8.286 Lecture 19
November 19, 2013

THE COSMOLOGICAL CONSTANT
CMB Data in 1975
Data from Berkeley-Nagoya Rocket Flight, 1987
A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE COSMIC BACKGROUND EXPLORER (COBE) SATELLITE

J.C. Mather, E. S. Cheng, R. E. Epke, R. B. Isaacman, S. S. Meyer,
R. A. Shafer, R. Weiss, E. L. Wright, C. L. Bennett, N. W. Boggess,
E. Dwek, S. Galikis, M. G. Hauser, M. Janssen, T. Kellett, P. M. Lubin,
S. H. Moseley, Jr., T. L. Murdoch, F. Silverberg, G. F. Smoot,
and D. T. Wilkinson.
Original COBE Measurement of the CMB Spectrum, Jan 1990. Energy density is in units of electron volts per cubic meter per gigahertz.
Summary of Lecture 18:
Gravitational Effect of Pressure

\[
\frac{d^2a}{dt^2} = -\frac{4\pi}{3} G \left( \rho + \frac{3p}{c^2} \right) a .
\]
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Gravitational Effect of Pressure

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\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G \left( \rho + \frac{3p}{c^2} \right) a.
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Vacuum Energy and the Cosmological Constant:

\[
\mathcal{U}_{\text{vac}} = \rho_{\text{vac}}c^2 = \frac{\Lambda c^4}{8\pi G}.
\]
Summary of Lecture 18:
Gravitational Effect of Pressure

\[
\frac{d^2 a}{dt^2} = -\frac{4\pi}{3} G \left( \rho + \frac{3p}{c^2} \right) a .
\]

Vacuum Energy and the Cosmological Constant:

\[
u_{\text{vac}} = \rho_{\text{vac}} c^2 = \frac{\Lambda c^4}{8\pi G} .
\]

\[
\dot{\rho}_{\text{vac}} = 0 \implies p_{\text{vac}} = -\rho_{\text{vac}} c^2 = -\frac{\Lambda c^4}{8\pi G} .
\]
Defining \( \rho = \rho_n + \rho_{\text{vac}} \) and \( p = p_n + p_{\text{vac}} \),

\[
\frac{d^2 a}{dt^2} = -\frac{4\pi}{3}G \left( \rho_n + \frac{3p_n}{c^2} - 2\rho_{\text{vac}} \right) a. 
\]

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G (\rho_n + \rho_{\text{vac}}) - \frac{k c^2}{a^2}. 
\]
Defining $\rho = \rho_n + \rho_{\text{vac}}$ and $p = p_n + p_{\text{vac}}$, 

\[
\frac{d^2a}{dt^2} = - \frac{4\pi}{3} G \left( \rho_n + \frac{3p_n}{c^2} - 2\rho_{\text{vac}} \right) a.
\]

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G (\rho_n + \rho_{\text{vac}}) - \frac{k c^2}{a^2}.
\]

Dominance of vacuum energy at late time implies

\[
H \rightarrow H_{\text{vac}} = \sqrt{\frac{8\pi}{3} G \rho_{\text{vac}}},
\]

\[
a(t) \propto e^{H_{\text{vac}}t}.
\]
The age of an open ($\Omega < 1$), closed ($\Omega > 1$), or flat ($\Omega = 1$) universe containing only nonrelativistic matter.
The age of a flat universe containing nonrelativistic matter and vacuum energy.