8.286 Lecture 16 November 7, 2018

THE COSMIC MICROWAVE BACKGROUND

Black-Body Radiation

Energy Density:

$$u = g \frac{\pi^2}{30} \frac{(kT)^4}{(\hbar c)^3} ,$$

where

 $k = \text{Boltzmann's constant} = 1.381 \times 10^{-16} \text{ erg/K}$ $= 8.617 \times 10^{-5} \text{ eV/K} ,$

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-27} \text{ erg-sec}$$

 $= 6.582 \times 10^{-16} \text{ eV-sec}$,

and

$$g=2$$
 (for photons).

Pressure and Number Density

Pressure:

$$p = \frac{1}{3}u$$
 .

Number density:

$$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} ,$$

where ζ is the Riemann zeta function, with

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \dots \approx 1.202.$$

and

$$g^*=2$$
 (for photons).



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Entropy density (entropy per unit volume):

$$s = g \frac{2\pi^2}{45} \frac{k^4 T^3}{(\hbar c)^3} .$$

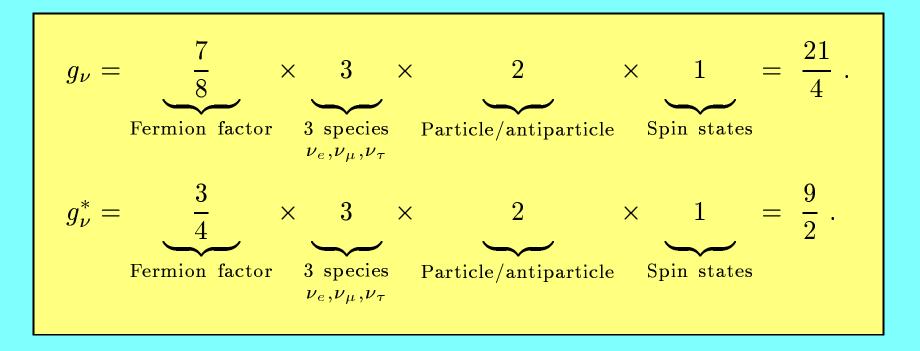
Entropy is a measure of "disorder". If a system is known to be in one of N quantum states, with equal probability, then its entropy is $k \log_e N$.

To an excellent approximation, entropy is CONSERVED during early universe evolution. Exceptions: ending of inflation, and (much) later the formation of structure.





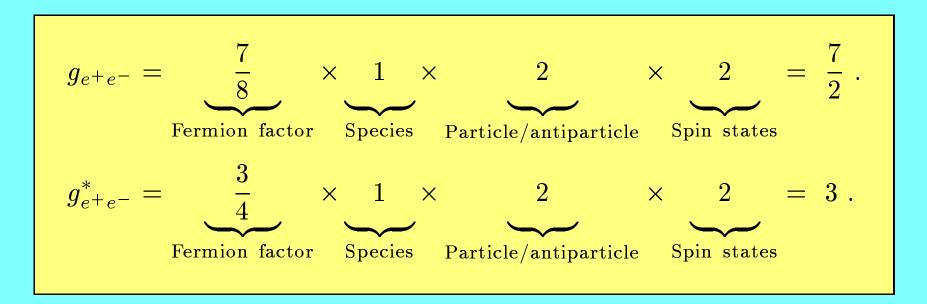
For early universe purposes (first three minutes and more), neutrinos can be treated as massless. In this approximation,







When $kT \gg m_e c^2 \approx 0.511$ MeV, electrons and positrons are produced (in pairs) as if they were massless particles.





Neutrino Masses

The values of the neutrino masses are not known, but neutrino oscillation experiments tell us the differences of the squares of the masses:

$$\Delta m_{21}^2 c^4 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2 ,$$

$$\Delta m_{32}^2 c^4 = (2.51 \pm 0.05) \times 10^{-3} \text{ eV}^2 ,$$

or

$$\Delta m_{32}^2 c^4 = (-2.56 \pm 0.04) \times 10^{-3} \,\mathrm{eV}^2 \,\mathrm{(inverted)},$$

where the second option for Δm_{32}^2 applies if $m_3^2 < m_2^2$.

Quantum complications: neutrinos are produced as states of definite flavor, ν_e , ν_{μ} , or ν_{τ} . But the states of definite mass, ν_1 , ν_2 , and ν_3 , are superpositions of these, with ν_1 being mostly (~ 2/3) ν_e , while the others are more evenly mixed.

Source: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).

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