8.286 Lecture 3
September 14, 2020

THE KINEMATICS of a HOMOGENEOUSLY EXPANDING UNIVERSE

Hubble's Law

$$v = Hr \; .$$

Here

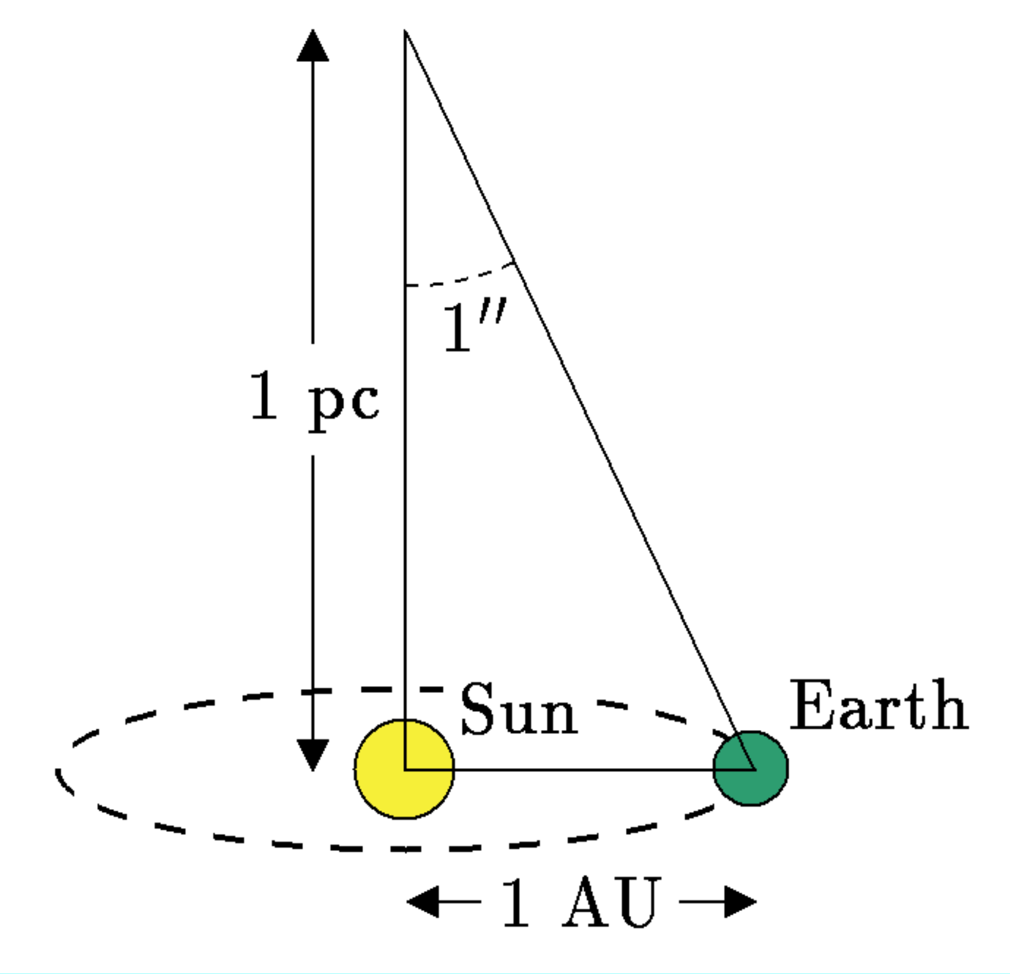
$$v \equiv \text{recession velocity} \; ,$$

$$H \equiv \text{Hubble expansion rate} \; ,$$

and

$$r \equiv \text{distance to galaxy} \; .$$

The Parsec



Units for the Hubble Expansion Rate

$v = Hr \quad \Longrightarrow \quad [H] = [v]/[r] = (L/T)/L = 1/T.$

Astronomers invariably think in terms of velocity/distance, which they measure in km-s^{-1}-Mpc^{-1}.

1 pc = 3.2616 light-yr

Relation to inverse time:

$$\frac{1}{10^{10}\ \text{yr}} = 97.8\ \text{km-s}^{-1}\text{-Mpc}^{-1}.$$

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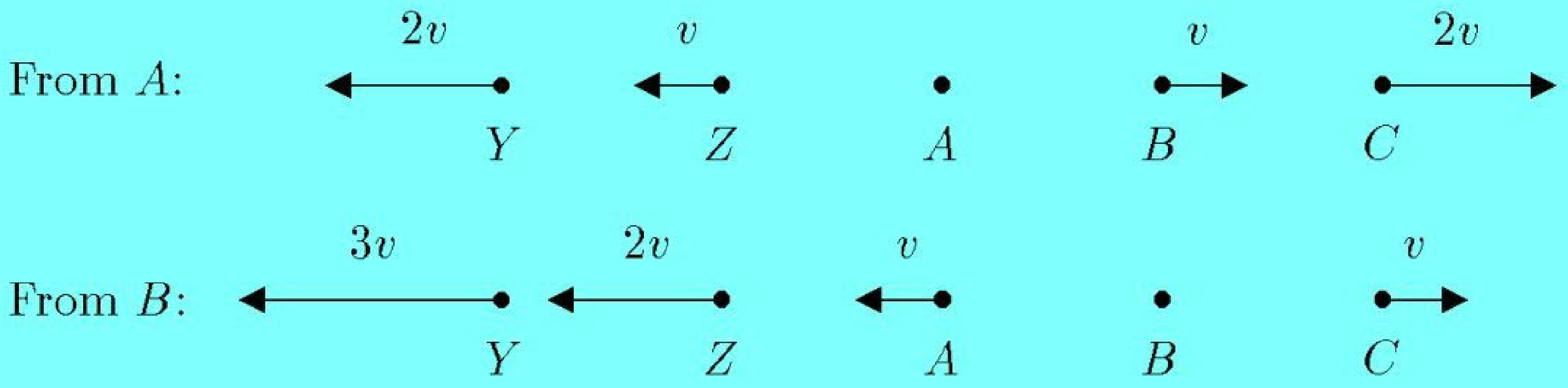
- ☆ As Weinberg explains it in *The First Three Minutes:*



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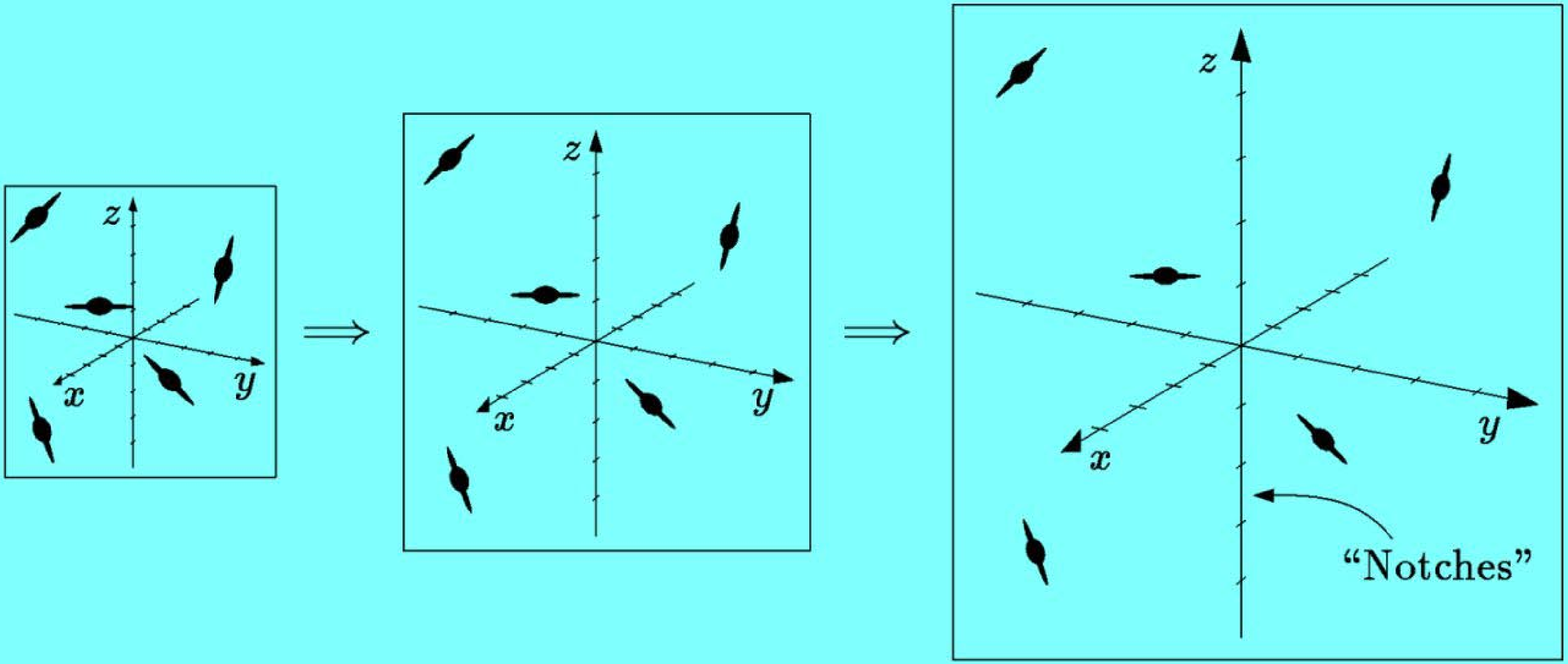
- We imagine a fixed 3D map of the universe, with distances marked in some arbitrary unit: I call them "notches," to make it clear that they have no fixed meaning in terms of any standard units of length. The "scale," or scale factor, is denoted by $a(t)$, where $a(t)$ is measured in notches per meter (or light-year, or Mpc, or whatever). The relation is then

$$\ell_p(t) = a(t)\,\ell_c \ ,$$

where $\ell_p(t)$ is the **physical** distance, measured in meters (or light-years, etc.), and ℓ_c is the **coordinate** distance, measured in notches.


z
x
y
⟹
z
x
y
⟹
z
x
y
"Notches"

Hubble's Law as a Consequence of Uniform Expansion

$$\ell_p(t) = a(t)\,\ell_c \ ,$$

So how fast does $\ell_p(t)$ change?

$$v = \frac{\mathrm{d}\ell_p}{\mathrm{d}t} = \frac{\mathrm{d}a}{\mathrm{d}t}\ell_c = \left[\frac{1}{a(t)}\,\frac{\mathrm{d}a(t)}{\mathrm{d}t}\right] a(t)\ell_c \ .$$

Note that this can be rewritten as

$$v = \frac{\mathrm{d}\ell_p}{\mathrm{d}t} = H\ell_p \ ,$$

where

$$H(t) = \frac{1}{a(t)}\,\frac{\mathrm{d}a(t)}{\mathrm{d}t} \ .$$