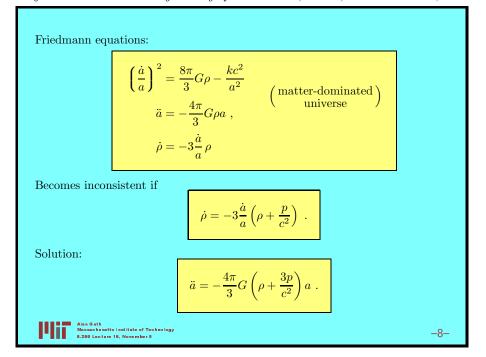
Modification of the Friedmann Equations  $\rho \propto \frac{1}{a^3} \implies \dot{\rho} = -3\frac{\dot{a}}{a}\rho, \quad \rho(t) \propto \frac{1}{a^4(t)} \implies \dot{\rho} = -4\frac{\dot{a}}{a}\rho.$ 

 $\dot{
ho}$  and pressure p:

$$\begin{split} dU &= -p \, dV \quad \Longrightarrow \quad \frac{d}{dt} \left( a^3 \rho c^2 \right) = -p \frac{d}{dt} (a^3) \\ &\implies \quad \dot{\rho} = -3 \frac{\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right) \; . \end{split}$$

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Alan Guth, Black-Body Radiation and the Early History of the Universe, Part 2, 8.286 Lecture 16, November 5, 2013, p. 3.