8.286 Lecture 6
September 26, 2016

THE DYNAMICS
OF
NEWTONIAN COSMOLOGY,
PART 1

Isaac Newton to Richard Bentley, Letter 1
Newton on the Infinite Universe

As to your first query, it seems to me that if the matter of our sun and planets and all the matter of the universe were evenly scattered throughout all the heavens, and every particle had an innate gravity toward all the rest, and the whole space throughout which this matter was scattered was but finite, the matter on the outside of this space would, by its gravity, tend toward all the matter on the inside and, by consequence, fall down into the middle of the whole space and there compose one great spherical mass. But if the matter was evenly disposed throughout an infinite space, it could never convene into one mass; but some of it would convene into one mass and some into another, so as to make an infinite number of great masses, scattered at great distances from one to another throughout all that infinite space. And thus might the sun and fixed stars be formed, supposing the matter were of a lucid nature.

— December 10, 1692

But how the matter should divide itself into two sorts, and that part of it which is to compose a shining body should fall down into one mass and make a sun and the rest which is fit to compose an opaque body should coalesce, not into one great body, like the shining matter, but into many little ones; or if the sun at first were an opaque body like the planets or the planets lucid bodies like the sun, how he alone should be changed into a shining body whilst all they continue opaque, or all they be changed into opaque ones whilst he remains unchanged, I do not think explicable by mere natural causes, but am forced to ascribe it to the counsel and contrivance of a voluntary Agent.

— December 10, 1692

Isaac Newton to Richard Bentley, Letter 2
Newton on Infinities

But you argue, in the next paragraph of your letter, that every particle of matter in an infinite space has an infinite quantity of matter on all sides, and, by consequence, an infinite attraction every way, and therefore must rest in equilibrio, because all infinites are equal. Yet you suspect a paralogism in this argument; and I conceive the paralogism lies in the position, that all infinites are equal. The generality of mankind consider infinites no other ways than indefinitely; and in this sense they say all infinites are equal; though they would speak more truly if they should say, they are neither equal nor unequal, nor have any certain difference or proportion one to another. In this sense, therefore, no conclusions can be drawn from them about the equality, proportions, or differences of things; and they that attempt to do it usually fall into paralogisms.

— January 17, 1693

So, when men argue against the infinite divisibility of magnitude, by saying, that if an inch may be divided into an infinite number of parts, the sum of those parts will be an inch; and if a foot may be divided into an infinite number of parts, the sum of those parts must be a foot; and therefore, since all infinites are equal, those sums must be equal, that is, an inch equal to a foot. The falseness of the conclusion shews an error in the premises ; and the error lies in the position, that all infinites are equal.

— January 17, 1693

Web references: http://www.newtonproject.sussex.ac.uk/view/texts/normalized/THEM00255
http://books.google.com/books?id=8DkCAAAAQAAJ&pg=PA201

Can a Uniform Infinite Distribution of Mass Be Stable?

Gauss's Law of Gravity:

$$\vec{g} = -\frac{GM}{r^2}\hat{r} \quad \Longrightarrow \quad \oint \vec{g} \cdot d\vec{a} = -4\pi G M_{\text{enclosed}}$$

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Poisson's Equation:

$$\nabla^2 \phi = 4\pi G \rho, \quad \text{where } \vec{g} = -\vec{\nabla}\phi.$$

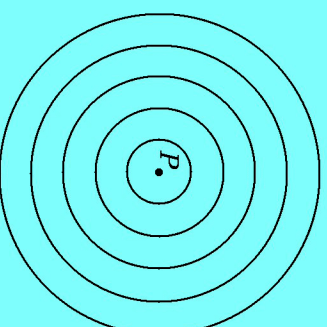where ρ is the mass density, $\vec{\nabla}\phi$ is the gradient of ϕ:

$$\vec{\nabla}\phi \equiv \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z},$$
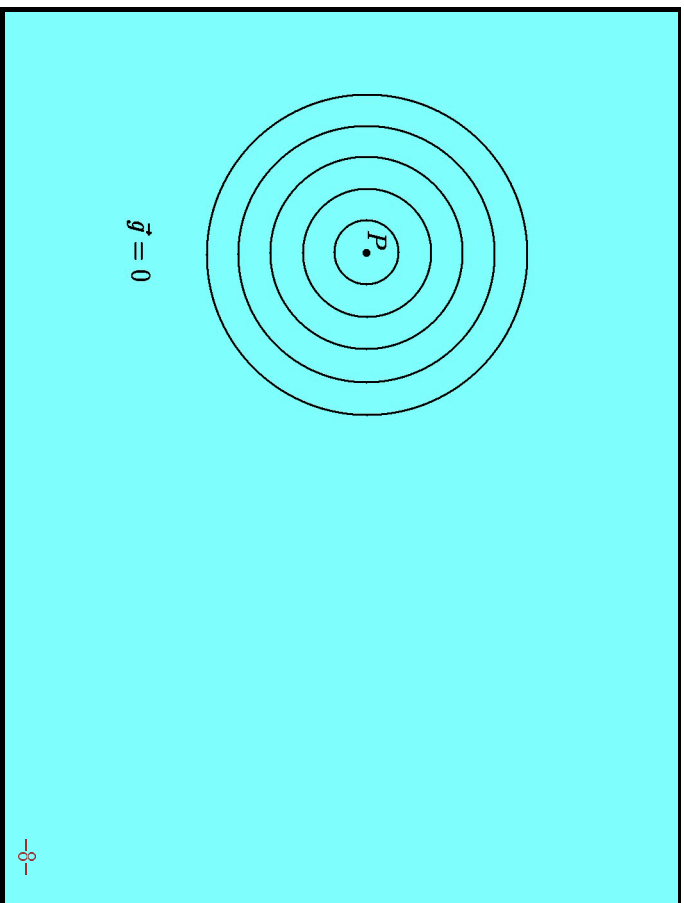
and $\nabla^2 \phi$ is the Laplacian of ϕ:

$$\nabla^2 \phi \equiv \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}.$$
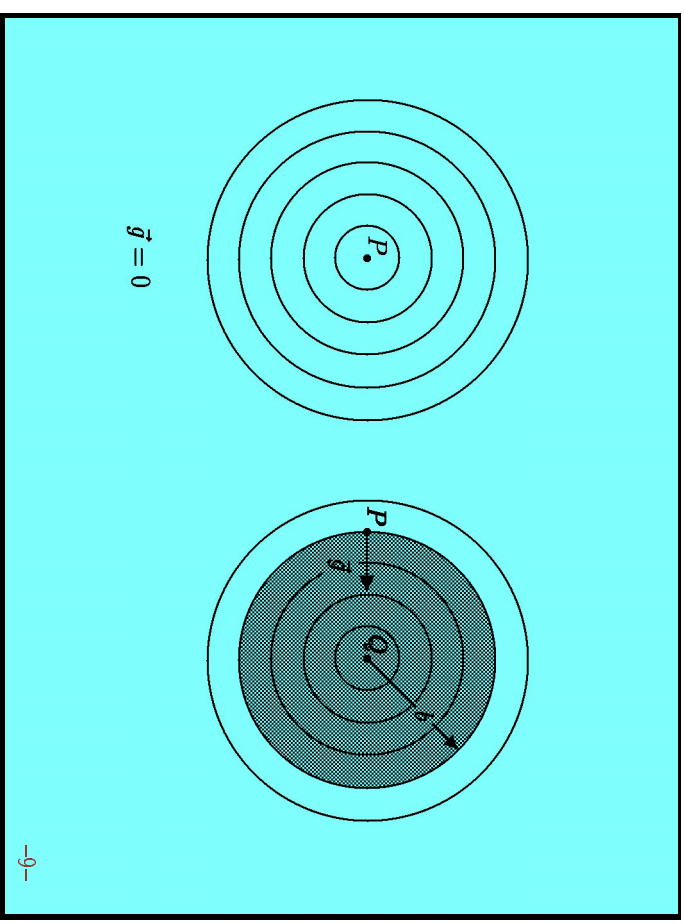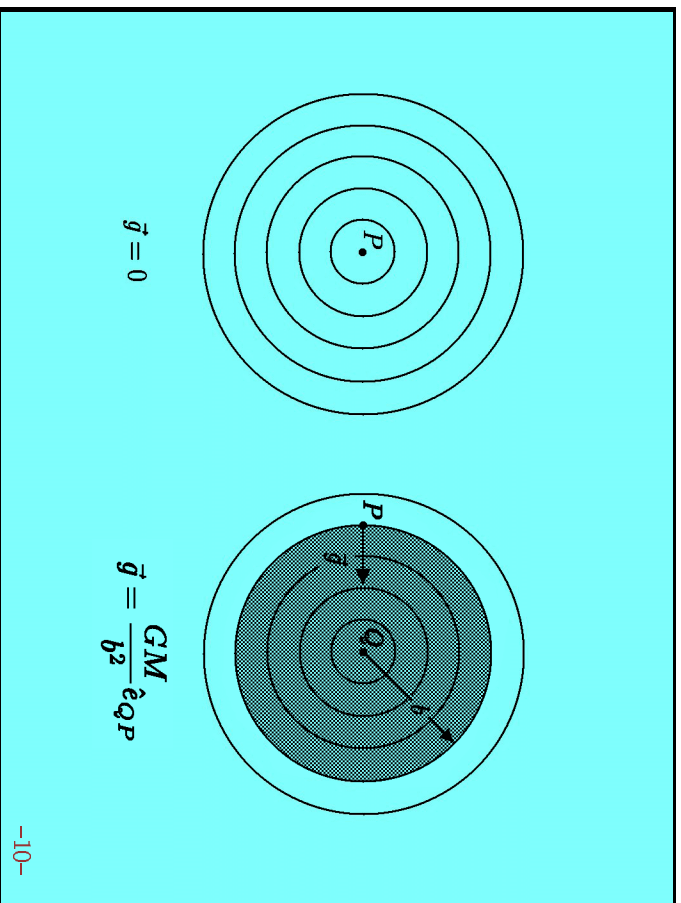
and

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$\vec{g} = 0$

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$\vec{g} = 0$

–9–

$\vec{g} = 0$

$$\vec{g} = \frac{GM}{b^2}\,\hat{e}_{QP}$$

–10–

But What About Symmetry?

✪ Newton argued that there could be no acceleration, because there is no preferred direction for it to point.

✪ Complication: acceleration is measured relative to an inertial frame, which Newton defined as the frame of the "fixed stars". But if the universe collapses, then there are no fixed stars.

✪ In the absence of an inertial frame, all accelerations, like velocities, are relative.

✪ When all accelerations are relative, any observer can consider herself to be non-accelerating. She would then see all other objects accelerating radially toward herself. Like the velocities of Hubble expansion, this picture looks like it has a unique center, but really it is homogenous.