8.286 Class 15
October 26, 2020

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 3

Announcements

Quiz 2 will be this Wednesday, October 28. Procedures will be the same as for Quiz 1. Review Problems for Quiz 2 have been posted, and they contain a complete description of what will be covered on the quiz.

WARNING: Once again, I shout: **Don't let your wonderful success on Quiz 1 cause you to become complacent!** In 2018 the class average plummeted from 85.0 to 69.7 in going from the first quiz to the second. Don't let that happen this year!

Review session by Bruno Scheihing: Today, Monday 10/26/20, at 7:30 pm. If you have any problems or topics that you would particularly like Bruno to discuss, then email him!

Special office hours this week:

Bruno: Today, Monday 10/26/20 at 4:00 pm

Me: Tomorrow, Tuesday 10/27/20 at 6:00 pm

Recent posting: Compiled course documents (lecture notes, problem sets, solutions, quiz review problems, Quiz 1), on Lecture Notes page. Compiled lecture slides, on main web page and Lecture Notes page.

Exit Poll, Last Class

Polling 2: Exit poll Edit

Polling is closed 16 voted

1. How well were you able to follow this lecture?

Very well	(1) 6%
Well	(11) 69%
Borderline	(3) 19%
Badly	(0) 0%
Was mostly lost	(1) 6%

Share Results Re-launch Polling

Dynamics of the Radiation-Dominated Era

$$\rho \propto \frac{1}{a^3} \implies \dot\rho = -3\frac{\dot a}{a}\,\rho\ , \quad \rho(t) \propto \frac{1}{a^4(t)} \implies \dot\rho = -4\frac{\dot a}{a}\,\rho\ .$$

$\dot\rho$ and pressure p: (Problem 4, Problem Set 6)

$$dU = -p\,dV \qquad \implies \qquad \frac{d}{dt}\left(a^3\rho c^2\right) = -p\frac{d}{dt}(a^3)$$

$$\implies \qquad \dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right)\ .$$

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}$$

$$\ddot{a} = -\frac{4\pi}{3}G\rho a \ ,$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho$$

$$\left(\begin{array}{c}\text{matter-dominated}\\ \text{universe}\end{array}\right)$$

Any two of the above equations implies the third. So they become inconsistent if

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \ .$$

Review from last class

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$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)\ .$$

So, if we believe the equation for $\dot{\rho}$, we must modify one of the two Friedmann equations. First order equation represents conservation of energy: pressure does not belong! (Pressures can change suddenly, as when dynamite explodes, so it does not make sense to have pressure in a conservation equation.) So modify the 2nd order equation, deriving it from the first order equation and the $\dot{\rho}$ equation:

$$\ddot{a} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)a\ .$$

Summary: Complete Friedmann Equations and Energy Conservation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}$$

$$\ddot{a} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)a$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \ .$$

The items in red are new.

Dynamics of a Flat Radiation-dominated Universe

$$H^2 = \frac{8\pi G}{3}\rho \ , \ \rho \propto 1/a^4 \quad \Longrightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{\text{const}}{a^4} \ .$$

Then

$$a\,\mathrm{d}a = \sqrt{\text{const}}\,\mathrm{d}t \quad \Longrightarrow \quad \frac{1}{2}a^2 = \sqrt{\text{const}}\,t + \text{const}' \ .$$

So, setting our clocks so that $\text{const}' = 0$,

$$\boxed{a(t) \propto \sqrt{t} \qquad \text{(flat radiation-dominated)} \ .}$$

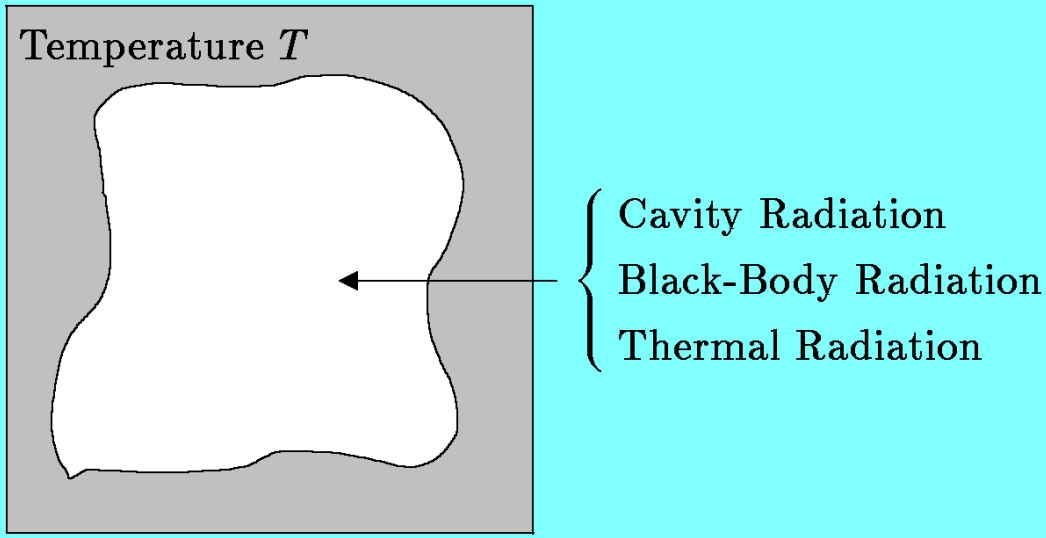
$$\boxed{H(t) = \frac{\dot{a}}{a} = \frac{1}{2t} \qquad \text{(flat radiation-dominated)} \ .}$$

$$\ell_{p,\text{horizon}}(t) = a(t) \int_0^t \frac{c}{a(t')} dt'$$

$$= \boxed{2ct \qquad \text{(flat radiation-dominated)} \ .}$$

$$H^2 = \frac{8\pi G}{3}\rho \quad \Longrightarrow \quad \boxed{\rho = \frac{3}{32\pi G t^2} \ .}$$

Black Body Radiation



- If a cavity is carved out of any material, and the walls are kept at a uniform temperature T, then the cavity will fill with radiation.
- If no radiation can get through the wall, then the energy density and spectrum of the radiation is determined by T alone — the material of the wall is irrelevant.
- The radiation is known as cavity radiation, black-body radiation, or thermal radiation.
- It can be thought of simply as radiation at temperature T.

Why Is It Called Black-Body?

- A black body at temperature T in empty space emits radiation with exactly this intensity and spectrum.
- Definitions:
  - A *black* object absorbs all light that hits it, reflecting none.
  - Reflection vs. emission: *reflection* is immediate. If the body aborbs radiation and emits it later, that is *emission*.
- Equilibrium: if a black body were placed in the cavity, it would reach an equilibrium in which no further energy would be exchanged. The body would be at the same temperature T as the box and the cavity radiation.
- Since the black body absorbs all the radiation that hits it, it must emit exactly this much radiation.

- Furthermore, in every frequency interval the block must emit exactly as much radiation as it absorbs.

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  Otherwise, we could imagine surrounding the body by a filter that transmits only in this frequency interval, and otherwise reflects. If the emission in this interval did not match the aborption, the body would then become hotter or colder than T, which violates a basic property of thermal equilibrium — once it is reached, the temperature will remain uniform, unless energy is exchanged with some external mechanism.

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- Thus, a black body at temperature T will emit with exactly the same intensity and spectrum as the radiation in the cavity.

Vague Description of the Black-Body Radiation Calculation

- We will leave the full derivation of black-body radiation to some stat mech class.
- But here we will summarize the basic ideas.
- **Prelude: The "equipartition theorem" of classical stat mech:** each degree of freedom of a system at temperature T acquires a mean thermal energy of $\frac{1}{2}kT$, where k = Boltzmann constant $= 8.617 \times 10^{-5}$ eV/K. For example, a gas of spinless particles has 3 degrees of freedom per atom: the x, y, and z components of velocity. In thermal equilbrium, the thermal energy is $\frac{3}{2}kT$ per particle. A harmonic oscillator has 2 degrees of freedom: its kinetic and potential energies.

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- **Jeans Catastrophe:** The number of modes is **infinite**, since there is no shortest wavelength. If classical physics applied, the electromagnetic field could never reach thermal equilibrium. Instead, it would continue to absorb energy, exciting shorter and shorter wavelength modes. It would be an infinite heat sink, absorbing all thermal energy.

☆ Quantum Theory to the Rescue:

- Classically, each mode can be excited by any amount.
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- For modes with $h\nu \gg kT$, the typical energy available ($\sim kT$) is much smaller than the minimum possible excitation ($h\nu$). These modes are excited only very rarely. The Jeans catastrophe is avoided, and the total energy density is finite.

Black-Body Radiation: Results

Energy Density:

$$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3}\ ,$$

where

$$\hbar = \frac{h}{2\pi} = 1.055\times 10^{-27}\ \text{erg-sec} = 6.582\times 10^{-16}\ \text{eV-sec}\ ,$$

and

$$g = 2 \quad (\text{for photons})\ .$$

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For photons, $g = 2$ because the photon has two polarizations, or equivalently, two spin states.

Other Properties

Pressure: $$p = \frac{1}{3} u \ .$$

Number Density: $$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \ ,$$

where $\zeta(3)$ is the Riemann zeta function evaluated at 3,

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \ ,$$

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g^* is used in the equation for the number density, rather than g, again to maximize reusability. For photons, $g^* = g$, but that won't be true for all particles.

ENTROPY!!

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- In our model of the universe, a huge amount of entropy was produced at the end of the period of inflation (to be discussed later), but the subsequent expansion and cooling of the universe happens at nearly constant entropy. Once stars form, entropy production resumes.

Entropy Density of Black-Body Radiation

The entropy density s of black-body radiation is given by

$$s = g\frac{2\pi^2}{45}\,\frac{k^4T^3}{(\hbar c)^3}\ .$$

The factor of g that appears here is the same g that occurs in the formulas for energy density and pressure. For photons, g is (still) 2.

Note that the entropy density, like the number density, is proportional to T^3. Thus the ratio

$$\frac{s}{n} = \frac{g}{g^*}3.60157\,k\ .$$

For the black-body radiation of photons, entropy is just another way to count photons, with 3.6 k units of entropy per photon.

Neutrinos — A Brief History

- In 1930, Wolfgang Pauli proposed the existence of the neutrino — an unseen particle that he theorized to explain how beta decay ($n \longrightarrow p + e^-$, inside a nucleus) could be consistent with energy conservation. (Niels Bohr, by contrast, proposed that energy conservation was only valid statistically.) Pauli called it a neutron, while the particle that we know as a neutron was not discovered until 1932, by James Chadwick.
- In 1934 Enrico Fermi developed a full theory of beta decay, and gave the neutrino its current name ("little neutral one").
- The neutrino was not seen observationally until 1956 by Clyde Cowan and Frederick Reines at the Savannah River nuclear reactor.
- Cowan died in 1974 at the age of 54, and Reines was awarded the Nobel Prize for this work in 1995, at the age of 77.

Neutrino Mass, Take 1

- During the 20th century, neutrinos were thought to be massless (rest mass $= 0$). We now know that they have a very small but nonzero mass, but for the period that we will be discussing now, the masses are negligible. As long as $mc^2 \ll kT$, the particle will act as if it is massless.
- So, for now (Take 1), we will pretend neutrinos are massless.

Photons are Bosons, Neutrinos are Fermions

- All particles can be divided into these two classes.
- For bosons, any number of particles can exist in the same quantum state. This is what allows photons to build up a classical electromagnetic field, which involves a very large number of photons. A laser in particular concentrates a huge number of photons in a single quantum state.
- For fermions, by contrast, there can be no more than one particle in a given quantum state. Electrons are also fermions — the one-electron-per-quantum-state rule is called the *Pauli Exclusion Principle*, and is responsible for essentially all of chemistry.
- In relativistic quantum field theory, one can prove the *spin-statistics theorem*: all particles with integer spin (in units of $\hbar$) are bosons, and all particles with half-integer spin ($\frac{1}{2}$, $\frac{3}{2}$, etc.) are fermions. (And those are the only possibilities.)

Consequences of Fermi Statistics

Reminder:

$$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3}\ , \qquad n = g^*\frac{\zeta(3)}{\pi^2}\,\frac{(kT)^3}{(\hbar c)^3}\ .$$

- ☆ Because there are fewer states that fermions can occupy, the number density, energy density, pressure, and entropy density for fermions are all reduced.
- ☆ For fermions,

g is reduced by a factor of 7/8.

g^* is reduced by a factor of 3/4.

Neutrino Flavors

Neutrinos come in 3 different species, or *flavors*:

Electron neutrino ν_e:	$e^- + p \longrightarrow n + \nu_e$
Muon neutrino ν_μ:	$\mu^- + p \longrightarrow n + \nu_\mu$
Tau neutrino ν_τ:	$\tau^- + p \longrightarrow n + \nu_\tau$.

A muon is essentially a heavy electron, with $m_\mu c^2 = 105.7$ MeV, compared to $m_e c^2 = 0.511$ MeV. A tau is a still heavier version of the electron, with $m_\tau c^2 = 1776.9$ MeV.

Neutrino States

- 3 flavors implies a factor of 3 in g and g^*.
- Neutrinos exist as particles and antiparticles, unlike photons, which are their own antiparticles. The particle/antiparticle option leads to a factor of 2 in g and g^*
- While photons can be left or right circularly polarized, neutrinos are always seen to be *left-handed*: the spin is opposite the direction of the momentum. Antineutrinos are always right-handed.

An Aside on Discrete Symmetries

- Before left-handed property of neutrinos was discovered, it was thought that that the laws of physics were invariant under *parity transformations* ($x \to -x, y \to -y, z \to -z$). But the parity transform of a left-handed neutrino would be a right-handed neutrino, which has never been seen, so the laws of physics are **NOT** parity-invariant.
- The handedness of neutrinos is consistent with CP symmetry, charge conjugation time parity. The CP transform of a left-handed neutrino is a right-handed antineutrino — both exist and, as far as we know, behave identically. However, CP symmetry is known to be violated by neutral kaons.
- However, CPT symmetry — charge conjugation times parity times time-reversal — is required by relativistic quantum field theory and is believed to be a symmetry of nature.

g and g^* for Neutrinos

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4}\,.$$

$$g_\nu^* = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2}\,.$$

Hotter Still

If we follow the universe further back in time, we will find that at some point kT becomes large compared to $m_e c^2 = 0.511$ MeV, the rest energy of an electron. Then electron-positron pairs start to behave as massless particles, and contribute to the black-body radiation.

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2}\,.$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3\,.$$