8.286 Lecture 18
November 19, 2018

THE COSMOLOGICAL CONSTANT

Gravitational Effect of Pressure

$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)a \ .$$

Vacuum Energy and the Cosmological Constant:

$$u_{\rm vac} = \rho_{\rm vac}c^2 = \frac{\Lambda c^4}{8\pi G} \ .$$

$$\dot\rho_{\rm vac} = 0 \quad \Longrightarrow \quad p_{\rm vac} = -\rho_{\rm vac}c^2 = -\frac{\Lambda c^4}{8\pi G} \ .$$

Defining $\rho = \rho_n + \rho_{\rm vac}$ and $p = p_n + p_{\rm vac}$,

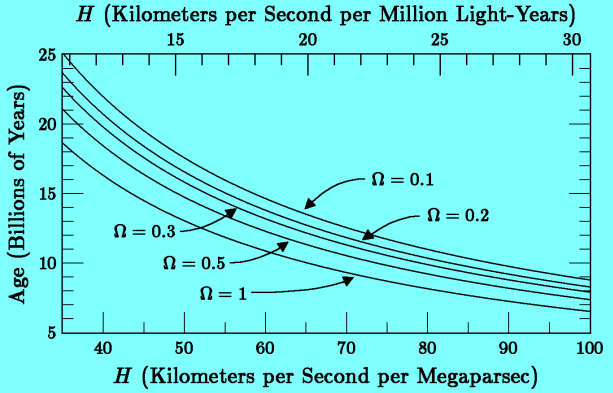
$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G\left(\rho_n + \frac{3p_n}{c^2} - 2\rho_{\rm vac}\right)a \ .$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G(\rho_n + \rho_{\rm vac}) - \frac{kc^2}{a^2} \ .$$

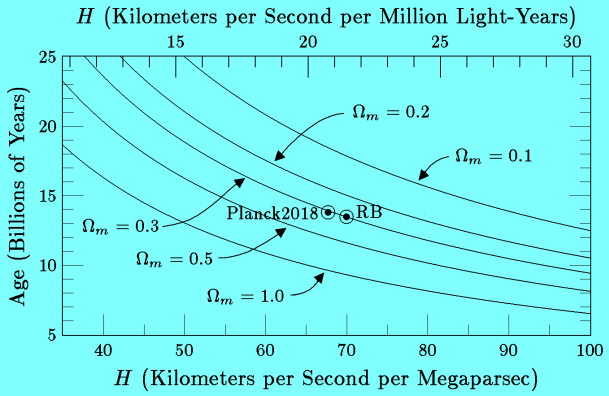
Dominance of vacuum energy at late time implies

$$H \to H_{\rm vac} = \sqrt{\frac{8\pi}{3}G\rho_{\rm vac}} \ ,$$

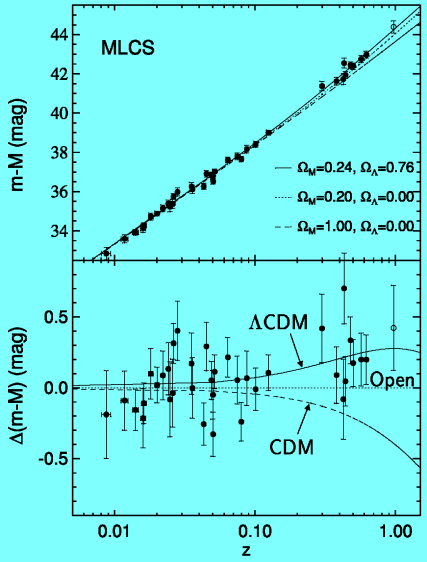
$$a(t) \propto e^{H_{\rm vac}t} \ .$$



The age of an open ($\Omega < 1$), closed ($\Omega > 1$), or flat ($\Omega = 1$) universe containing only nonrelativistic matter.



The age of a flat universe containing nonrelativistic matter and vacuum energy.



Hubble diagram from Riess *et al.*, *Astronomical Journal* **116**, No. 3, 1009 (1998) [http://arXiv.org/abs/astro-ph/9805201].