8.286 Class 17
November 2, 2020

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 4

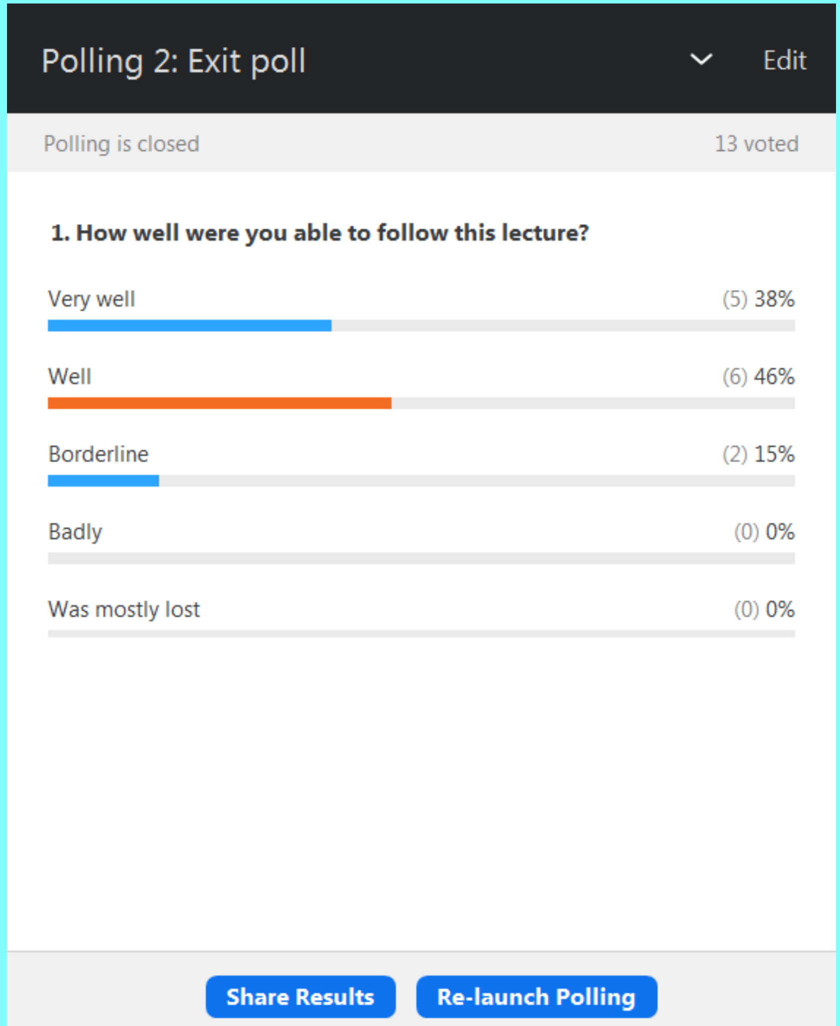
Announcements

Quiz 2 Results: **Wonderful!** Class average was 85.9. There was one perfect paper, one 98, two 95's, one 94, and three 93's.

Grades are posted. Quiz solutions are posted, and also a histogram of class grades, with letter grade cuts.

In this case the letter grade cuts are the same as Quiz 1.

Exit Poll, Last Class



Exit Poll Preview for Today

1. How well were you able to follow this lecture?

   1: Very well
   2: Well
   3: Borderline
   4: Badly
   5: Was mostly lost

2. How was the pace of the lecture?

   1: Too fast
   2: About right
   3: Too slow
   4: Uneven: parts too fast, parts too slow

Review from last class:

Black-Body Radiation: Results

Energy Density:

$$u = g\frac{\pi^2}{30}\ \frac{(kT)^4}{(\hbar c)^3}\ ,$$

where $g = 2$ for photons. The factor of g is included to make the formula reusable. To discuss the black-body radiation of neutrinos, e^+e^- pairs, muon-antimuon pairs, etc., we will only have to change the value of g.

g is taken to be 2 for photons because the photon has two polarizations, or equivalently, two spin states.

Review from last class:

Pressure and Number Density

Pressure: $$p = \frac{1}{3} u \ .$$

Number Density: $$n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \ ,$$

where $\zeta(3)$ is the Riemann zeta function evaluated at 3,

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \ ,$$

and $g^* = 2$ for photons. g^* is again introduced for reusability. For photons $g^* = g$, but that won't always be the case.

ENTROPY

- Entropy is often described as a measure of the "disorder" of the state of a physical system. Roughly, the entropy of a system is k times the logarithm of the number of microscopic quantum states that are consistent with its macroscropically observed state.
- Good news: we will not really need to know what entropy means! However, we will make much use of the fact that, as long as a system remains very close to thermal equilibrium, entropy is conserved. When departures from thermal equilibrium occur, the entropy always increases (a principle called the *second law of thermodynamics*).
- In our model of the universe, a huge amount of entropy was produced at the end of the period of inflation (to be discussed later), but the subsequent expansion and cooling of the universe happens at nearly constant entropy. Once stars form, entropy production resumes.

Entropy Density of Black-Body Radiation

The entropy density s of black-body radiation is given by

$$\boxed{s = g \frac{2\pi^2}{45} \, \frac{k^4 T^3}{(\hbar c)^3} \; .}$$

For photons, g is (still) 2.

Note that the entropy density, like the number density, is proportional to T^3. Thus the ratio

$$\boxed{\frac{s}{n} = \frac{g}{g^*} 3.60157 \, k \; .}$$

For the black-body radiation of photons, entropy is just another way to count photons, with 3.6 k units of entropy per photon.

Neutrinos — A Brief History

- ☆ In 1930, Wolfgang Pauli proposed the existence of the neutrino — an unseen particle that he theorized to explain how beta decay ($n \longrightarrow p + e^-$, inside a nucleus) could be consistent with energy conservation. (Niels Bohr, by contrast, proposed that energy conservation was only valid statistically.) Pauli called it a neutron, while the particle that we know as a neutron was not discovered until 1932, by James Chadwick.
- ☆ In 1934 Enrico Fermi developed a full theory of beta decay, and gave the neutrino its current name ("little neutral one").
- ☆ The neutrino was not seen observationally until 1956 by Clyde Cowan and Frederick Reines at the Savannah River nuclear reactor.
- ☆ Cowan died in 1974 at the age of 54, and Reines was awarded the Nobel Prize for this work in 1995, at the age of 77.

Neutrino Mass, Take 1

- During the 20th century, neutrinos were thought to be massless (rest mass $= 0$). We now know that they have a very small but nonzero mass, but for the period that we will be discussing now, the masses are negligible. As long as $mc^2 \ll kT$, the particle will act as if it is massless.
- So, for now (Take 1), we will pretend neutrinos are massless.

Review from last class:

Photons are Bosons, Neutrinos are Fermions

- ☆ All particles can be divided into these two classes.
- ☆ **For bosons, any number of particles can exist in the same quantum state.** This is what allows photons to build up a classical electromagnetic field, which involves a very large number of photons. A laser in particular concentrates a huge number of photons in a single quantum state.
- ☆ **For fermions, by contrast, there can be no more than one particle in a given quantum state.** Electrons are also fermions — the one-electron-per-quantum-state rule is called the *Pauli Exclusion Principle*, and is responsible for essentially all of chemistry.
- ☆ In relativistic quantum field theory, one can prove the *spin-statistics theorem*: all particles with integer spin (in units of $\hbar$) are bosons, and all particles with half-integer spin ($\frac{1}{2}$, $\frac{3}{2}$, etc.) are fermions. (And those are the only possibilities.)

Consequences of Fermi Statistics

Reminder:

$$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3}\;, \qquad n = g^*\frac{\zeta(3)}{\pi^2}\,\frac{(kT)^3}{(\hbar c)^3}\;.$$

- Because there are fewer states that fermions can occupy, the number density, energy density, pressure, and entropy density for fermions are all reduced.
- For fermions,

g is reduced by a factor of $7/8$.

g^* is reduced by a factor of $3/4$.

Neutrino Flavors

Neutrinos come in 3 different species, or *flavors*:

$$\begin{aligned} &\text{Electron neutrino } \nu_e: && e^- + p \longrightarrow n + \nu_e \\ &\text{Muon neutrino } \nu_\mu: && \mu^- + p \longrightarrow n + \nu_\mu \\ &\text{Tau neutrino } \nu_\tau: && \tau^- + p \longrightarrow n + \nu_\tau \ . \end{aligned}$$

A muon is essentially a heavy electron, with $m_\mu c^2 = 105.7$ MeV, compared to $m_e c^2 = 0.511$ MeV. A tau is a still heavier version of the electron, with $m_\tau c^2 = 1776.9$ MeV.

Neutrino States

- 3 flavors implies a factor of 3 in g and g^*.
- Neutrinos exist as particles and antiparticles, unlike photons, which are their own antiparticles. The particle/antiparticle option leads to a factor of 2 in g and g^*
- While photons can be left or right circularly polarized, neutrinos are always seen to be *left-handed*: the spin is opposite the direction of the momentum. Antineutrinos are always right-handed.

An Aside on Discrete Symmetries

- Before left-handed property of neutrinos was discovered, it was thought that that the laws of physics were invariant under *parity transformations* $(x \to -x, y \to -y, z \to -z)$. But the parity transform of a left-handed neutrino would be a right-handed neutrino, which has never been seen, so the laws of physics are **NOT** parity-invariant.

- The handedness of neutrinos is consistent with CP symmetry, charge conjugation time parity. The CP transform of a left-handed neutrino is a right-handed antineutrino — both exist and, as far as we know, behave identically. However, CP symmetry is known to be violated by neutral kaons.

- However, CPT symmetry — charge conjugation times parity times time-reversal — is required by relativistic quantum field theory and is believed to be a symmetry of nature.

g and g^* for Neutrinos

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4} \; .$$

$$g_\nu^* = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e, \nu_\mu, \nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2} \; .$$

Hotter Still

If we follow the universe further back in time, we will find that at some point kT becomes large compared to $m_e c^2 = 0.511$ MeV, the rest energy of an electron. Then electron-positron pairs start to behave as massless particles, and contribute to the black-body radiation.

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2}\,.$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3\,.$$

For 0.511 MeV $\ll kT \ll$ 106 MeV

For electrons, $m_e c^2 = 0.511$ MeV.

For muons, $m_\mu c^2 = 106$ MeV.

For 0.511 MeV $\ll kT \ll$ 106 MeV, electrons and positrons act like massless particles, and only a negligible number of muons would be produced.

The energy density can therefore be calculated from

$$\boxed{g_{\text{tot}} = \underbrace{2}_{\text{photons}} + \underbrace{\frac{21}{4}}_{\text{neutrinos}} + \underbrace{\frac{7}{2}}_{e^+e^-} = 10\,\frac{3}{4}}$$

Energy Density of Radiation Today

- Temperature of the cosmic microwave background (CMB) today: $\boxed{T_\gamma = 2.7255 \pm 0.0006 \text{ K.}^*}$ This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.

*D.J. Fixsen, Ap. J. **707,** 916 (2009). Based mainly on the COBE (Cosmic Background Explorer) data, 1989 – 1993.

Energy Density of Radiation Today

- ☆ Temperature of the cosmic microwave background (CMB) today: $\boxed{T_\gamma = 2.7255 \pm 0.0006 \text{ K.}^*}$ This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.

- ☆ Continuing our "Take 1" pretense that neutrinos are massless, the radiation that exists in the universe today includes photons and neutrinos.

*D.J. Fixsen, Ap. J. **707,** 916 (2009). Based mainly on the COBE (Cosmic Background Explorer) data, 1989 – 1993.

Energy Density of Radiation Today

- Temperature of the cosmic microwave background (CMB) today: $\boxed{T_\gamma = 2.7255 \pm 0.0006 \text{ K.}^*}$ This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.
- Continuing our "Take 1" pretense that neutrinos are massless, the radiation that exists in the universe today includes photons and neutrinos. **But $T_\nu \neq T_\gamma$.**

*D.J. Fixsen, Ap. J. **707**, 916 (2009). Based mainly on the COBE (Cosmic Background Explorer) data, 1989 – 1993.

Energy Density of Radiation Today

- Temperature of the cosmic microwave background (CMB) today: $\boxed{T_\gamma = 2.7255 \pm 0.0006 \text{ K.}^*}$ This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.

- Continuing our "Take 1" pretense that neutrinos are massless, the radiation that exists in the universe today includes photons and neutrinos. **But $T_\nu \neq T_\gamma$.**

- The complication occurs when the e^+e^- pairs "freeze out," (i.e., disappear), as kT falls below 0.511 MeV. This happens around $t = 1$ second. Neutrino interactions become weaker as the temperature falls, and by this time they have become so weak that the neutrinos absorb only a negligible amount of the e^+e^- energy. It essentially all goes into heating the photons, which then become hotter than the neutrinos.

*D.J. Fixsen, Ap. J. **707**, 916 (2009). Based mainly on the COBE (Cosmic Background Explorer) data, 1989 – 1993.

- The complication occurs when the e^+e^- pairs "freeze out," (i.e., disappear), as kT falls below 0.511 MeV. This happens around $t = 1$ second. Neutrino interactions become weaker as the temperature falls, and by this time they have become so weak that the neutrinos absorb only a negligible amount of the e^+e^- energy. It essentially all goes into heating the photons, which then become hotter than the neutrinos.

- You will calculate this on Problem Set 7. The key is to use *entropy*, not energy, since entropy is simply conserved. Energy density, by contrast, obeys

$$\boxed{\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \ ,}$$

so one needs to calculate the pressure p as the e^+e^- pairs freeze out. That's complicated.

- The result (that you will find) is

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \ .}$$

- This ratio is maintained to the present day, so the total radiation energy density today is

$$\boxed{\begin{aligned} u_{\mathrm{rad},0} &= \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3} \\ &= 7.01 \times 10^{-14}\ \mathrm{J/m^3}\ , \end{aligned}}$$

which is what we used when we estimated t_{eq}, the time of matter-radiation equality.

- We (crudely) found $\sim 75,000$ years. Ryden gives 47,000 years. The Particle Data Group (2020) gives $51,100 \pm 800$ years.

The Real Story of Neutrino Masses

- ☆ We have not yet measured the mass of a neutrino, but we have seen neutrinos "oscillate" from one flavor to another:
  - Electron neutrinos from the Sun arrive at Earth as a mixture of all three flavors.
  - Neutrinos produced by cosmic rays in the upper atmosphere have been found to undergo oscillations on their way to ground level.
  - Neutrinos produced by reactors and accelerators have been seen to oscillate.
- ☆ Oscillations require a nonzero mass: esentially because a massless particle experiences an infinite time dilation, so time stops.
- ☆ The oscillations measure the differences of the squares of the masses.

Neutrino Masses and Quantum Superpositions

- ☆ Quantum theory allows for states that are superpositions of other states.
- ☆ Neutrinos are produced in states of definite flavor, called ν_e, ν_μ, and ν_τ. But these are not states of definite mass!
- ☆ The states of definite mass are called ν_1, ν_2, and ν_3.
- ☆ Each flavor state is a superposition of all three states of definite mass, and each state of definite mass is a superposition of all three flavor states.

Differences of Squares of Neutrino Masses

As of 2020, the Particle Data Group reports:

$$\Delta m_{21}^2\, c^4 = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2\ ,$$

$$\Delta m_{32}^2\, c^4 = \left(2.546^{+0.034}_{-0.040}\right) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

or

$$\Delta m_{32}^2\, c^4 = (2.453 \pm 0.034) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

where the two options for Δm_{32}^2 depend on assumptions about the ordering of the masses. Note that $\sqrt{\Delta m_{21}^2\, c^4} = 8.68 \times 10^{-3}$ eV, and $\sqrt{\Delta m_{32}^2\, c^4} = 0.0505$ eV or 0.0495 eV. Recall that $kT_\gamma = 2.35 \times 10^{-4}$ eV, which is much smaller.

Does Neutrino Mass Affect Our Calculation of $t_{\rm eq}$?

Does Neutrino Mass Affect Our Calculation of t_{eq}?

- No.

Does Neutrino Mass Affect Our Calculation of $t_{\rm eq}$?

- No.
- But we calculated the present neutrino energy density assuming that the neutrinos were massless?

Does Neutrino Mass Affect Our Calculation of t_{eq}?

- No.
- But we calculated the present neutrino energy density assuming that the neutrinos were massless?
- But the neutrinos were effectively massless in the early universe, and that justifies our calculation. Our calculation of the present radiation energy density was fictional. But we could have done the calculation correctly by calculating the ratio of the neutrino to photon energy densities after e^+e^- freeze-out, using the $(4/11)^{1/3}$ temperature ratio, and using the present T_γ to determine the amount of expansion between then and now. This calculation would get exactly the same answer as we got.

Cosmological Bound on the Sum of ν Masses

- ☆ From cosmology of large-scale structure, we know that

$$\boxed{(m_1 + m_2 + m_3)c^2 \leq 0.17 \text{ eV.}}$$

- ☆ Why? Because neutrinos "free-stream" easily from one place to another. If they carried too much mass, they would even out the mass density and suppress large-scale structure.

Neutrino Mass and Spin States

- ☆ The measurements of the mass differences imply that at least 2 of the 3 neutrino masses must be nonzero.
- ☆ If the mass of a neutrino is nonzero, then it **cannot** always be left-handed.
- ☆ To see this, consider a left-handed neutrino moving in the z direction, with spin in the $-z$ direction. With $m > 0$, it must move slower than c. So an observer can move along the z-axis faster than the neutrino. To such an observer, the momentum of the ν will in the $-z$ direction, the spin will be in the $-z$ direction, and the ν will appear right-handed.
- ☆ How could this right-handed neutrino fit into our theory?

Majorana and Dirac Masses

There are two possibilities for neutrino mass:

Dirac Mass: Right-handed neutrino would be a new as-yet unseen type of particle. But it would interact so weakly that it would not have been produced in significant numbers during the big bang.

Majorana Mass: If *lepton number* is not conserved (which seems plausible), so the neutrino is absolutely neutral, then the right-handed neutrino could be the particle that we have called the anti-neutrino.

Neutrino Masses and Neutrinoless Double Beta Decay

☆ Key experiment to distinguish Majorana from Dirac mass: neutrinoless double beta decay. Standard double beta decay looks like

$$\boxed{(A, Z) \to (A, Z+2) + 2e^- + 2\bar{\nu}_e \ .}$$

If the ν has a Majorana mass, and therefore it is its own antiparticle, then the reaction could happen without the two final $\bar{\nu}_e$'s, which can essentially annihilate each other. (The annihilation could happen as part of the interaction, so the energy is given to the $(A, Z+2)$ and $2e^-$ particles, with no other particles emitted.)