8.286 Lecture 16
November 7, 2018

THE COSMIC MICROWAVE BACKGROUND

Black-Body Radiation

Energy Density:

$$u = g \frac{\pi^2}{30} \frac{(kT)^4}{(\hbar c)^3} \ ,$$

where

$$k = \text{Boltzmann's constant} = 1.381 \times 10^{-16}\,\text{erg/K}$$
$$= 8.617 \times 10^{-5}\,\text{eV/K} \ ,$$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-27}\ \text{erg-sec}$$

$$= 6.582 \times 10^{-16}\ \text{eV-sec} \ ,$$

and

$$g = 2 \quad \text{(for photons)} \ .$$

Pressure and Number Density

Pressure:

$$p = \frac{1}{3} u \ .$$

Number density:

$$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \ ,$$

where ζ is the Riemann zeta function, with

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202.$$

and

$$g^* = 2 \quad \text{(for photons)} \ .$$

Entropy

Entropy density (entropy per unit volume):

$$s = g\frac{2\pi^2}{45}\;\frac{k^4T^3}{(\hbar c)^3}\;.$$

Entropy is a measure of “disorder”. If a system is known to be in one of N quantum states, with equal probability, then its entropy is $k \log_e N$.

To an excellent approximation, entropy is CONSERVED during early universe evolution. Exceptions: ending of inflation, and (much) later the formation of structure.

Neutrinos

For early universe purposes (first three minutes and more), neutrinos can be treated as massless. In this approximation,

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4} \; .$$

$$g^*_\nu = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2} \; .$$

e^+-e^- Pairs

When $kT \gg m_e c^2 \approx 0.511$ MeV, electrons and positrons are produced (in pairs) as if they were massless particles.

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2} \,.$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3 \,.$$

Neutrino Masses

The values of the neutrino masses are not known, but neutrino oscillation experiments tell us the differences of the squares of the masses:

$$\Delta m_{21}^2\, c^4 = (7.53 \pm 0.18) \times 10^{-5}\ \text{eV}^2\ ,$$

$$\Delta m_{32}^2\, c^4 = (2.51 \pm 0.05) \times 10^{-3}\, \text{eV}^2\ ,$$

or

$$\Delta m_{32}^2\, c^4 = (-2.56 \pm 0.04) \times 10^{-3}\, \text{eV}^2\ \text{(inverted)},$$

where the second option for Δm_{32}^2 applies if $m_3^2 < m_2^2$.

Quantum complications: neutrinos are produced as states of definite flavor, ν_e, ν_μ, or ν_τ. But the states of definite mass, ν_1, ν_2, and ν_3, are superpositions of these, with ν_1 being mostly ($\sim 2/3$) ν_e, while the others are more evenly mixed.

Source: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).