8.286 Lecture 19
November 21, 2018

THE COSMOLOGICAL CONSTANT, CONTINUED

Summary of Last Lecture: Age of Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\,(\underbrace{\rho_m}_{\propto\frac{1}{a^3(t)}} + \underbrace{\rho_{\rm rad}}_{\propto\frac{1}{a^4(t)}} + \rho_{\rm vac}) - \frac{kc^2}{a^2} \ .$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{\Omega_{m,0}}{x^3} + \frac{\Omega_{\rm rad,0}}{x^4} + \Omega_{\rm vac}\right) - \frac{kc^2}{a^2} \ ,$$

where $x \equiv a(t)/a(t_0)$.

Define

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0)H_0^2} \ .$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{x}}{x}\right)^2 = \frac{H_0^2}{x^4}\left(\Omega_{m,0}x + \Omega_{\mathrm{rad},0} + \Omega_{\mathrm{vac},0}x^4 + \Omega_{k,0}x^2\right) .$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\mathrm{rad},0} - \Omega_{\mathrm{vac},0} .$$

Finally,

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0}x + \Omega_{\mathrm{rad},0} + \Omega_{\mathrm{vac},0}x^4 + \Omega_{k,0}x^2}} .$$

Numerical Integration with Mathematica

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IN: t0[H0_,Ωm0_,Ωrad0_,Ωvac0_,Ωk0_] := (1/H0) *
      NIntegrate[x/Sqrt[Ωm0 x + Ωrad0 + Ωvac0 x^4 + Ωk0 x^2], {x,0,1}]
IN: PlanckH0 := Quantity[67.66,"km/sec/Mpc"]
IN: PlanckΩm0 := 0.311
IN: PlanckΩvac0 := 0.689
IN: UnitConvert[t0[PlanckH0,PlanckΩm0,0,PlanckΩvac0,0],"Years"]
```

OUT: 1.38022×10^{10} years

Radiation Flux versus Redshift

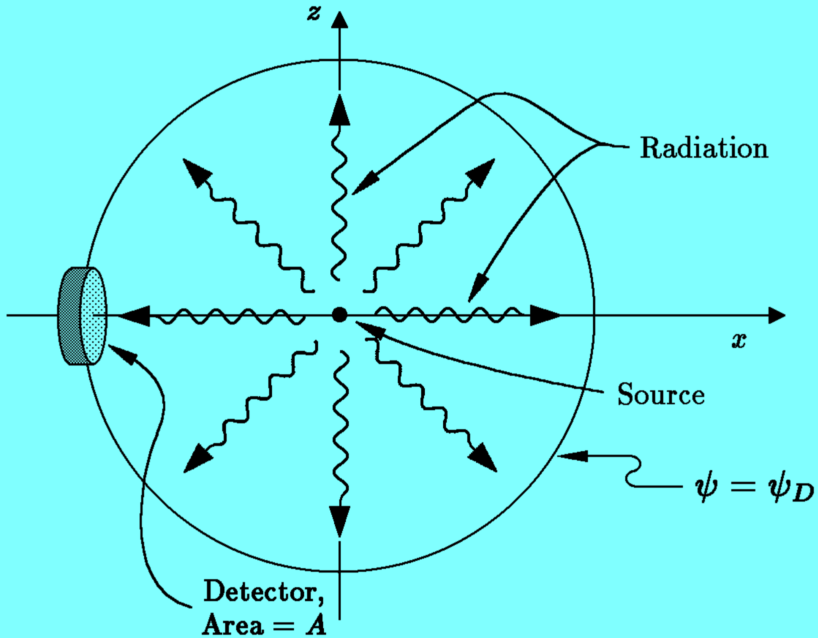
Closed universe case:

(Open and flat cases on Problem Set 8)

$$ds^2 = -c^2\,dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} .$$

Take $k = 1$ and define $\sin\psi \equiv r$:

$$ds^2 = -c^2\,dt^2 + a^2(t)\left\{d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} .$$



$$\text{fraction} = \frac{\text{area of detector}}{\text{area of sphere}} = \frac{A}{4\pi a^2(t_0)\sin^2\psi_D} \; .$$

Why? $\mathrm{d}s^2 \supset a^2(t)\sin^2\psi\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$ implies that angular metric $=$ $a^2(t)\sin^2\psi$ times the metric for a unit sphere. So $R = a(t)\sin\psi_D$, and area $= 4\pi R^2$.

Power of source, P, suppressed by two factors of $(1 + z_S)$: one for redshift of photons, one for rate of arrival of photons.

$$J = \frac{P_{\text{received}}}{A} = \frac{P \times \text{fraction}}{(1+z_S)^2 A} = \frac{P}{4\pi(1+z_S)^2 a^2(t_0)\sin^2\psi_D} \ .$$

But we must evaluate $a(t_0)$ and $\sin\psi_D$.

Recall that $\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\text{rad},0} - \Omega_{\text{vac},0}$ is defined by

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0)H_0^2} \ ,$$

so if $k = 1$ then

$$a(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} \ .$$

To find $\psi(z_S)$, follow the trajectory of the photon. Photon travels on a null geodesic, so

$$0 = -c^2\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\psi^2 \quad \Longrightarrow \quad \frac{\mathrm{d}\psi}{\mathrm{d}t} = \frac{c}{a(t)}\ .$$

Then $\psi(z_S)$ is the change in ψ during the flight of the photon:

$$\psi(z_S) = \int_{t_S}^{t_0} \frac{c}{a(t)}\,\mathrm{d}t\ .$$

We want the answer in terms of z_S, so we can change the variable of integration from t to z, where

$$1 + z = \frac{a(t_0)}{a(t)}\ .$$

Then

$$\mathrm{d}z = -\frac{a(t_0)}{a(t)^2}\dot{a}(t)\,\mathrm{d}t = -a(t_0)H(t)\,\frac{\mathrm{d}t}{a(t)}\ .$$

The integral then becomes

$$\psi(z_S) = \frac{1}{a(t_0)}\int_0^{z_S} \frac{c}{H(z)}\,\mathrm{d}z\ .$$

But we learned from the first Friedmann equation that

$$H(x) = \frac{H_0}{x^2}\sqrt{F(x)} \ ,$$

where

$$F(x) = \Omega_{m,0}x + \Omega_{\rm rad,0} + \Omega_{\rm vac,0}x^4 + \Omega_{k,0}x^2 \ ,$$

where $x \equiv a(t)/a(t_0) = 1/(1+z)$. So we know $H(z)$. Putting these equations together,

$$\psi(z_S) = \sqrt{|\Omega_{k,0}|} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\rm rad,0}(1+z)^4 + \Omega_{\rm vac,0} + \Omega_{k,0}(1+z)^2}} \ .$$

Finally, recalling that

$$J = \frac{P}{4\pi(1+z_S)^2 a^2(t_0) \sin^2 \psi_D}$$

and that

$$a(t_0) = \frac{cH_0^{-1}}{\sqrt{-\Omega_{k,0}}} \ ,$$

we have

$$\boxed{J = \frac{PH_0^2|\Omega_{k,0}|}{4\pi(1+z_S)^2 c^2 \sin^2 \psi(z_S)} \ .}$$

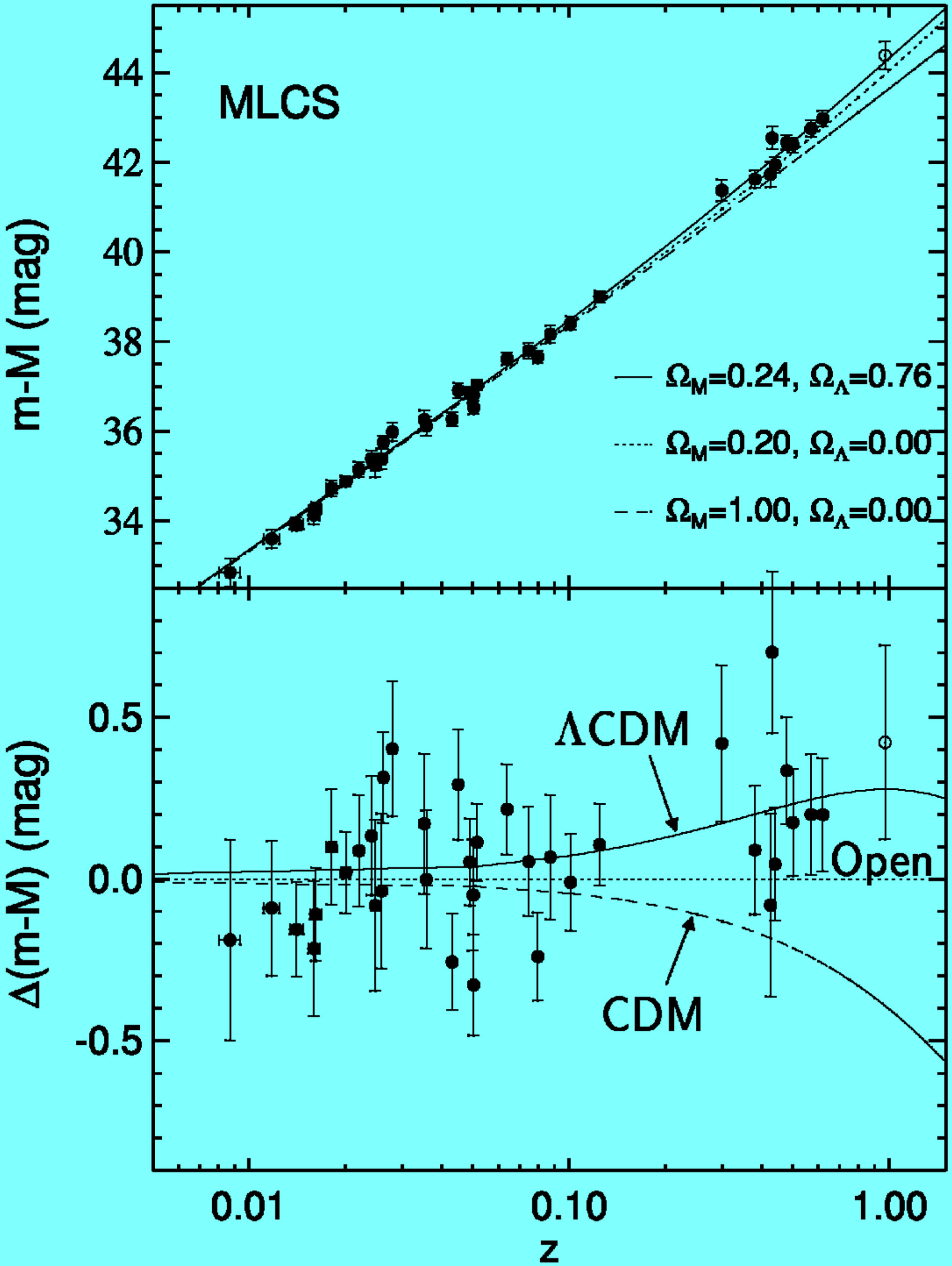
Supernovae Type Ia as Standard Candles

See Ryden, Section 7.5 (which we skipped).

Supernovae Type Ia are believed to be the result of a binary system containing a white dwarf — a stellar remnant that has burned its nuclear fuel, and is supported by electron degeneracy pressure. As the white dwarf accretes gas from it companion star, its mass builds up to 1.4 $M_\odot$, the Chandrasekhar limit, the maximum mass that can be supported by electron degeneracy pressure. The star then collapses, leading to a supernova explosion. Because the Chandrasekhar limit is fixed by physics, all SN Ia are very similar in power output.

There are still some known variations in power output, but they are found to be correlated with the shape of the light curve: if the light curve rises and falls slowly, the supernova is brighter than average.

The properties of SN Ia are known best from observation — theory lags behind.



Hubble diagram from Riess *et al.*, *Astronomical Journal* **116**, No. 3, 1009 (1998) [http://arXiv.org/abs/astro-ph/9805201].

Dimmer Supernovae Imply Acceleration

- ☆ The acceleration of the universe is deduced from the fact that distant supernovae appear to be 20-30% dimmer than expected.
- ☆ Why does dimness imply acceleration?
  - Consider a supernova of specified apparent brightness.
  - "Dimmer" implies data point is to the left of where expected — at lower z.
  - Lower z implies slower recession, which implies that the universe was expanding slower than expected in the past — hence, acceleration!

Other Possible Explanations for Dimness

☆ Absorption by dust.

- But absorption usually reddens the spectrum. This would have to be "gray" dust, absorbing uniformly at all observed wavelengths. Such dust is possible, but not known to exist anywhere.
- Dust would most likely be in the host galaxy, which would cause variable absorption, depending on SN location in galaxy. Such variability is not seen.

☆ Chemical evolution of heavy element abundance.

- But nearby and distant SN Ia look essentially identical.
- For nearby SN Ia, heavy element abundance varies, and does not appear to affect brightness.

☆ SN Ia at $z = 1.7$ is consistent with deceleration, as expected for vacuum energy, but not consistent with either models of absorption or chemical evolution.

Evidence for the Accelerating Universe

1) Supernova Data: distant SN Ia are dimmer than expected by about 20–30%.

2) Cosmic Microwave Background (CMB) anisotropies: gives Ω_{vac} close to SN value. Also gives $\Omega_{\text{tot}} = 1$ to 1/2% accuracy, which cannot be accounted for without dark energy.

3) Inclusion of $\Omega_{\text{vac}} \approx 0.70$ makes the age of the universe consistent with the age of the oldest stars.

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- ☆ With the 3 arguments together, the case for the accelerating universe and $\Omega_{\rm dark\ energy} \approx 0.70$ has persuaded almost everyone.
- ☆ The simplest explanation for dark energy is vacuum energy, but "quintessence" is also possible.

Particle Physics of a Cosmological Constant

☆ $$u_{\text{vac}} = \rho_{\text{vac}} c^2 = \frac{\Lambda c^4}{8\pi G}$$

☆ Contributions to vacuum energy density:

1) Quantum fluctuations of the photon and other bosonic fields: positive and divergent.

2) Quantum fluctuations of the electron and other fermionic fields: negative and divergent.

3) Fields with nonzero values in the vacuum, like the Higgs field.

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