8.286 Class 18
November 4, 2020

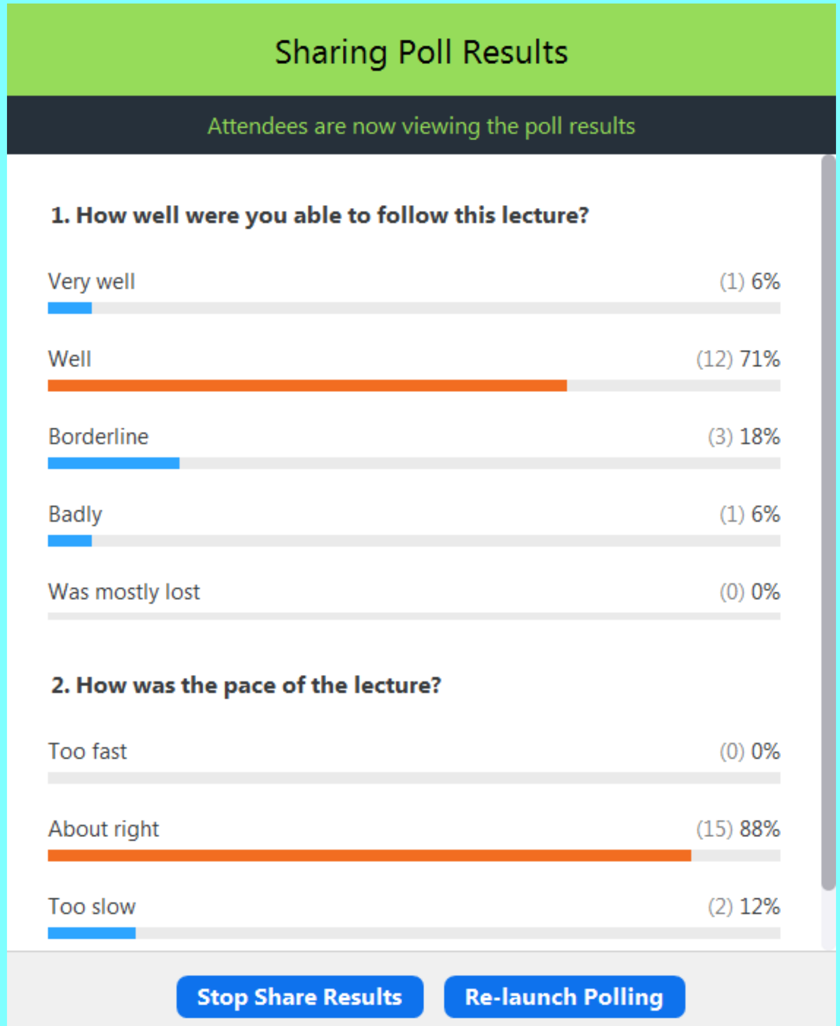
BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 5

(Modified 12/27/20 to fix minor typos on pages 9 and 13, and to add a reference on p. 12.)

Announcements

Problem Set 7 is due this Friday.

Exit Poll, Last Class



Review from last class:

Black-Body Radiation

Energy density: $$u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3}$$

Pressure: $$p = \frac{1}{3}u$$

Number density: $$n = g^*\frac{\zeta(3)}{\pi^2}\,\frac{(kT)^3}{(\hbar c)^3}$$

Entropy density: $$s = g\frac{2\pi^2}{45}\,\frac{k^4T^3}{(\hbar c)^3}\ ,$$

where

$g =$ number of spin states, times $7/8$ for fermions

$g^* =$ number of spin states, times $3/4$ for fermions .

Review from last class:

When $kT \gg m_e c^2$

If we follow the universe further back in time, we will find that at some point kT becomes large compared to $m_e c^2 = 0.511$ MeV, the rest energy of an electron. Then electron-positron pairs start to behave as massless particles, and contribute to the black-body radiation.

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2} \,.$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3 \,.$$

Neutrinos

- ☆ Neutrinos are fermions (only one particle in the same quantum state, as opposed to bosons)
- ☆ For early universe calculations (until the time of structure formation), neutrinos can be treated as if they are massless, and always left-handed (spin is opposite momentum). Anti-neutrinos are right-handed.
- ☆ Left-handedness of neutrinos violates P symmetry (parity), but is consistent with CP (charge-conjugation $\times$ parity). CP is not exact, but CPT (T = time-reversal symmetry) is required by relativistic quantum field theory and appears to be exact.
- ☆ Neutrinos have three possible flavors: ν_e, ν_μ, and ν_τ.

g and g^* for Neutrinos

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4} \,.$$

$$g^*_\nu = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2} \,.$$

Review from last class:

Energy Density of Radiation Today

- ☆ Temperature of the cosmic microwave background (CMB) today: $\boxed{T_\gamma = 2.7255 \pm 0.0006 \text{ K.}^*}$ This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.

- ☆ Continuing our "Take 1" pretense that neutrinos are massless, the radiation that exists in the universe today includes photons and neutrinos, but $T_\nu \neq T_\gamma$.

- ☆ The complication occurs when the e^+e^- pairs "freeze out," (i.e., disappear), as kT falls below 0.511 MeV. This happens around $t = 1$ second. Neutrino interactions become weaker as the temperature falls, and by this time they have become so weak that the neutrinos absorb only a negligible amount of the e^+e^- energy. It essentially all goes into heating the photons, which then become hotter than the neutrinos. The heating of the photons is calculated by using conservation of entropy (not energy).

- You will find on Problem Set 7 that

$$\boxed{T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \ .}$$

- This ratio is maintained to the present day, so the total radiation energy density today is

$$\boxed{\begin{aligned} u_{\text{rad},0} &= \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3} \\ &= 7.01 \times 10^{-14}\ \text{J/m}^3 \ , \end{aligned}}$$

  which is what we used when we estimated t_{eq}, the time of matter-radiation equality.

- We (crudely) found $\sim 75{,}000$ years. Ryden gives 47,000 years. The Particle Data Group (2020) gives $51{,}100 \pm 800$ years.

The Real Story of Neutrino Masses

- ☆ We know that neutrinos have a nonzero mass, not because we have measured it, but because we see neutrinos oscillate: one flavor can evolve into the other flavors.
- ☆ Oscillations require a nonzero mass: essentially because a massless particle experiences an infinite time dilation, so time stops.
- ☆ Quantum theory allows for states that are superpositions of other states.
- ☆ Neutrinos are produced in states of definite flavor, called ν_e, ν_μ, and ν_τ. But these are not states of definite mass!
- ☆ The states of definite mass are called ν_1, ν_2, and ν_3.
- ☆ Each flavor state is a superposition of all three states of definite mass, and each state of definite mass is a superposition of all three flavor states.

Differences of Squares of Neutrino Masses

As of 2020, the Particle Data Group reports:

$$\Delta m^2_{21}\, c^4 = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2\ ,$$

$$\Delta m^2_{32}\, c^4 = \left(2.546^{+0.034}_{-0.040}\right) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

or

$$\Delta m^2_{32}\, c^4 = (2.453 \pm 0.034) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

where the two options for Δm^2_{32} depend on assumptions about the ordering of the masses. Note that $\sqrt{\Delta m^2_{21}\, c^4} = 8.68 \times 10^{-3}$ eV, and $\sqrt{\Delta m^2_{32}\, c^4} = 0.0505$ eV or 0.0495 eV. Recall that $kT_\gamma = 2.35 \times 10^{-4}$ eV, which is much smaller.

Does Neutrino Mass Affect Our Calculation of $t_{\rm eq}$?

No!

We wrote

$$u_{\rm rad,0} = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right]\frac{\pi^2}{30}\frac{(kT_\gamma)^4}{(\hbar c)^3}$$
$$= 7.01 \times 10^{-14}\ \mathrm{J/m^3}\ ,$$

but what we really used was

$$u_{\rm rad}(t) = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right]\frac{\pi^2}{30}\frac{(kT_\gamma)^4}{(\hbar c)^3}\left(\frac{a(t_0)}{a(t)}\right)^4\ ,$$

which is valid for t anywhere near the time $t_{\rm eq}$.

Cosmological Bound on the Sum of ν Masses

☆ From cosmology of large-scale structure, we know that*

$$(m_1 + m_2 + m_3)c^2 \leq 0.17 \text{ eV}.$$

☆ Why? Because neutrinos "free-stream" easily from one place to another. If they carried too much mass, they would even out the mass density and suppress large-scale structure.

*S. R. Choudhury and S. Hannestad, JCAP 2020, No. 7, 037 (2020), arXiv:1907.12598.

Neutrino Mass and Spin States

- ☆ The measurements of the mass differences imply that at least 2 of the 3 neutrino masses must be nonzero.
- ☆ If the mass of a neutrino is nonzero, then it **cannot** always be left-handed.
- ☆ To see this, consider a left-handed neutrino moving in the z direction, with spin in the $-z$ direction. With $m > 0$, it must move slower than c. So an observer can move along the z-axis faster than the neutrino. To such an observer, the momentum of the ν will be in the $-z$ direction, the spin will be in the $-z$ direction, and the ν will appear right-handed. What is this particle?
- ☆ There are two possibilities for neutrino mass:
  - Dirac Mass: Right-handed neutrino would be a new as-yet unseen type of particle. But it would interact so weakly that it would not have been produced in significant numbers during the big bang.
  - Majorana Mass: If *lepton number* is not conserved (which seems plausible), so the neutrino is absolutely neutral, then the right-handed neutrino could be the particle that we have called the anti-neutrino.

Thermal History of the Universe

Assuming that the early universe can be described as radiation-dominated and flat (excellent approximations), then

$$H^2 = \frac{8\pi}{3} G\rho \,, \quad a(t) \propto t^{1/2} \,, \quad H = \frac{\dot{a}}{a} = \frac{1}{2t} \,,$$

which implies

$$\boxed{\rho = \frac{3}{32\pi G t^2} \,.}$$

We also know

$$u = g\frac{\pi^2}{30} \frac{(kT)^4}{(\hbar c)^3} \,, \quad \text{and} \quad \rho = u/c^2 \,,$$

so

$$\boxed{kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}} \,.}$$

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Assuming $0.511 \text{ MeV} \ll kT \ll 106 \text{ MeV}$ (i.e., assuming kT is between mc^2 for the electron and muon),

$$g_{\text{tot}} = \underbrace{2}_{\text{photons}} + \underbrace{\frac{21}{4}}_{\text{neutrinos}} + \underbrace{\frac{7}{2}}_{e^+e^-} = 10\frac{3}{4} \ .$$

For $t = 1$ second, this gives $kT = 0.860$ MeV.

Assuming 0.511 MeV $\ll kT \ll$ 106 MeV (i.e., assuming kT is between mc^2 for the electron and muon), we find that at $t = 1$ second, $kT = 0.860$ MeV.

Since $T \propto 1/\sqrt{t}$, we can write

$$\boxed{kT = \frac{0.860 \text{ MeV}}{\sqrt{t \text{ (in sec)}}} \quad ,}$$

or equivalently

$$\boxed{T = \frac{9.98 \times 10^9 \text{ K}}{\sqrt{t \text{ (in sec)}}} \quad .}$$

Relation Between a and T

- Conservation of entropy implies that

$$\boxed{s \propto 1/a^3(t)\ .}$$

- But we also know that

$$\boxed{s \propto gT^3\ .}$$

- It follows that

$$\boxed{g^{1/3}T \propto \frac{1}{a(t)}\ .}$$

Recombination

- “Baryonic” matter is matter made of protons, neutrons, and electrons. I.e., it is ordinary matter, as opposed to dark matter or dark energy.
- About 80% of baryonic matter is hydrogen. Most of the rest is helium. Elements heavier then helium make up a very small fraction. So we mostly have hydrogen.
- At high T, hydrogen atoms ionize, become free protons and electrons. The ionization temperature depends on density, but for the density of the early universe, it is about 4,000 K. (Ryden calculates it on p. 154 as 3760 K.)
- When T falls below 4,000 K, the protons and electrons combine to form neutral H. This is called “recombination,” but “combination” would be more accurate.

Decoupling

- Photons interact strongly with free electrons.
- The reason can be understood classically: when an electromagnetic wave hits a free electron, the electron experiences the $\vec{F} = e\vec{E}$ force of the electric field. Since its mass is very small, it oscillates rapidly, and sends electromatic radiation in all directions, using energy that it removes from the incoming wave. Thus, the incoming wave is scattered.
- The result is that the universe was opaque to photons in the ionized phase (plasma phase), but became transparent when the ionized gas became neutral atoms.
- The transition to a transparent universe is called "decoupling" (i.e., the photons "decouple" from the matter of the universe).

Time of Decoupling t_d

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- The transition to a transparent universe is called "decoupling" (i.e., the photons "decouple" from the matter of the universe).

- At $T_{\rm rec} = 4,000$ K, about half of the hydrogen is ionized. Note that $KT_{\rm rec} \approx 0.34$ eV, while the ionization energy of H is 13.6 eV.
- Since even a very small density of free electrons is enough to make the universe opaque, photon decoupling does not occur until T falls to $T_{\rm dec} \approx 3,000$ K.

★ Since even a very small density of free electrons is enough to make the universe opaque, photon decoupling does not occur until T falls to $T_{\rm dec} \approx 3,000$ K.

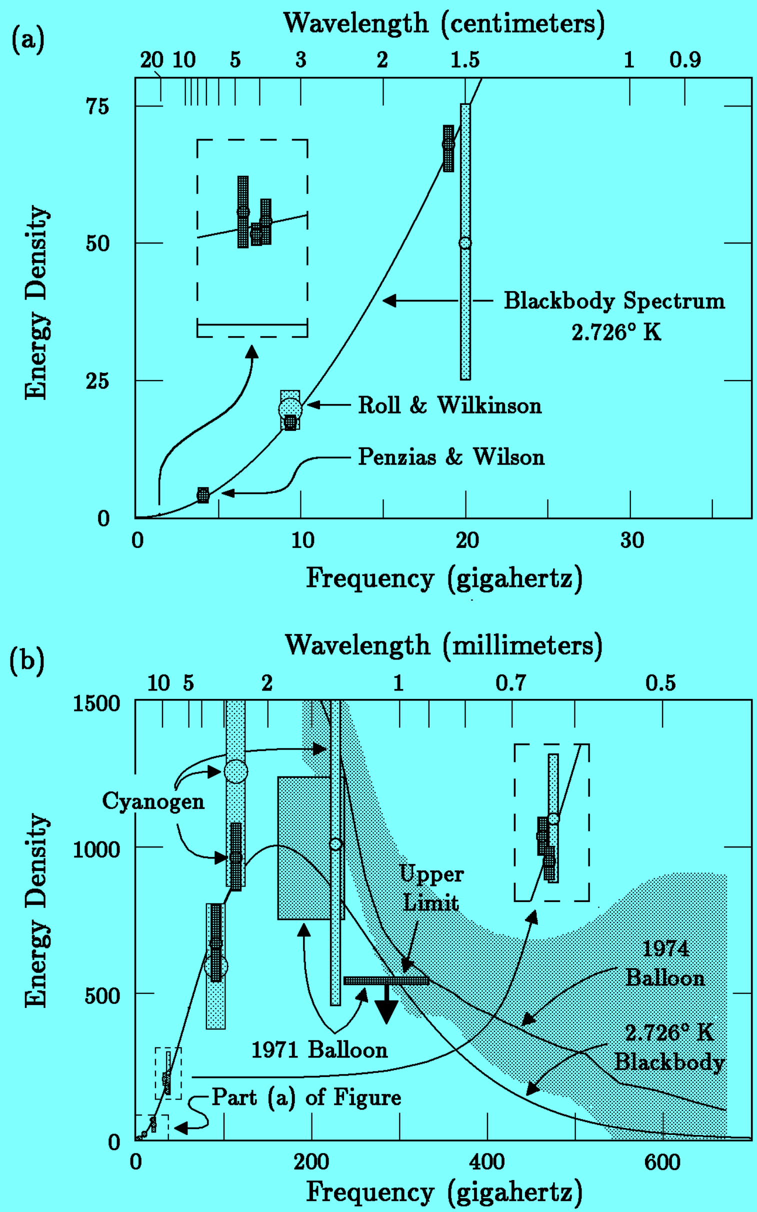
★ Approximating the universe as flat and matter-dominated from $T_{\rm dec}$ to today, we can estimate the time of decoupling by

$$t_d = \left(\frac{T_0}{T_d}\right)^{3/2} t_0$$

$$\approx \left(\frac{2.7\,\mathrm{K}}{3000\,\mathrm{K}}\right)^{3/2} \times \left(13.7 \times 10^9 \text{ yr}\right) \approx 370,000 \text{ yr} .$$

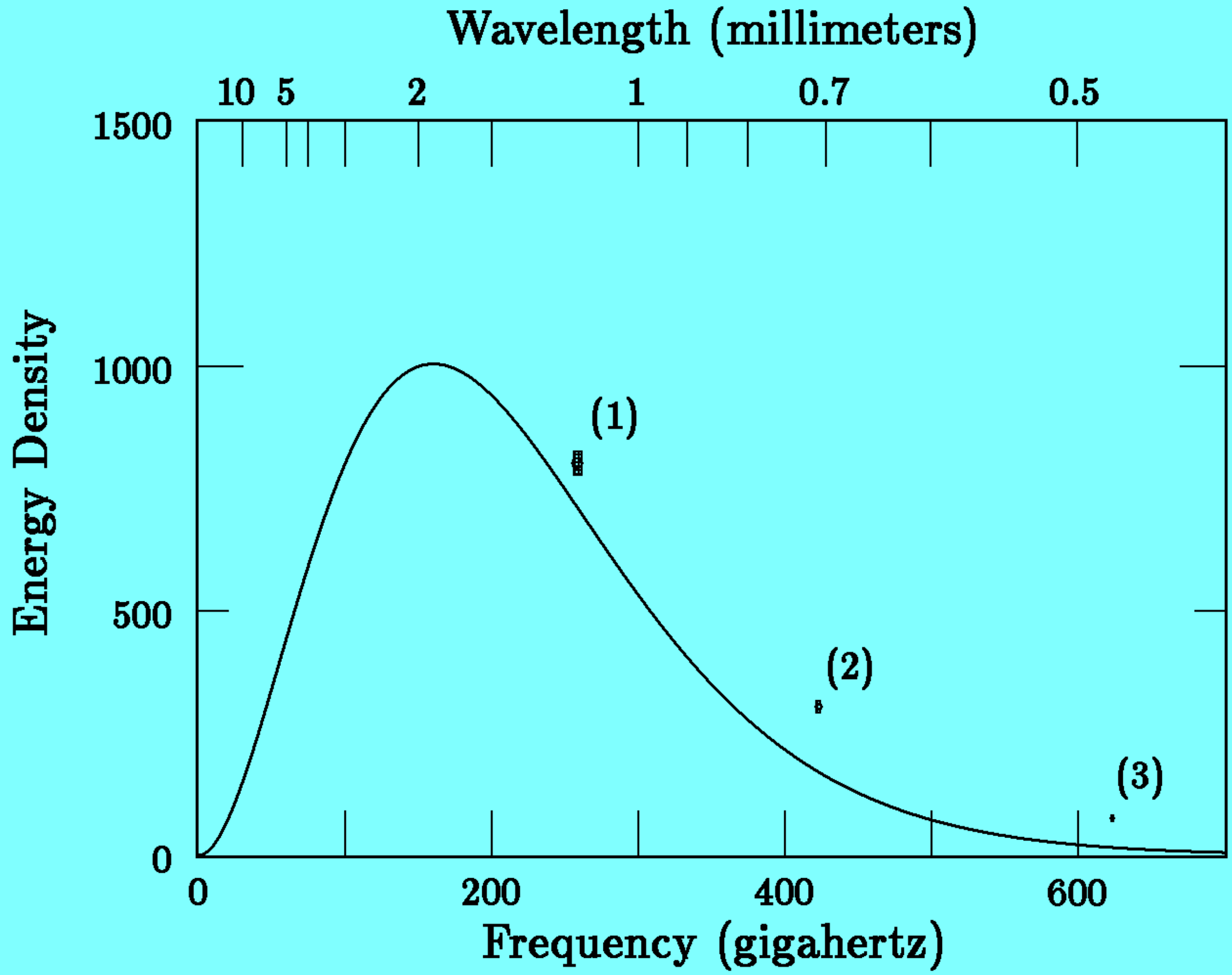
Spectrum of the Cosmic Microwave Background

$$\rho_\nu(\nu)d\nu = \frac{16\pi^2\hbar\nu^3}{c^3}\frac{1}{e^{2\pi\hbar\nu/kT}-1}d\nu \ .$$



CMB Data in 1975

This and the following slides were not reached, but will be discussed in the next class.



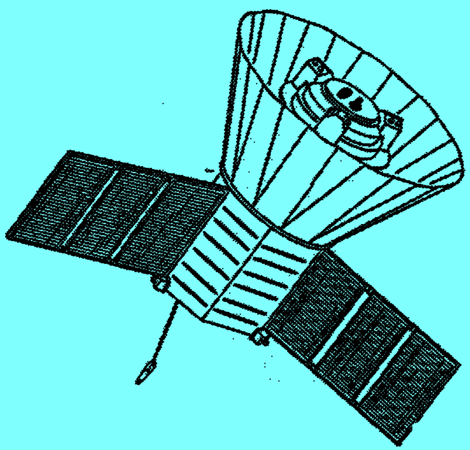
Data from Berkeley-Nagoya Rocket Flight, 1987

Preprint No. 90–01

COBE PREPRINT

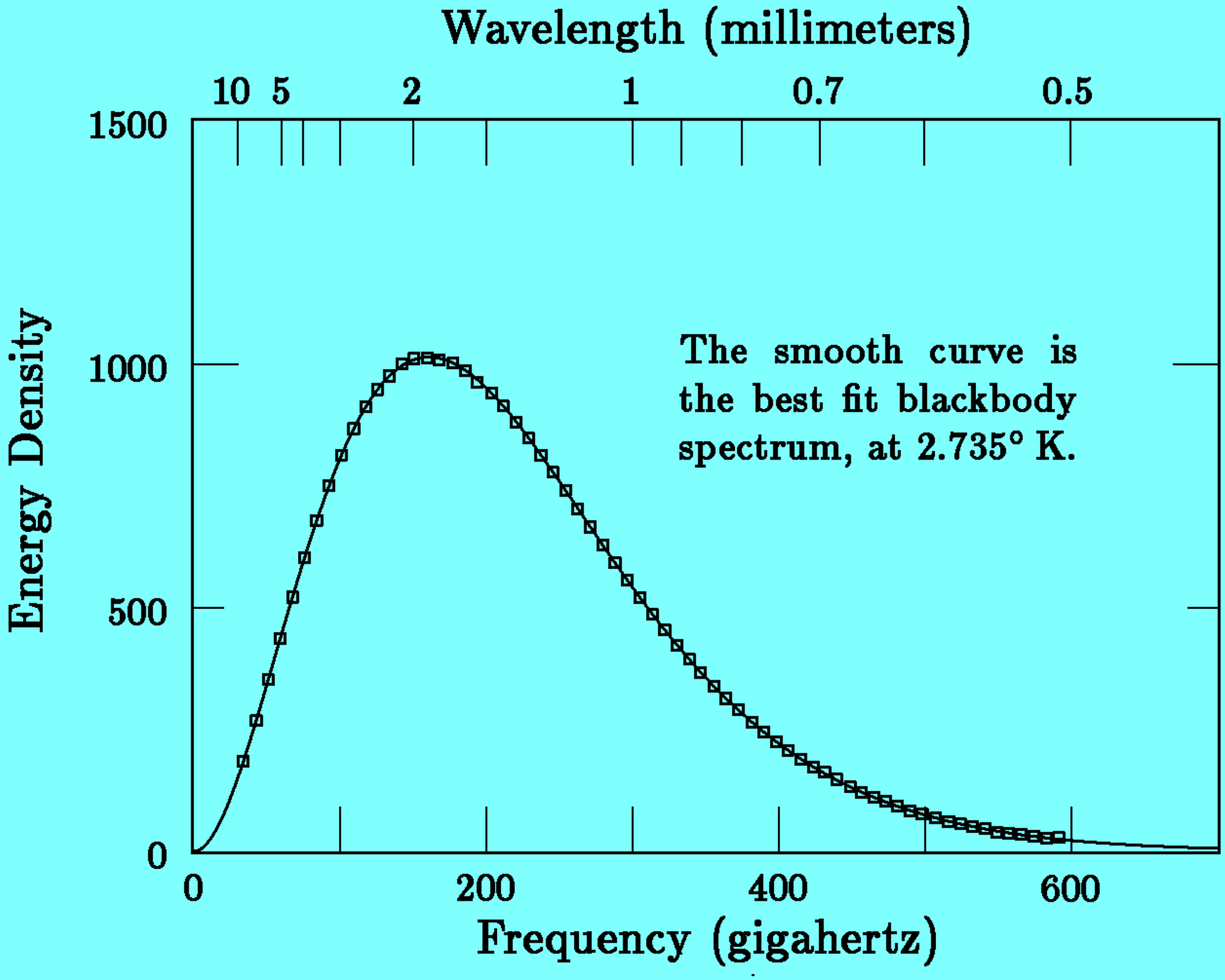
A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE COSMIC BACKGROUND EXPLORER (COBE) SATELLITE

J.C. Mather, E. S. Cheng, R. E. Eplee, R. B. Isaacman, S. S. Meyer, R. A. Shafer, R. Weiss, E. L. Wright, C. L. Bennett, N. W. Boggess, E. Dwek, S. Gulkis, M. G. Hauser, M. Janssen, T. Kelsall, P. M. Lubin, S. H. Moseley, Jr., T. L. Murdock, R. F. Silverberg, G. F. Smoot, and D. T. Wilkinson.



COSMIC BACKGROUND EXPLORER

Cover Page of Original Preprint of the COBE Measurement of the CMB Spectrum, 1990



Original COBE Measurement of the CMB Spectrum, Jan 1990. Energy density is in units of electron volts per cubic meter per gigahertz.