*8.286 Lecture 20*
*November 21, 2013*

# SUPERNOVAE Ia
# and
# VACUUM ENERGY DENSITY

# Summary of Lecture 19: The Age of the Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\,(\underbrace{\rho_m}_{\propto\frac{1}{a^3(t)}} + \underbrace{\rho_{\rm rad}}_{\propto\frac{1}{a^4(t)}} + \rho_{\rm vac}) - \frac{kc^2}{a^2}\ .$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{\Omega_{m,0}}{x^3} + \frac{\Omega_{\rm rad,0}}{x^4} + \Omega_{\rm vac}\right) - \frac{kc^2}{a^2}\ ,$$

where $x \equiv a(t)/a(t_0)$ .

Define

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0)H_0^2}\ .$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{x}}{x}\right)^2 = \frac{H_0^2}{x^4}\left(\Omega_{m,0}x + \Omega_{\mathrm{rad},0} + \Omega_{\mathrm{vac},0}x^4 + \Omega_{k,0}x^2\right) \; .$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\mathrm{rad},0} - \Omega_{\mathrm{vac},0} \; .$$

Finally,

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0}x + \Omega_{\mathrm{rad},0} + \Omega_{\mathrm{vac},0}x^4 + \Omega_{k,0}x^2}} \; .$$

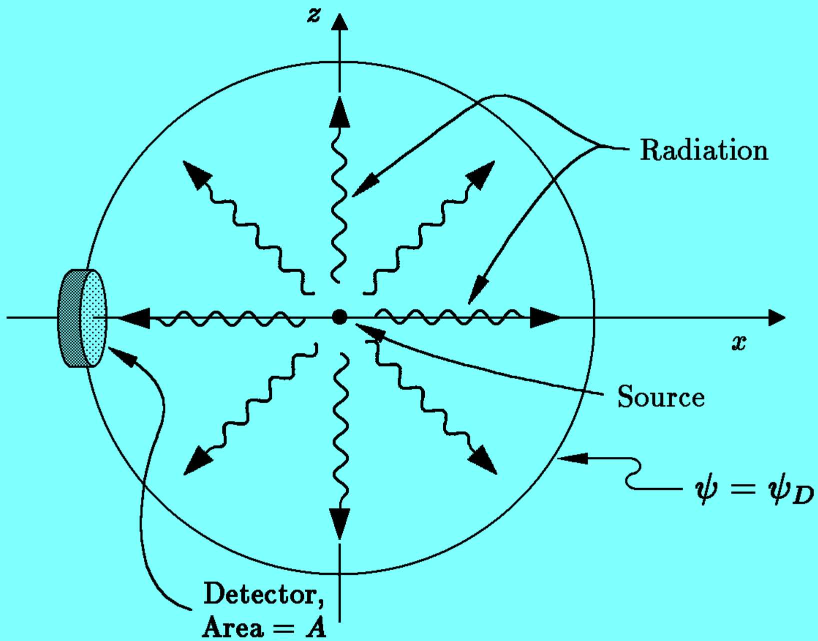
## Summary of Lecture 19:

# Radiation Flux versus Redshift

Closed Universe:

$$ds^2 = -c^2\,dt^2 + \tilde{a}^2(t)\left\{d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} \; ,$$

where $\sin\psi \equiv \sqrt{k}\,r$ .




$$\text{fraction} = \frac{\text{area of detector}}{\text{area of sphere}} = \frac{A}{4\pi\tilde{a}^2(t_0)\sin^2\psi_D} \; .$$

Power suppressed by two factors of $(1 + z_S)$: one for redshift of photons, one for rate of arrival of photons.

$$J = \frac{P_{\text{received}}}{A} = \frac{P}{4\pi(1+z_S)^2\tilde{a}^2(t_0)\sin^2\psi_D} \; .$$

$$\tilde{a}(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} \ .$$

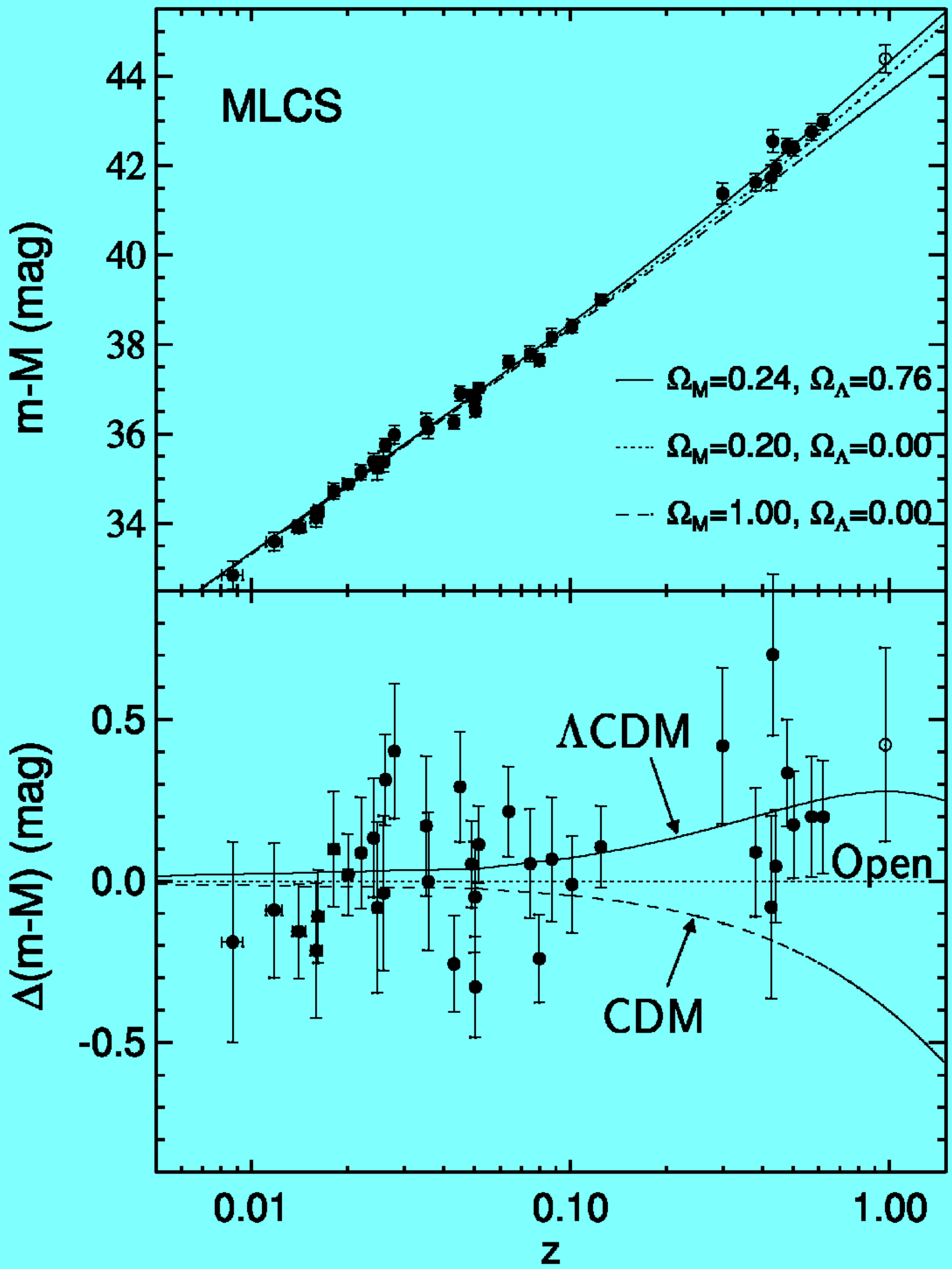
$$0 = -c^2\, dt^2 + \tilde{a}^2(t)\, d\psi^2 \quad \Longrightarrow \quad \frac{d\psi}{dt} = \frac{c}{\tilde{a}(t)} \ .$$

$$\psi(z_S) = \int_{t_S}^{t_0} \frac{c}{\tilde{a}(t)}\, dt \ .$$

$$\psi(z_S) = \frac{1}{\tilde{a}(t_0)} \int_0^{z_S} \frac{c}{H(z)}\, dz \ .$$

$$\psi(z_S) = \sqrt{|\Omega_{k,0}|} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\mathrm{rad},0}(1+z)^4 + \Omega_{\mathrm{vac},0} + \Omega_{k,0}(1+z)^2}} \ .$$

$$J = \frac{P H_0^2 |\Omega_{k,0}|}{4\pi(1+z_S)^2 c^2 \sin^2 \psi(z_S)} \ ,$$




Alan Guth
Massachusetts Institute of Technology