8.286 Lecture 20
November 21, 2013

SUPERNOVAE Ia
and
VACUUM ENERGY DENSITY

Summary of Lecture 19: The Age of the Universe

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\,(\underbrace{\rho_m}_{\propto\frac{1}{a^3(t)}} + \underbrace{\rho_{\rm rad}}_{\propto\frac{1}{a^4(t)}} + \rho_{\rm vac}) - \frac{kc^2}{a^2}\ .$$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left(\frac{\Omega_{m,0}}{x^3} + \frac{\Omega_{{\rm rad},0}}{x^4} + \Omega_{\rm vac}\right) - \frac{kc^2}{a^2}\ ,$$

where $x \equiv a(t)/a(t_0)$.

Define

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0)H_0^2}\ .$$

Summary of Lecture 19

$$\left(\frac{\dot a}{a}\right)^2 = \left(\frac{\dot x}{x}\right)^2 = \frac{H_0^2}{x^4}\left(\Omega_{m,0}x + \Omega_{{\rm rad},0} + \Omega_{{\rm vac},0}x^4 + \Omega_{k,0}x^2\right)\ .$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{{\rm rad},0} - \Omega_{{\rm vac},0}\ .$$

Finally,

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0}x + \Omega_{{\rm rad},0} + \Omega_{{\rm vac},0}x^4 + \Omega_{k,0}x^2}}\ .$$

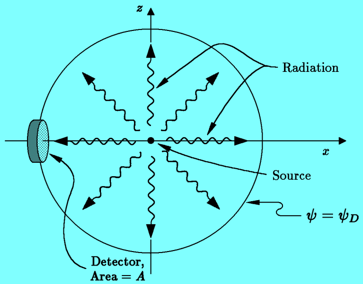
Summary of Lecture 19: Radiation Flux versus Redshift

Closed Universe:

$$ds^2 = -c^2\,dt^2 + \tilde a^2(t)\left\{d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\}\ ,$$

where $\sin\psi \equiv \sqrt{k}\,r$.

Summary of Lecture 19



$$\text{fraction} = \frac{\text{area of detector}}{\text{area of sphere}} = \frac{A}{4\pi\tilde{a}^2(t_0)\sin^2\psi_D} .$$

Power suppressed by two factors of $(1+z_S)$: one for redshift of photons, one for rate of arrival of photons.

$$J = \frac{P_{\text{received}}}{A} = \frac{P}{4\pi(1+z_S)^2\tilde{a}^2(t_0)\sin^2\psi_D} .$$

Summary of Lecture 19

$$\tilde{a}(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} .$$

$$0 = -c^2\,dt^2 + \tilde{a}^2(t)\,d\psi^2 \quad\Longrightarrow\quad \frac{d\psi}{dt} = \frac{c}{\tilde{a}(t)} .$$

$$\psi(z_S) = \int_{t_S}^{t_0} \frac{c}{\tilde{a}(t)}\,dt .$$

$$\psi(z_S) = \frac{1}{\tilde{a}(t_0)}\int_0^{z_S} \frac{c}{H(z)}\,dz .$$

$$\psi(z_S) = \sqrt{|\Omega_{k,0}|} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\text{rad},0}(1+z)^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1+z)^2}} .$$

Summary of Lecture 19

$$J = \frac{PH_0^2|\Omega_{k,0}|}{4\pi(1+z_S)^2c^2\sin^2\psi(z_S)} ,$$