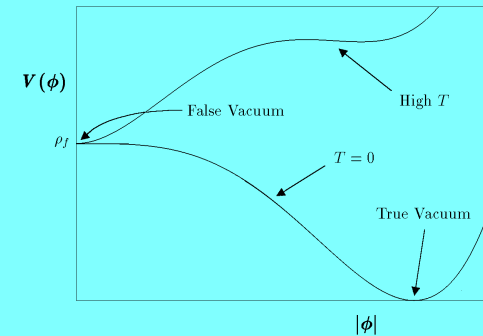*8.286 Lecture 24 (Last!)*
December 12, 2018

THE INFLATIONARY UNIVERSE

The Inflationary Universe Scenario

Assume an "inflaton" field ϕ with a potential energy density resembling

More general potential energy functions are possible, but the curve above — describing "new inflation" — is a good starting point for discussion.

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Start of Inflation

There is no accepted (or even persuasive) theory of the origin of the universe, so the starting point is uncertain. Inflation starts when the scalar field is at the top of the hill, no matter how it got there.

The scalar field can reach the top of the hill by:

1) Cooling from high temperature ("new" inflation: Linde 1982, Albrecht & Steinhardt, 1982). But: there is not enough time for thermal equilibrium to be reached, so it must be *assumed*.

2) With spatially dependent "chaotic" initial conditions, it will happen somewhere (Linde, 1983). This is probably the dominant point of view today.

3) Creation of the universe by "tunneling from nothing" (Vilenkin, 1983, Linde 1984).

4) Initial conditions for the "wave function of the universe" (Hartle & Hawking, 1983).

5) Who knows?

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The Inflationary Era

Once the inflaton is at the top of the hill, the mass/energy density is fixed, leading to a large negative pressure and gravitational repulsion:

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \; ; \;\; \dot{\rho} = 0 \;\; \implies \;\; p = -\rho c^2 \ .$$

Assuming approximate Friedmann-Robertson-Walker evolution,

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right) = \frac{8\pi}{3}G\rho_f,$$

where ρ_f = mass density of the false vacuum. Thus, ρ_f produces gravitational repulsion.

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The de Sitter Solution

The homogeneous isotropic solution can be described as a Robertson-Walker flat universe:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2 \ ,$$

where

$$a(t) \propto e^{\chi t} \ , \ \chi = \sqrt{\frac{8\pi}{3}G\rho_{\mathrm{f}}} \ .$$

This is called de Sitter spacetime.

By a change of coordinates, de Sitter spacetime can, surprisingly, be described as an open universe, a closed universe, or a static universe!

Cosmological "No-Hair" Conjecture

Conjecture: For "reasonable" initial conditions, even if far from homogeneous and isotropic, $\rho = \rho_f$ implies that the region will approach de Sitter space.

Conjectured by Hawking & Moss (1982). Can be proven for linearized perturbations about de Sitter spacetime. Was shown by Wald (1983) to hold for a class of very large perturbations.

Analogous to the Black Hole No-Hair Theorem, which implies that gravitationally collapsing matter approaches a stationary black hole state that depends only on the mass, angular momentum, and charge.

Qualitative behavior: any distortion of the metric is stretched by the expansion to look smooth and flat. Any initial matter distribution is diluted away by the expansion.

De Sitter Event Horizon

In the de Sitter metric, with $a(t) = be^{\chi t}$, the coordinate distance that light can travel between times t_1 and t_2 is

$$\Delta r(t_1, t_2) = \int_{t_1}^{t_2} \frac{c}{a(t)}\,\mathrm{d}t = \frac{c}{b}\int_{t_1}^{t_2} e^{-\chi t}\,\mathrm{d}t = \frac{c}{b\chi}\left[e^{-\chi t_1} - e^{-\chi t_2}\right] \ ,$$

which is bounded as $t \to \infty$. If we multiply by $a(t_1)$ and take the limit,

$$\lim_{t_2 \to \infty} a(t_1)\,\Delta r(t_1, t_2) = c\chi^{-1} \ ,$$

which means that if two objects have a physical separation larger than $c\chi^{-1}$, the Hubble length, at any time, light from the first will never reach the second. This is called an event horizon. Event horizons protect an inflating patch from the rest of the universe: once the patch is large compared to $c\chi^{-1}$, nothing from outside can penetrate further than $c\chi^{-1}$.

The Ending of Inflation

A standard scalar field in a flat FRW universe obeys the equation of motion:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla_i^2\phi = -\frac{\partial V}{\partial \phi} \ ,$$

where ∇_i^2 is the Laplacian operator in comoving coordinates x^i.

The spatial derivative piece soon becomes negligible, due to the $(1/a^2)$ suppression, which is reflects the fact that the stretching of space causes ϕ to become nearly uniform over huge regions. The equation is then identical to that of a ball sliding on a hill described by $V(\phi)$, but with a viscous damping (i.e., friction) described by the term $-3(\dot{a}/a)\dot{\phi}$.

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \ .$$

Fluctuations in ϕ due to thermal and/or quantum effects will cause the field to start to slide down the hill. This will not happen globally, but in regions, typically of size $c\chi^{-1}$.

Within a region, ϕ will start to oscillate about the true vacuum value, at the bottom of the hill. Interactions with other fields will allow ϕ to give its energy to the other fields, producing a "hot soup" of other particles, which is exactly the starting point of the conventional hot big bang theory. This is called *reheating*.

$V(\phi)$

ρ_f

False Vacuum

True Vacuum

$|\phi|$
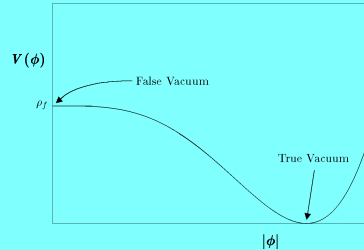
The standard hot big bang scenario begins. Inflation has played to role of a prequel, setting the initial conditions for conventional cosmology.

Numerical Estimates

The energy scale at which inflation happened is not known. One plausible guess is the GUT scale, $E_{\text{GUT}} \approx 10^{16}$ GeV. It cannot be higher (too much gravitational radiation), but can be as low as about 10^3 GeV.

For E_{GUT}, we can estimate

$$\rho_f \approx \frac{E_{\text{GUT}}^4}{\hbar^3 c^5} = 2.3 \times 10^{81} \text{g/cm}^3 \ .$$

Then

$$\chi^{-1} \approx 2.8 \times 10^{-38} \text{ s} \ , \quad c\chi^{-1} = 8.3 \times 10^{-28} \text{ cm} \ ,$$

and the mass of a minimal region of inflation would be about

$$M \approx \frac{4\pi}{3}(c\chi^{-1})^3 \rho_f \approx 5.6 \text{ gram}.$$

BUT Where Does the Energy Come From?

★ The energy of a gravitational field is negative (both in Newtonian gravity and in general relativity).

★ The negative energy of gravity cancelled the positive energy of matter, so the total energy was constant and possibly zero.

★ The total energy of the universe today is consistent with zero. Schematically,

Matter
&
Radiation

Total Energy = ———— + ———— = 0.

Gravity

★ Warning: the concept of total energy in GR is controversial. Some authors would just say that total energy is not defined.

Solutions to the Cosmological Problems

1) **Horizon Problem:** In inflationary models, uniformity is achieved in a tiny region BEFORE inflation starts. Without inflation, such regions would be far too small to matter. But inflation can stretch a tiny region of uniformity to become large enough to include the entire visible universe and more. Need expansion by about 10^{28}, which is about 65 time constants of the exponential expansion.

2) **Flatness Problem:** Just look at Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \ .$$

"Flatness" is the statement that the final term in this equation is negligible. But during inflation, $\rho \approx \rho_v = $ const, while $a(t)$ grows exponentially. If $a(t)$ grows by at least 10^{28} during inflation, the final term is suppressed by a factor of $\left(10^{28}\right)^2 = 10^{56}$.

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3) **Monopole Problem:** Solved by dilution, as long as the inflation occurs during or after the process of monopole production. During inflation the volume of any comoving region increases by a factor of about $\left(10^{28}\right)^3 = 10^{84}$ or more! That is plenty enough to make monopoles impossible to find.

Some small number of monopoles could be produced during reheating, so it makes sense to look for them. But except for the irreproducible event seen by Cabrera in 1982, magnetic monopoles have not been seen.

Ripples in the Cosmic Microwave Background

The CMB is uniform in all directions to an accuracy of a few parts in 100,000. Nonetheless, at the level of a few parts in 100,000 there ARE anisotropies, and they have now been measured to high precision. Since the CMB is essentially a snapshot of the universe at $t \approx 380,000$ yr, these ripples are interpreted as perturbations in the cosmic mass density at this time.

In the early days of inflation, such density perturbations were a cause for worry. (The ripples had not yet been seen, but cosmologists knew that the early universe must have had density perturbations, or else galaxies and stars could never have formed.) Inflation smooths out the universe so effectively, that it looked like no density perturbations could survive.

Quantum Mechanics to the Rescue (Again)

Why again? We spoke earlier about how quantum mechanics was necessary to save us from freezing to death. If classical mechanics ruled, all thermal energy would gradually disappear into shorter and shorter wavelength electromagnetic radiation.

If inflation happened with classical physics, it would smooth the universe so perfectly that stars and galaxies could never form.

But quantum mechanics is intrinsically probabilistic. While the classical version of inflation predicts an almost exactly uniform mass density, the intrinsic randomness of the quantum version implies that the mass density will be a little higher in some places, and a little lower in others.

In 1965, Andrei Sakharov, the Russian nuclear physicist and political activist, proposed in a rather wildly speculative paper that quantum fluctuations might account for the structure of the universe.

In 1981, Mukhanov and Chibisov tried to calculate the density fluctuations in pre-inflationary/inflationary model invented by Alexei Starobinsky in 1980.

In summer 1982, Gary Gibbons and Stephen Hawking organized the Nuffield Workshop on the Very Early Universe in Cambridge UK, where a number of physicists worked feverishly and argued through the night about how to calculate these perturbations in inflation. In the end, all agreed. Four papers emerged: Hawking, Starobinsky, Guth & Pi, and Bardeen, Steinhardt, & Turner.

Basic conclusion: the amplitude of the density perturbations is very "model-dependent," meaning that it depends on the unknown details of $V(\phi)$. But: the spectrum — the way in which the intensity of the ripples depends on the wavelength of the ripples — is the same for a wide range of "simple" inflationary models. Simple = "Single field / slow-roll models," i.e. models with a single inflaton field, and with small values for $\mathrm{d}V/\mathrm{d}\phi$ and $\mathrm{d}^2V/\mathrm{d}\phi^2$.

Observations of the Ripples in the CMB

In 1982, it seemed (at least to me) out of the question that these ripples would ever be seen.

There have now been 3 satellite experiments to measure the CMB, plus many many ground-based experiments. The three satellites were:

COBE: Cosmic Background Explorer, launched by NASA in 1989, after 15 years of planning. In 1992 it announced its first measurements of CMB anisotropies. The angular resolution was crude, about 7°, but the results agreed with inflation.

WMAP: The Wilkinson Microwave Anisotropy Probe, launched by NASA in 2001. 45 times more sensitive, with 33 times better angular resolution than COBE. Still consistent with inflation.

Planck: Launched in 2009 by ESA. Resolution about 2.5 times better than WMAP.

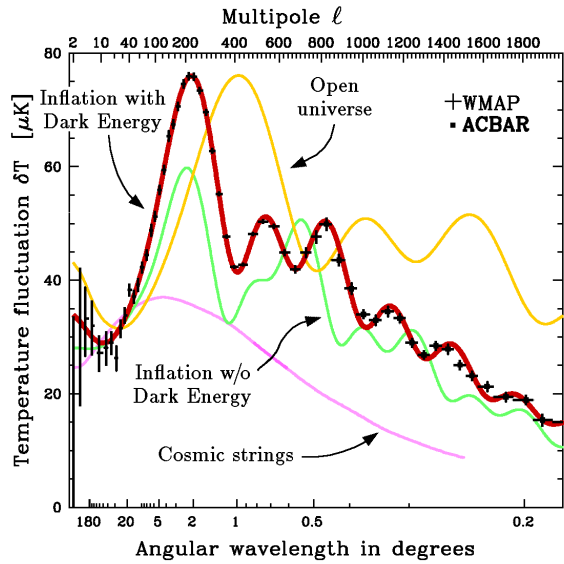## Ripples in the Cosmic Microwave Background

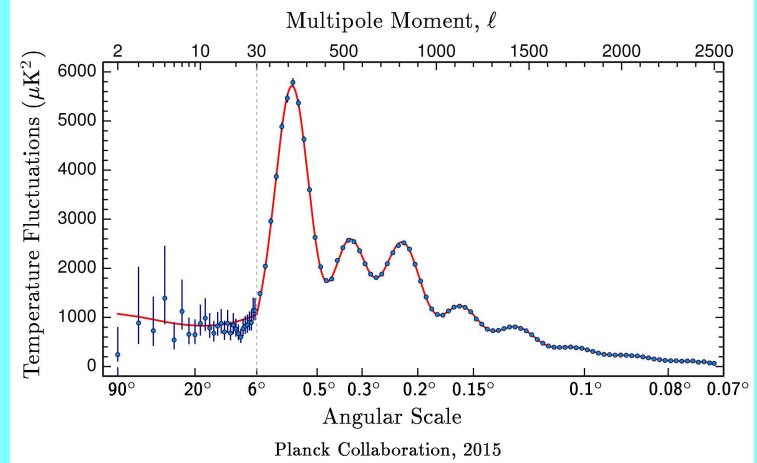Planck Collaboration: The *Planck* mission
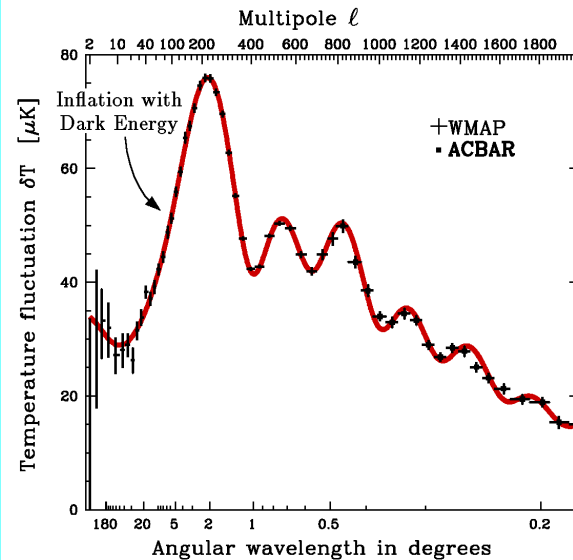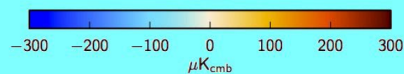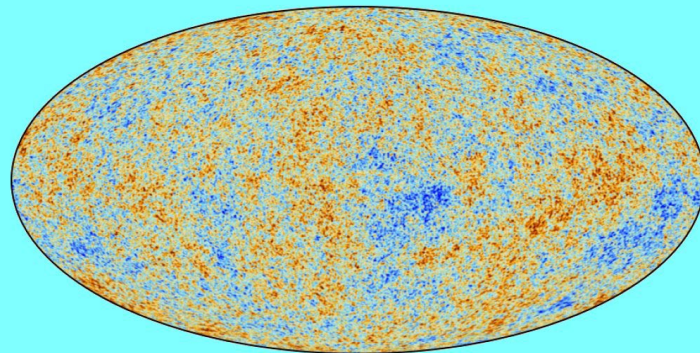
CMB: Comparison of Theory and Experiment

Graph by Max Tegmark, for A. Guth & D. Kaiser, *Science* **307, 884** (Feb 11, 2005), updated to include WMAP 7-year data (Jan 2010).
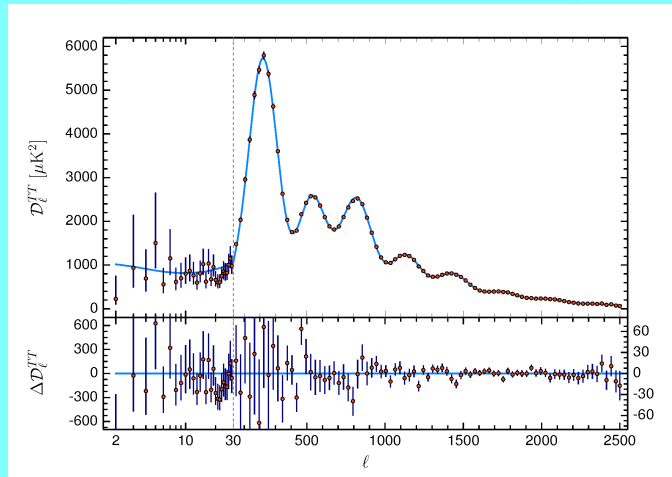
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Planck 2015 Spectrum

Planck 2018 Spectrum

Lower panel shows difference between data and model.