\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G \left( \frac{\rho_m}{a(t)} + \frac{\rho_{\text{rad}}}{a(t)^4} + \rho_{\text{vac}} \right) - \frac{k c^2}{a^2}.
\]

Define
\[
\Omega_{k,0} \equiv - \frac{k c^2}{a^2(t_0) H_0^2}.
\]

Finally,
\[
\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\text{rad,0}} - \Omega_{\text{vac,0}}.
\]

Closed Universe:
\[
ds^2 = -c^2 \, dt^2 + \bar{a}^2(t) \left\{ d\psi^2 + \sin^2 \psi \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right\},
\]
where \( \sin \psi \equiv \sqrt{k} \, r \).
fraction = \frac{\text{area of detector}}{\text{area of sphere}} = \frac{A}{4\pi \bar{a}^2(t_0) \sin^2 \psi_D}.

Power suppressed by two factors of \((1 + z_S)\): one for redshift of photons, one for rate of arrival of photons.

\[ J = \frac{P_{\text{received}}}{A} = \frac{P}{4\pi(1 + z_S)^2 \bar{a}^2(t_0) \sin^2 \psi_D}, \]

\[ \bar{a}(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} \]

\[ 0 = -c^2 dt^2 + \bar{a}^2(t) d\psi^2 \quad \Rightarrow \quad \frac{d\psi}{dt} = \frac{c}{\bar{a}(t)} \]

\[ \psi(z_S) = \int_{z_S}^{t_0} \frac{c}{\bar{a}(t)} dt \]

\[ \psi(z_S) = \frac{1}{\bar{a}(t_0)} \int_0^{z_S} \frac{c}{H(z)} dz \]

\[ \psi(z_S) = \sqrt{\Omega_{k,0}} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_m,0(1 + z) + \Omega_{\text{rad},0}(1 + z)^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1 + z)^2}}. \]