

Summary of Lecture 15: Relativistic Four-Momentum

$$p^0 = \frac{E}{c}$$
, $p^\mu = \left(\frac{E}{c}, \vec{p}\right)$.

 p^{μ} transforms from one inertial frame to another in the same way as $x^{\mu} = (ct, \vec{x})$.

$$\vec{p} = \gamma m_0 \vec{v}$$
, $E = \gamma m_0 c^2 = \sqrt{(m_0 c^2)^2 + |\vec{p}|^2 c^2}$,

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \ .$$

Lorentz-invariant square of p^{μ} :

$$p^2 \equiv |\vec{p}|^2 - (p^0)^2 = |\vec{p}|^2 - \frac{E^2}{c^2} = -(m_0 c)^2$$
.

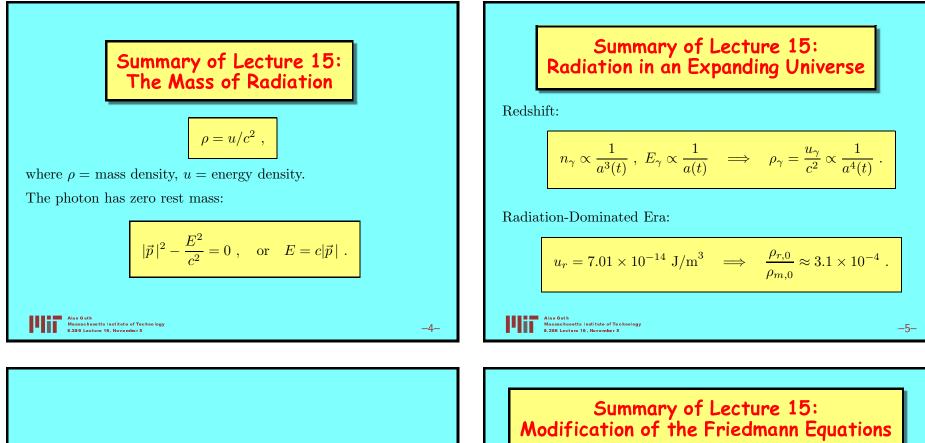
The Hydrogen Atom:

$$m_H = m_p + m_e - \Delta E/c^2 ,$$

where m_H , m_p , and m_e are the masses of the hydrogen atom, the proton, and the electron, and $\Delta E = 13.6$ eV is the binding energy.

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$$\frac{\rho_r(t)}{\rho_m(t)} = \frac{a(t_0)}{a(t)} \times 3.1 \times 10^{-4} ,$$

$$\frac{\rho_r(t_{\rm eq})}{\rho_m(t_{\rm eq})} \equiv 1 \quad \Longrightarrow \quad \frac{a(t_0)}{a(t_{\rm eq})} = \frac{1}{3.1 \times 10^{-4}} \approx 3200 \ . \label{eq:rho}$$

Assuming that $a(t) \propto t^{2/3}$ from $t_{\rm eq}$ until t_0 (crude approximation), find $t_{\rm eq} = (3.1 \times 10^{-4})^{3/2} t_0 \approx 75,000$ yr, for $t_0 = 13.8$ Gyr.

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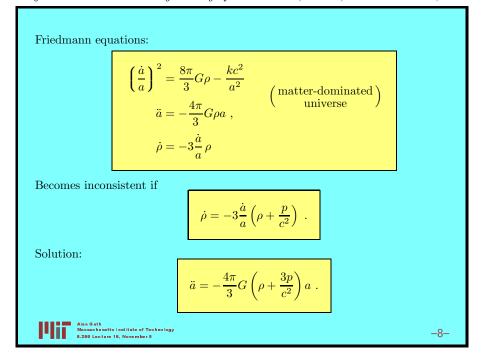
Modification of the Friedmann Equations $\rho \propto \frac{1}{a^3} \implies \dot{\rho} = -3\frac{\dot{a}}{a}\rho, \quad \rho(t) \propto \frac{1}{a^4(t)} \implies \dot{\rho} = -4\frac{\dot{a}}{a}\rho.$

 $\dot{
ho}$ and pressure p:

$$\begin{split} dU &= -p \, dV \quad \Longrightarrow \quad \frac{d}{dt} \left(a^3 \rho c^2 \right) = -p \frac{d}{dt} (a^3) \\ &\implies \quad \dot{\rho} = -3 \frac{\dot{a}}{a} \left(\rho + \frac{p}{c^2} \right) \; . \end{split}$$

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Alan Guth, Black-Body Radiation and the Early History of the Universe, Part 2, 8.286 Lecture 16, November 5, 2013, p. 3.