8.286 Class 10
October 7, 2020

INTRODUCTION TO NON-EUCLIDEAN SPACES PART 2

Announcements

- ☆ "Remote learning check-in" survey is up and running:

  https://forms.gle/4GjAhH5YBvpoema18

  If you have not already filled it in, please do so by midnight tonight (after the vice-presidential debate).

- ☆ The survey is only to help Bruno and me make improvements to the course. We VALUE your feedback and suggestions.

https://www.nobelprize.org/prizes/physics/

The 2020 Physics Laureates

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2020 with one half to Roger Penrose “for the discovery that black hole formation is a robust prediction of the general theory of relativity" and and the other half jointly to Reinhard Genzel and Andrea Ghez "for the discovery of a supermassive compact object at the centre of our galaxy".

Ill. Niklas Elmehed. © Nobel Media.

YouTube

Search

https://www.youtube.com/watch?v=5bmJalWKTj8&feature=emb_logo

Roger Penrose: "I had this strange feeling of elation"

15,429 views • Oct 6, 2020

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Nobel Prize
236K subscribers

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In this phone interview with Adam Smith, recorded just after the announcement of the 2020 Nobel Prize in Physics, Roger Penrose recounts the story of how a particular crossroads held the key to his seminal 1965 paper on the theoretical basis of black holes. In the second half of the conversation he goes on to explain some of his latest work, describing how black holes are the basis for the second law of thermodynamics, and suggesting that signatures of black holes from a previous universe might be faintly apparent in the cosmic microwave background radiation.

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

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Apparent evidence for Hawking points in the CMB Sky★

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ABSTRACT

This paper presents strong observational evidence of numerous previously unobserved anomalous circular spots, of significantly raised temperature, in the cosmic microwave background sky. The spots have angular radii between 0.03 and 0.04 rad (i.e. angular diameters between about 3° and 4°). There is a clear cut-off at that size, indicating that each anomalous spot would have originated from a highly energetic point-like source, located at the end of inflation – or else point-like at the conformally expanded Big Bang, if it is considered that there was no inflationary phase. The significant presence of these anomalous spots, was initially noticed in the *Planck* 70 GHz satellite data by comparison with 1000 standard simulations, and then confirmed by extending the comparison to 10 000 simulations. Such anomalous points were then found at precisely the same locations in the *WMAP (Wilkinson Microwave Anisotropy Probe)* data, their significance was confirmed by comparison with 1000 *WMAP* simulations. *Planck* and *WMAP* have very different noise properties and it seems exceedingly unlikely that the observed presence of anomalous points in the same directions on both maps may come entirely from the noise. Subsequently, further confirmation was found in the *Planck* data by comparison with 1000 FFP8.1 MC simulations (with $l \leq 1500$). The existence of such anomalous regions, resulting from point-like sources at the conformally stretched-out big bang, is a predicted consequence of conformal cyclic cosmology, these sources being the Hawking points of the theory, resulting from the Hawking radiation from supermassive black holes in a cosmic aeon prior to our own.

Key words: cosmic background radiation.

PACS: 04.20.Ha – 04.70.Dy – 98.80.Bp – 98.80.Ft.

[Submitted on 6 Aug 2018 (v1), last revised 2 Mar 2020 (this version, v4)]

Apparent evidence for Hawking points in the CMB Sky

Daniel An, Krzysztof A. Meissner, Pawel Nurowski, Roger Penrose

This paper presents strong observational evidence of numerous previously unobserved anomalous circular spots, of significantly raised temperature, in the CMB sky. The spots have angular radii between 0.03 and 0.04 radians (i.e. angular diameters between about 3 and 4 degrees). There is a clear cut-off at that size, indicating that each anomalous spot would have originated from a highly energetic point-like source, located at the end of inflation -- or else point-like at the conformally expanded Big Bang, if it is considered that there was no inflationary phase. The significant presence of these anomalous spots, was initially noticed in the Planck 70 GHz satellite data by comparison with 1000 standard simulations, and then confirmed by extending the comparison to 10000 simulations. Such anomalous points were then found at precisely the same locations in the WMAP data, their significance confirmed by comparison with 1000 WMAP simulations. Planck and WMAP have very different noise properties and it seems exceedingly unlikely that the observed presence of anomalous points in the same directions on both maps may come entirely from the noise. Subsequently, further confirmation was found in the Planck data by comparison with 1000 FFP8.1 MC simulations (with $l \leq 1500$). The existence of such anomalous regions, resulting from point-like sources at the conformally stretched-out big bang, is a predicted consequence of conformal cyclic cosmology (CCC), these sources being the Hawking points of the theory, resulting from the Hawking radiation from supermassive black holes in a cosmic aeon prior to our own.

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**

Cite as: arXiv:1808.01740 **[astro-ph.CO]**

(or arXiv:1808.01740v4 **[astro-ph.CO]** for this version)

Submission history

From: Krzysztof A. Meissner [view email]

[v1] Mon, 6 Aug 2018 06:16:38 UTC (9 KB)

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Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 20 Sep 2019 (v1), last revised 21 Jan 2020 (this version, v2)]

Re-evaluating evidence for Hawking points in the CMB

Dylan L. Jow, Douglas Scott

We investigate recent claims for a detection of "Hawking points" (positions on the sky with unusually large temperature gradients between rings) in the cosmic microwave background (CMB) temperature maps at the 99.98% confidence level. We find that, after marginalization over the size of the rings, an excess is detected in Planck satellite maps at only an 87% confidence level (i.e., little more than 1σ). Therefore, we conclude that there is no statistically significant evidence for the presence of Hawking points in the CMB.

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**
DOI: 10.1088/1475-7516/2020/03/021
Cite as: arXiv:1909.09672 **[astro-ph.CO]**
(or arXiv:1909.09672v2 **[astro-ph.CO]** for this version)

Submission history

From: Dylan Jow [view email]
[v1] Fri, 20 Sep 2019 18:40:14 UTC (1,529 KB)
[v2] Tue, 21 Jan 2020 16:38:20 UTC (1,719 KB)

Intrinsic Geometry

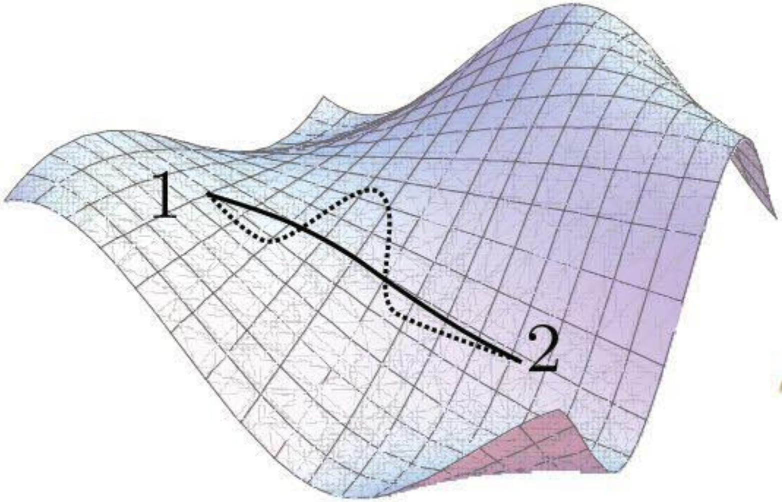

Image of ant from: http://www.termiteterry.com/

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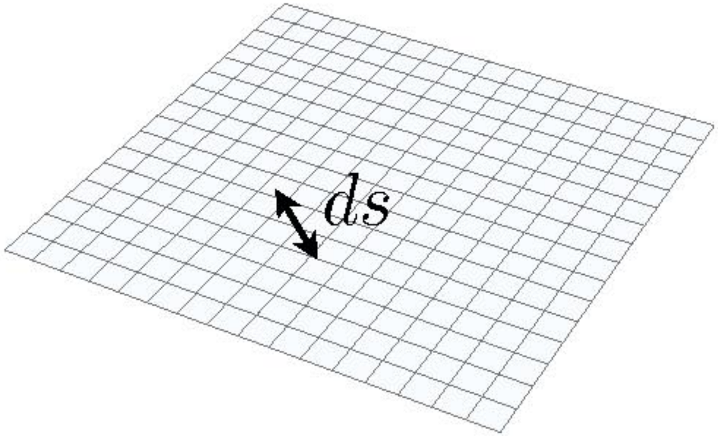
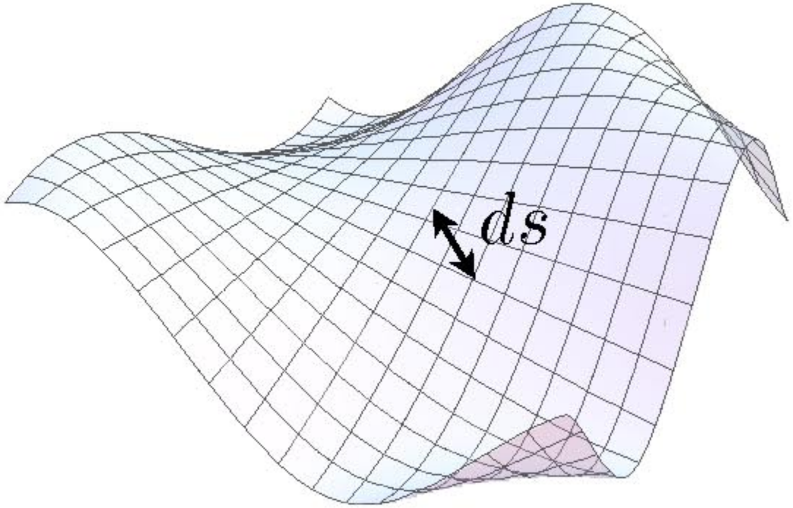
tiny distances





$$ds^2 = dx^2 + dy^2$$

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

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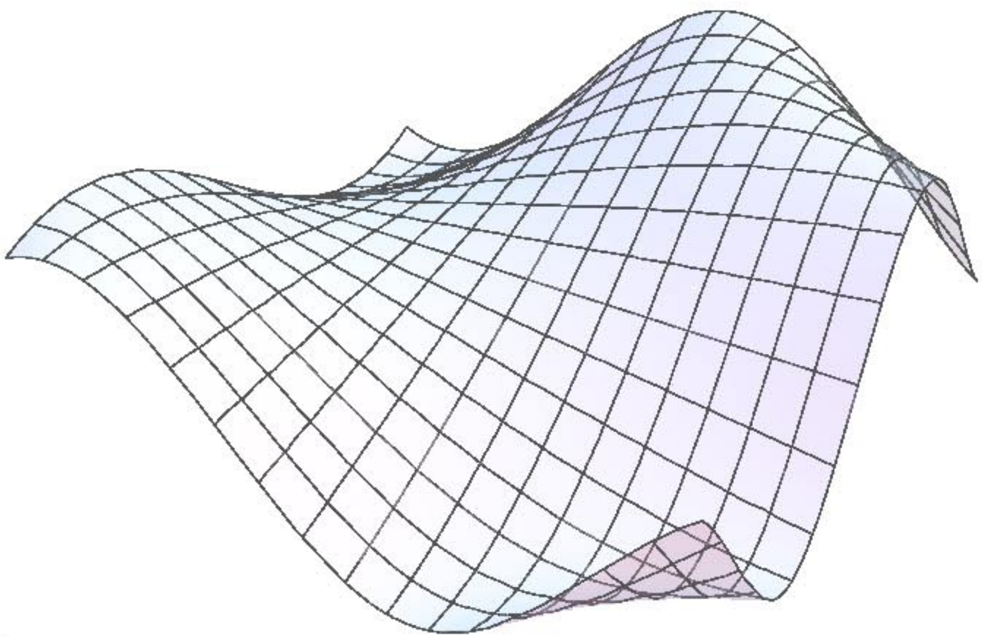
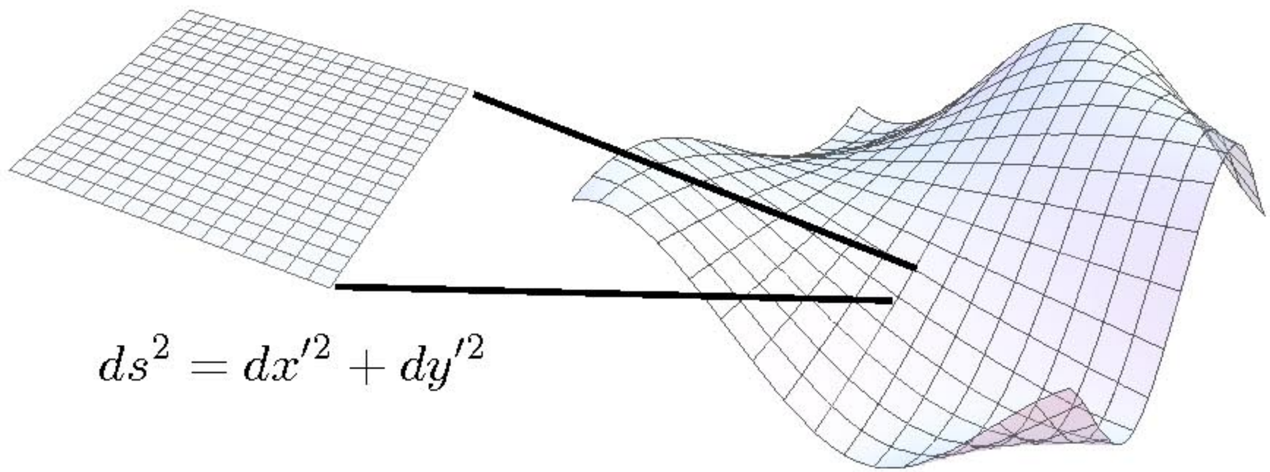
quadratic form

Image:www.easternct.edu/career/webresources.htm



$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

locally Euclidean

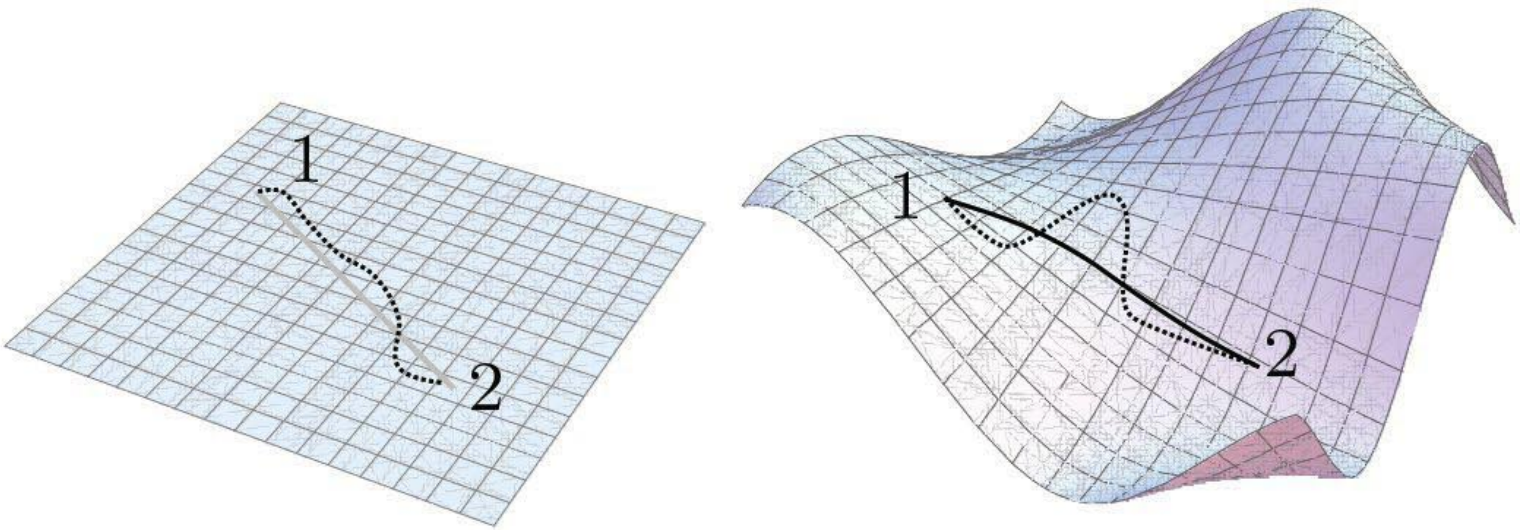


$$ds^2 = dx'^2 + dy'^2$$

$$ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

$$g_{xx}g_{yy} - g_{xy}^2 > 0$$

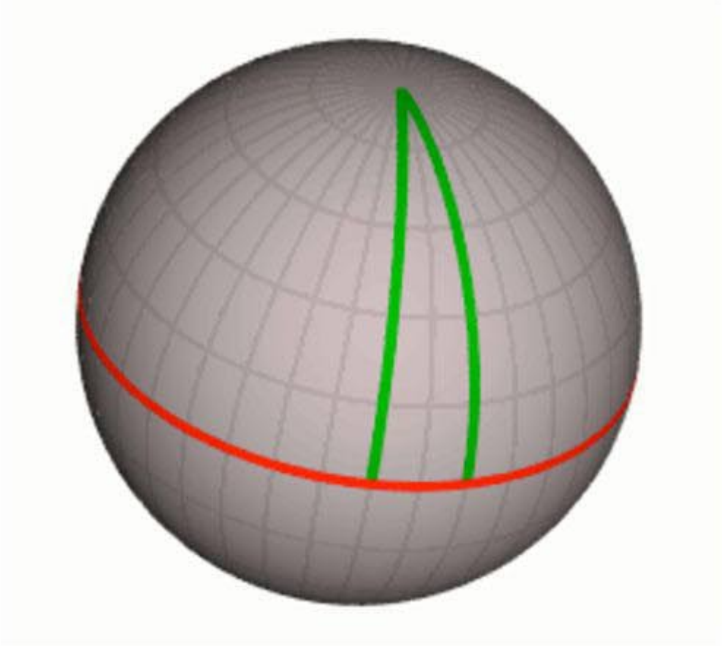
geodesics

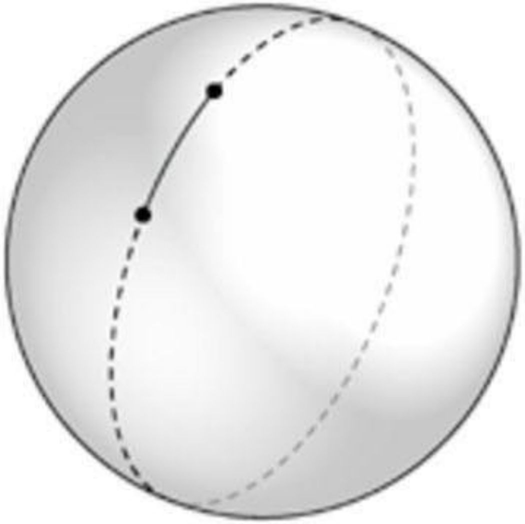


a geodesic is a curve along which the distance between two given points is extremised.

note: important!

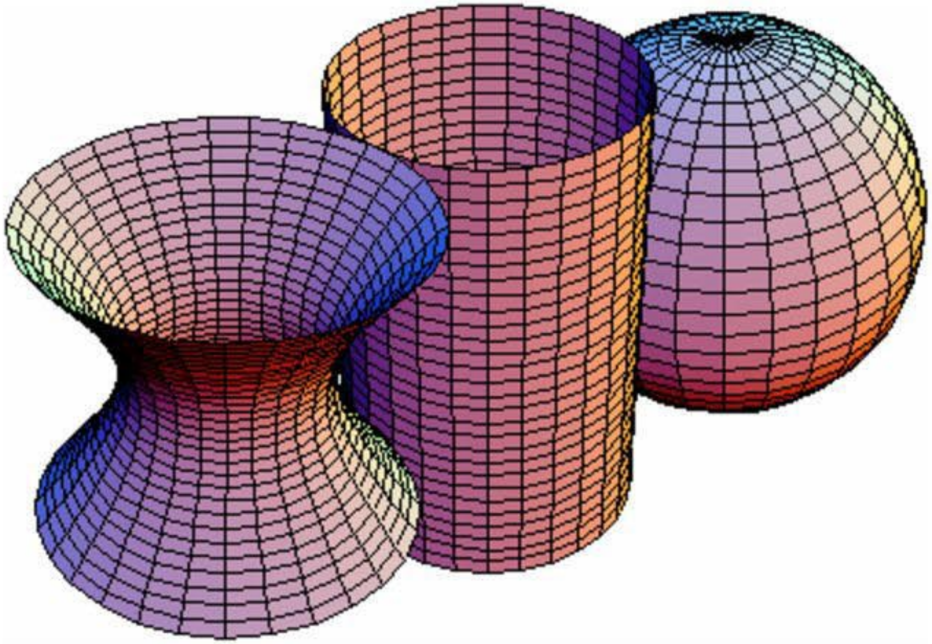
sphere: geodesics





longitudes: yes
latitudes: no

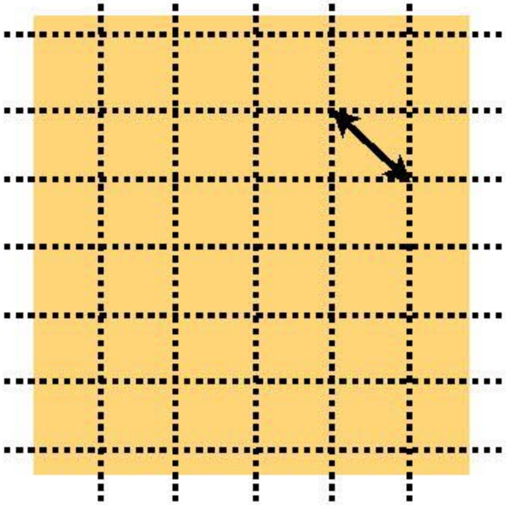
curved ?



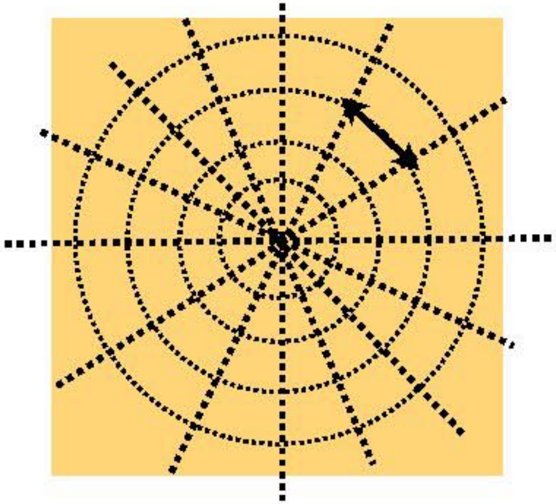
Above image:http://en.wikipedia.org/wiki/Gaussian_curvature

$$ds^2 = dx^2 + dy^2 \qquad ds^2 = g_{xx}(x,y)dx^2 + 2g_{xy}(x,y)dxdy + g_{yy}(x,y)dy^2$$

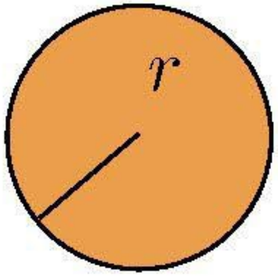
metric and co-ordinates



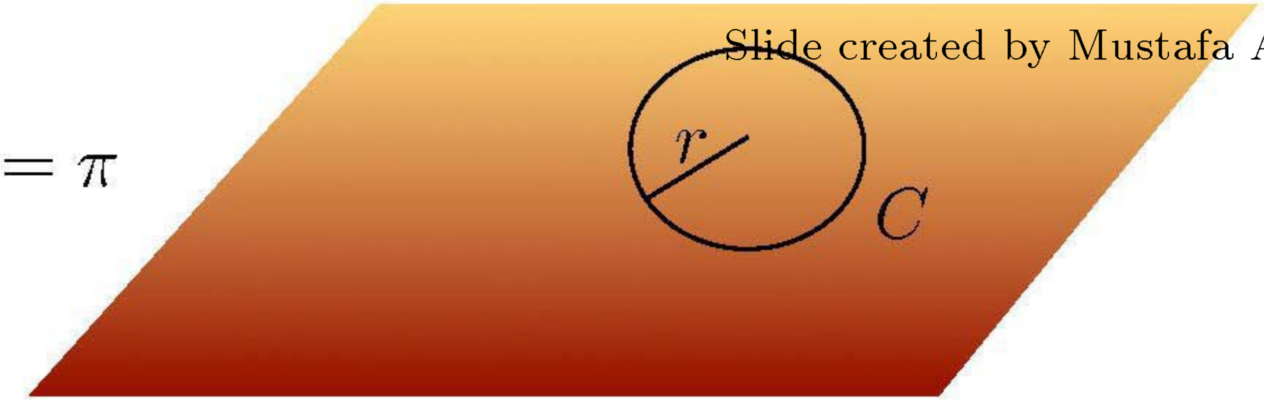
$$ds^2 = dx^2 + dy^2$$

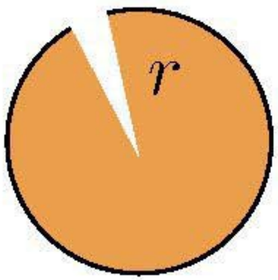


$$ds^2 = dr^2 + r^2 d\theta^2$$

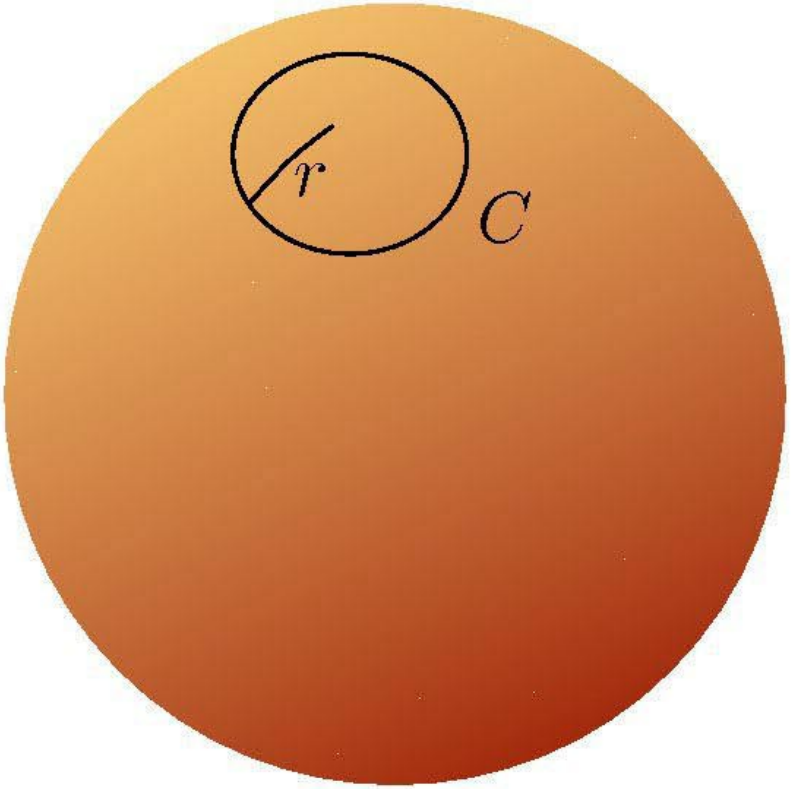


$$\frac{C}{2r} = \pi$$

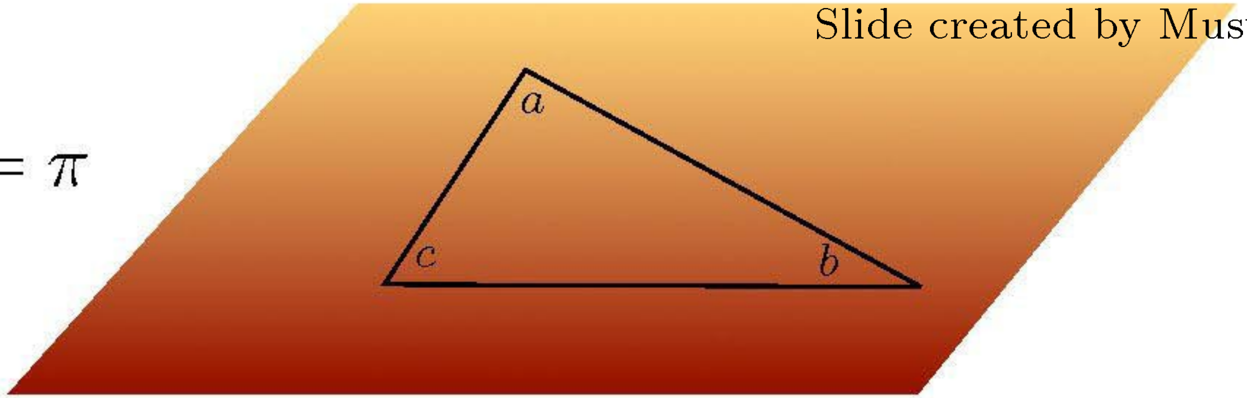




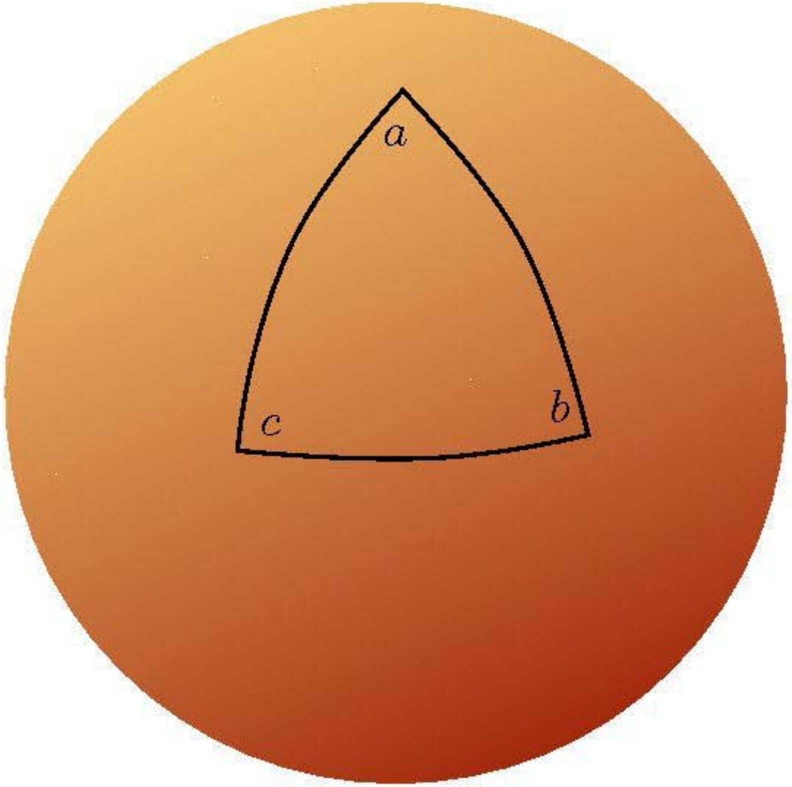
$$\frac{C}{2r} < \pi$$



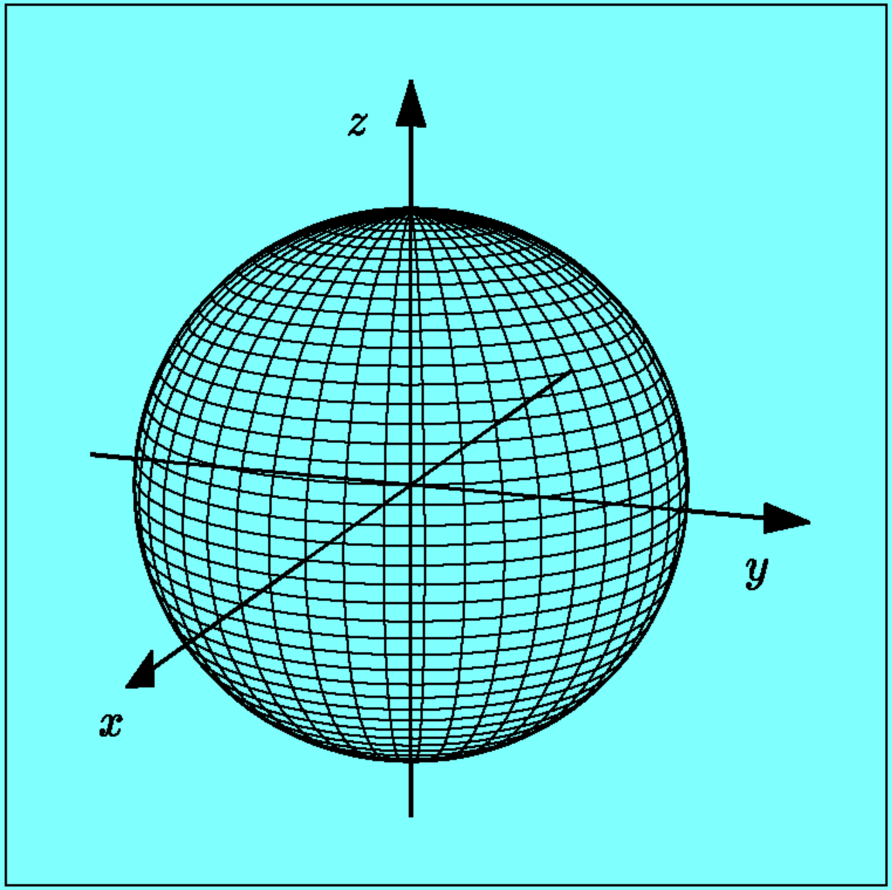
$$a + b + c = \pi$$



$$a + b + c > \pi$$



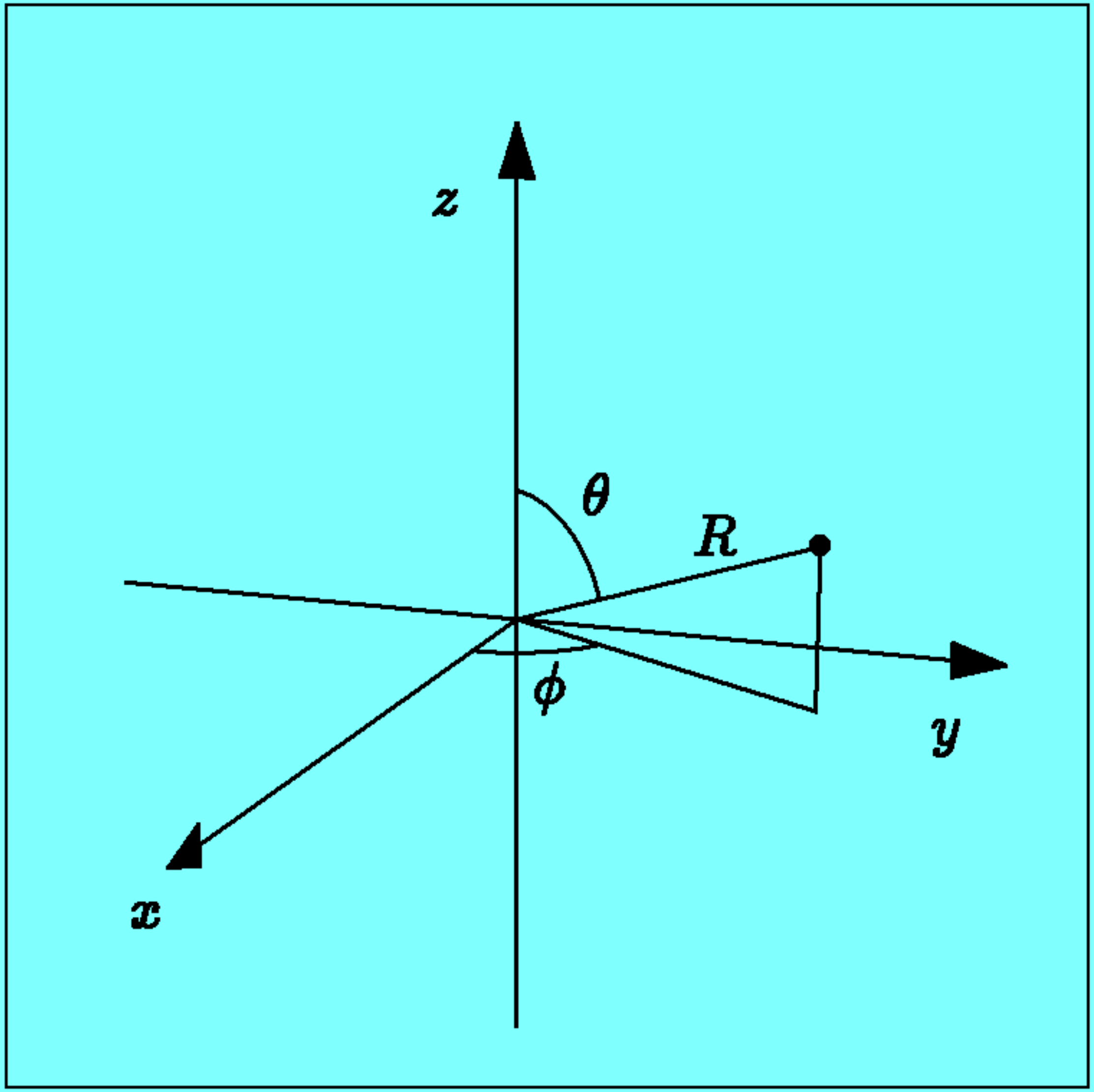
Non-Euclidean Geometry: The Surface of a Sphere



$$x^2 + y^2 + z^2 = R^2 \ .$$

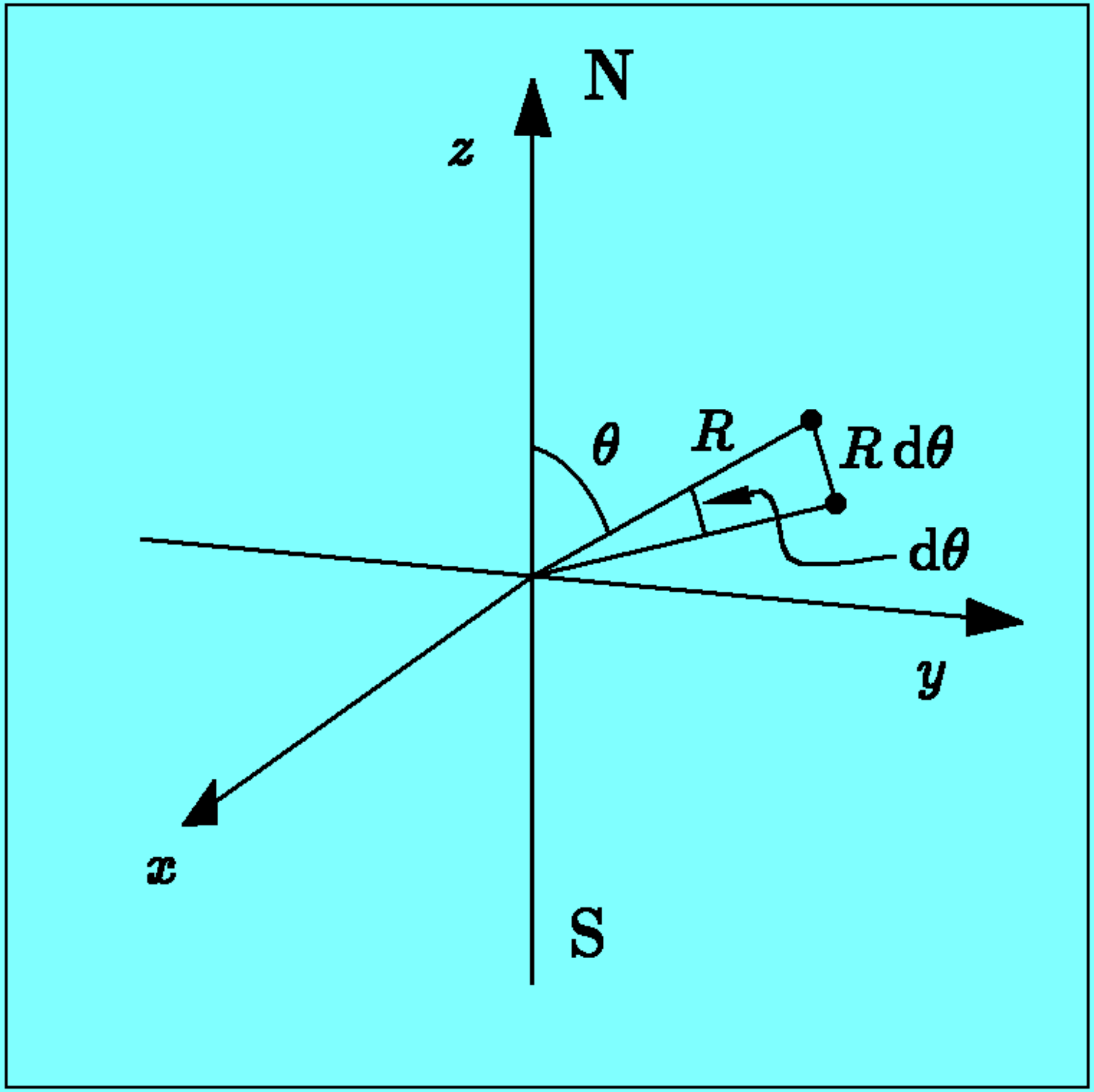
Polar Coordinates:

$$\boxed{\begin{aligned} x &= R\sin\theta\cos\phi \\ y &= R\sin\theta\sin\phi \\ z &= R\cos\theta\ , \end{aligned}}$$



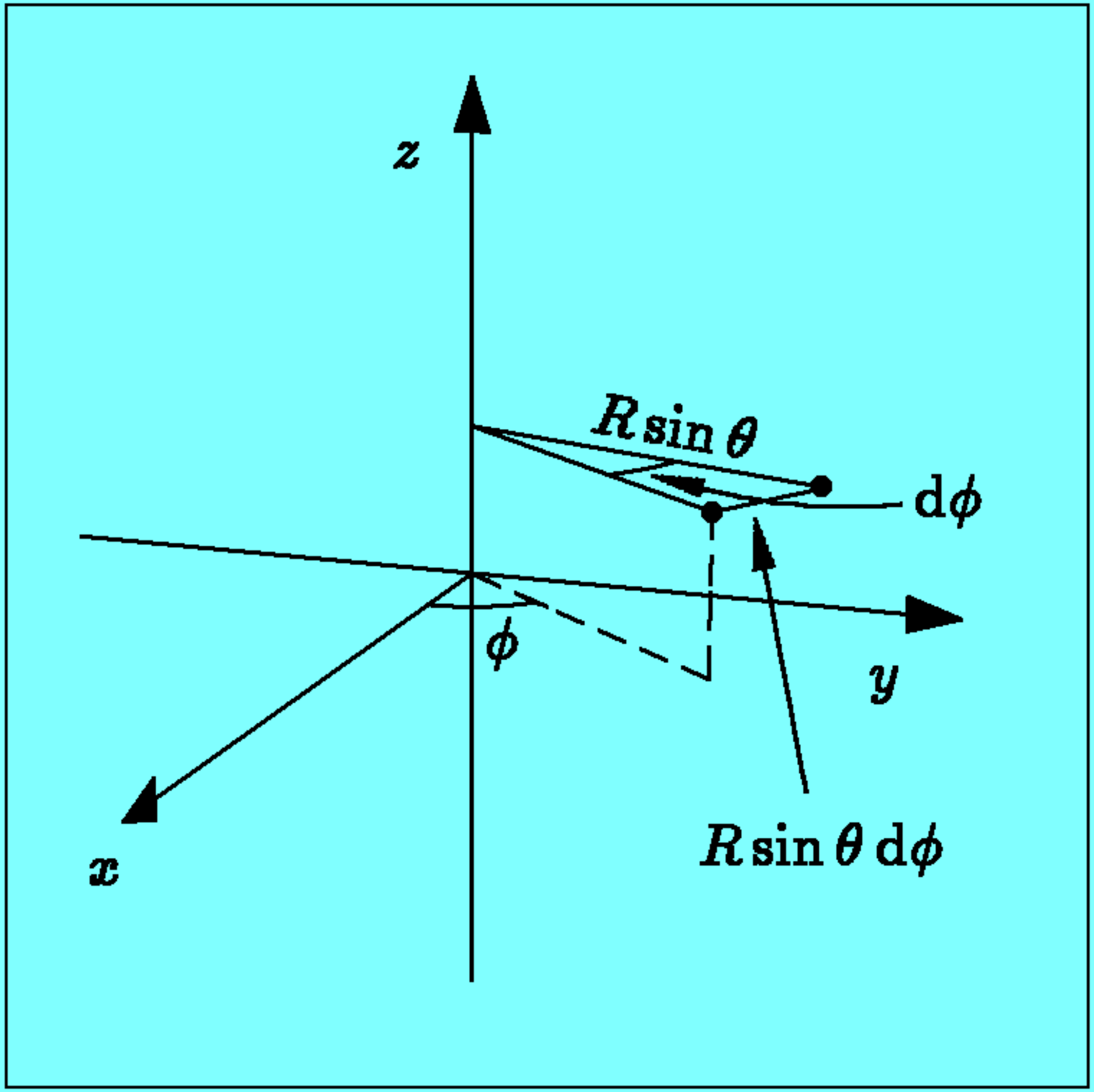
Varying θ:

$$ds = R\,d\theta$$



Varying ϕ:

$$ds = R \sin\theta \, d\phi$$



Varying θ and ϕ

Varying θ: $ds = R\,d\theta$

Varying ϕ: $ds = R\sin\theta\,d\phi$

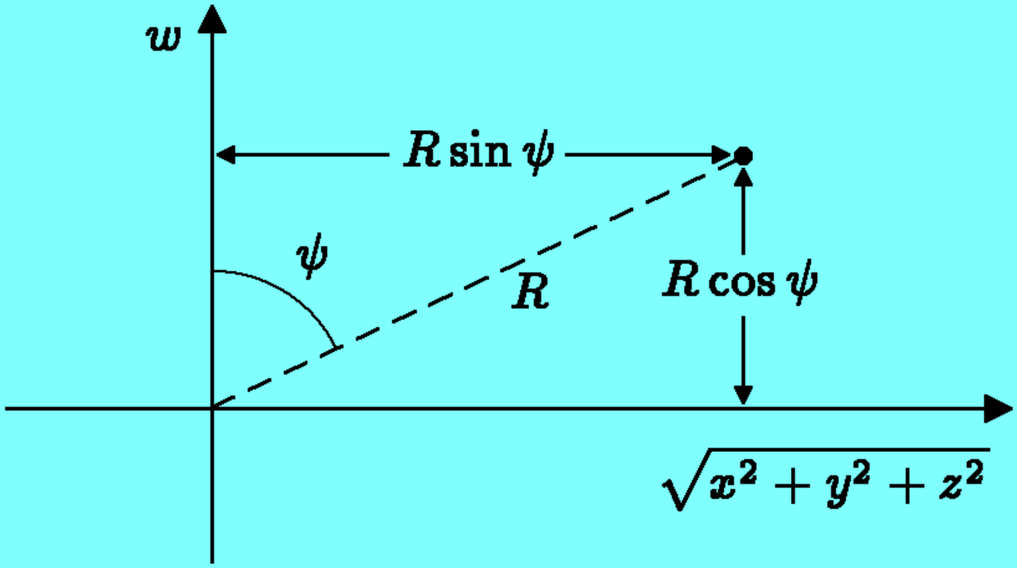
$$\boxed{ds^2 = R^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)}$$

A Closed Three-Dimensional Space

$$\boxed{x^2 + y^2 + z^2 + w^2 = R^2}$$

$$\begin{aligned} x &= R \sin\psi \sin\theta \cos\phi \\ y &= R \sin\psi \sin\theta \sin\phi \\ z &= R \sin\psi \cos\theta \\ w &= R \cos\psi \ , \end{aligned}$$

$$\boxed{ds = R\, d\psi}$$



Metric for the Closed 3D Space

Varying ψ: $\quad ds = R\, d\psi$

Varying θ or ϕ: $\quad ds^2 = R^2 \sin^2\psi (d\theta^2 + \sin^2\theta\, d\phi^2)$

If the variations are orthogonal to each other, then

$$ds^2 = R^2 \left[d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \right]$$

Proof of Orthogonality of Variations

Let $d\vec{r}_\psi$ = displacement of point when ψ is changed to $\psi + d\psi$.

Let $d\vec{r}_\theta$ = displacement of point when θ is changed to $\theta + d\theta$.

- $d\vec{r}_\theta$ has no w-component $\Longrightarrow$ $d\vec{r}_\psi \cdot d\vec{r}_\theta = d\vec{r}_\psi^{\,(3)} \cdot d\vec{r}_\theta^{\,(3)}$, where (3) denotes the projection into the x-y-z subspace.
- $d\vec{r}_\psi^{\,(3)}$ is radial; $d\vec{r}_\theta^{\,(3)}$ is tangential

$$\Longrightarrow \quad d\vec{r}_\psi^{\,(3)} \cdot d\vec{r}_\theta^{\,(3)} = 0$$

Implications of General Relativity

- $ds^2 = R^2 \left[d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$, where R is radius of curvature.
- According to GR, matter causes space to curve.
- R cannot be arbitrary. Instead, $R^2(t) = \dfrac{a^2(t)}{k}$.
- Finally,

$$\boxed{\mathrm{d}s^2 = a^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right\}} \quad ,$$

where $r = \dfrac{\sin\psi}{\sqrt{k}}$. Called the Robertson-Walker metric.

From Space to Spacetime

In special relativity,

$$s_{AB}^2 \equiv (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2 - c^2\,(t_A - t_B)^2 \quad .$$

s_{AB}^2 is Lorentz-invariant — it has the same value for all inertial reference frames.

Meaning of s_{AB}^2:

If positive, it is the distance between the two events in the inertial frame in which they are simultaneous. (Spacelike.)

If negative, it is the time interval between the two events in the inertial frame in which they occur at the same place. (Timelike.)

If zero, it implies that a light pulse could travel from the earlier to the later event.

Adding Time to the Robertson-Walker Metric

$$ds^2 = -c^2\,dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} .$$

Why does dt^2 term look like it does:

- The coefficient of dt^2 term must be independent of position, due to homogeneity.
- Terms such as $dt\,dr$ or $dt\,d\phi$ cannot appear, due to isotropy. That is, a term $dt\,dr$ would behave differently for $dr > 0$ and $dr < 0$, creating an asymmetry between the $+r$ and -r directions.
- The coefficient must be negative, to match the sign in Minkowski space for a locally free-falling coordinate system.

Adding Time to the Robertson-Walker Metric

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + a^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right\}\ .$$

Meaning:

- If $ds^2 > 0$, it is the square of the spatial separation measured by a local free-falling observer for whom the two events happen at the same time.
- If $ds^2 < 0$, it is $-c^2$ times the square of the time separation measured by a local free-falling observer for whom the two events happen at the same location.
- If $ds^2 = 0$, then the two events can be joined by a light pulse.