8.286 Class 19
November 9, 2020

THE COSMOLOGICAL CONSTANT

(Modified 12/27/20 to fix a minor typo on p. 23.)

Announcements

No class this Wednesday, due of Veteran's Day.

Problem Set 8 is due November 20, a week from this Friday.

No office hour this Wednesday, due to Veteran's Day. Instead I will have an office hour on Friday at 4:00 pm.

Bruno's office hours are unaffected. He will have two office hours this week, both on Thursday, at 10:00 am and at 6:00 pm.

Today's Nuclear/Particle Theory Seminar: Lepton number violation in nuclear physics

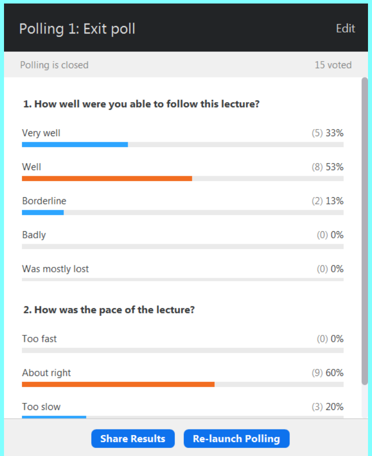
Seminar at 2:00 pm today.

Speaker: Jordy De Vries, University of Massachusetts Amherst

Abstract: Next-generation neutrinoless double-beta decay (0vbb) experiments aim to discover lepton number violation in order to shed light on the nature of neutrino masses. A non-zero signal would have profound implications by demonstrating the existence of elementary Majorana particles and possibly pointing towards a solution of matter-antimatter asymmetry in the universe. The interpretation of the experimental signal (or lack thereof) requires care as complicated hadronic input is required to connect the experimental data to a fundamental description of lepton-number violation. In this talk, I use effective field theory techniques to connect low-energy measurements to the fundamental lepton-number-violating source.

(If you want the Zoom link, email me [guth@ctp.mit.edu].)

Exit Poll, Last Class



Review from last class:

Thermal History of the Universe

Assuming that the early universe can be described as radiation-dominated and flat (excellent approximations), then

$$\rho = \frac{3}{32\pi G t^2} .$$

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}} .$$

Review from last class:

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}} .$$

Assuming 0.511 MeV $\ll kT \ll$ 106 MeV (i.e., assuming kT is between mc^2 for the electron and muon),

$$g_{\rm tot} = \underbrace{2}_{\text{photons}} + \underbrace{\frac{21}{4}}_{\text{neutrinos}} + \underbrace{\frac{7}{2}}_{e^+e^-} = 10\frac{3}{4} .$$

which implies

$$kT = \frac{0.860\text{ MeV}}{\sqrt{t\text{ (in sec)}}} , \quad T = \frac{9.98\times 10^9\text{ K}}{\sqrt{t\text{ (in sec)}}} .$$

Review from last class:

Relation Between a and T

- Conservation of entropy implies that $s \propto 1/a^3(t)$, but we also know that $s \propto gT^3$. It follows that

$$g^{1/3} T \propto \frac{1}{a(t)} .$$

Review from last class:

Recombination

- About 80% of baryonic matter is hydrogen. Most of the rest is helium. Elements heavier then helium make up a very small fraction. So we mostly have hydrogen.
- When T falls below $T_{\rm rec} \approx 4,000$ K, the protons and electrons combine to form neutral H. This is called "recombination," but "combination" would be more accurate.
- Note that $KT_{\rm rec} \approx 0.34$ eV, while the ionization energy of H is 13.6 eV. The reason for the big difference is that $n_{\rm baryon}/n_\gamma \approx 10^{-9}$, so it is rare for electrons and protons to find each other.

Review from last class:

Decoupling

- Photons interact strongly with free electrons.
- The result is that the universe was opaque to photons in the ionized phase (plasma phase), but became transparent when very few free electrons remained.
- The transition to a transparent universe is called “decoupling” (i.e., the photons “decouple” from the matter of the universe). $T_{\rm dec} \approx 3,000$ K. Since $T \propto 1/a$ and a is approximately proportional to $t^{2/3}$,

$$t_d = \left(\frac{T_0}{T_d}\right)^{3/2} t_0$$

$$\approx \left(\frac{2.7\,\mathrm{K}}{3000\,\mathrm{K}}\right)^{3/2} \times \left(13.7 \times 10^9 \text{ yr}\right) \approx 370,000 \text{ yr} .$$

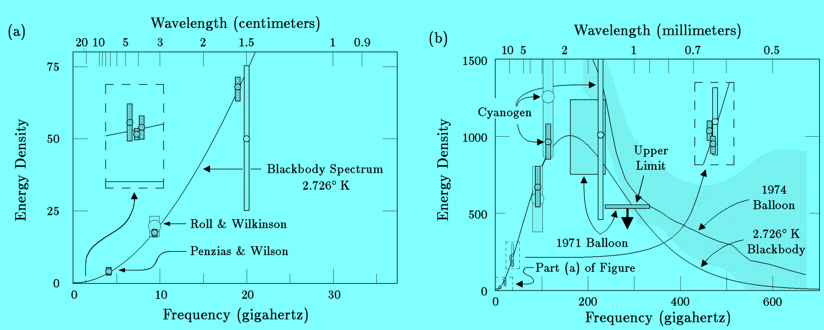
Review from last class:

Spectrum of the Cosmic Microwave Background

$$\rho_\nu(\nu)\,\mathrm{d}\nu = \frac{16\pi^2\hbar\nu^3}{c^3}\frac{1}{e^{2\pi\hbar\nu/kT}-1}\,\mathrm{d}\nu ,$$

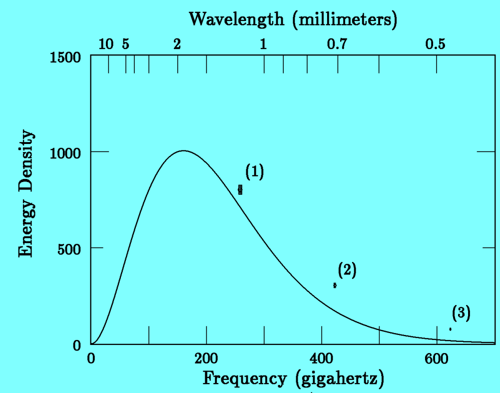
where $\rho_\nu(\nu)\,\mathrm{d}\nu$ is the energy density of photons in the frequency range from ν to $\nu + \mathrm{d}\nu$.

Review from last class:



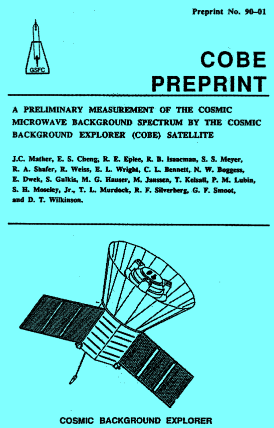
CMB Data in 1975

The situation got worse before it got better:



Data from Berkeley-Nagoya Rocket Flight, 1987

Points 2 and 3 differ from the blackbody curve by 12 and 16 standard deviations, respectively!


Preprint No. 90–01

GSFC

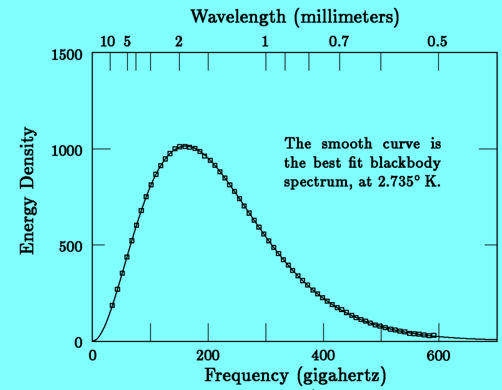
COBE PREPRINT

A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE COSMIC BACKGROUND EXPLORER (COBE) SATELLITE

J.C. Mather, E. S. Cheng, R. E. Eplee, R. B. Isaacman, S. S. Meyer, R. A. Shafer, R. Weiss, E. L. Wright, C. L. Bennett, N. W. Boggess, E. Dwek, S. Gulkis, M. G. Hauser, M. Janssen, T. Kelsall, P. M. Lubin, S. H. Moseley, Jr., T. L. Murdock, R. F. Silverberg, G. F. Smoot, and D. T. Wilkinson.

COSMIC BACKGROUND EXPLORER

The Cosmic Background Explorer (COBE) satellite was launched in the fall of 1989. In January 1990, at meeting of the American Astronomical Society in Washington, D.C., the first data on the spectrum of the cosmic microwave background was announced. Shown is the cover page of the original preprint.



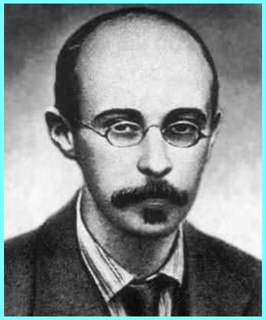
This is the original COBE measurement of the CMB spectrum, Jan 1990. The Energy density is in units of electron volts per cubic meter per gigahertz. The error bars are shown as 1% of the peak intensity. This graph was based on 9 minutes of data. Later data analysis reduced the error bars by a factor of 100, with still a perfect fit to the blackbody spectrum.

Historical Interlude:

Albert Einstein and Alexander Friedmann

Albert Einstein and the Friedmann Equations


Albert Einstein


Alexander A. Friedmann

Publication of the Friedmann Equations

On the Curvature of Space

A. Friedmann
Petersburg
Received June 29, 1922
Zeitschrift für Physik

Einstein's Reaction

REMARK ON THE WORK OF A. FRIEDMANN (FRIEDMANN 1922)
"ON THE CURVATURE OF SPACE"
A. Einstein, Berlin
Received September 18, 1922
Zeitschrift für Physik

The work cited contains a result concerning a non-stationary world which seems suspect to me. Indeed, those solutions do not appear compatible with the field equations (A). From the field equations it follows necessarily that the divergence of the matter tensor T_{ik} vanishes. This along with the anzatzes (C) and (D) leads to the condition

$$\partial\rho/\partial x_4 = 0$$

which together with (8) implies that the world-radius R is constant in time. The significance of the work therefore is to demonstrate this constancy.

REFERENCES: Friedmann, A. 1922, *Zs. f. Phys.*, 10, 377.

Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

Sequence of Events

June 29, 1922: Friedmann's paper received at *Zeitschrift für Physik.*

September 18, 1922: Einstein's refutation received at *Zeitschrift für Physik.*

December 6, 1922: Friedmann learns about Einstein's objection from his friend, Yuri A. Krutkov, who is visiting in Berlin. Friedmann writes a detailed letter to Einstein. Einstein is traveling and does not read it.

May, 1923: Einstein meets Krutkov in Leiden, both attending the farewell lecture by Lorentz, who was retiring.

Krutkov's letters to his sister: "On Monday, May 7, 1923, I was reading, together with Einstein, Friedmann's article in the *Zeitschrift für Physik.*" May 18: "I defeated Einstein in the argument about Friedmann. Petrograd's honor is saved!"*

May 31, 1923: Einstein's retraction of his refutation is received at *Zeitschrift für Physik.*

* Quoted in *Alexander A. Friedmann: the Man who Made the Universe Expand*, by E.A. Tropp, V. Ya. Frenkel, & A.D. Chernin.

Einstein's Retraction

A NOTE ON THE WORK OF A. FRIEDMANN
"ON THE CURVATURE OF SPACE"
A. Einstein, Berlin
Received May 31, 1923
Zeitschrift für Physik

I have in an earlier note (Einstein 1922) criticized the cited work (Friedmann 1922). My objection rested however — as Mr. Krutkoff in person and a letter from Mr. Friedmann convinced me — on a calculational error. I am convinced that Mr. Friedmann's results are both correct and clarifying. They show that in addition to the static solutions to the field equations there are time varying solutions with a spatially symmetric structure.

REFERENCES:

Einstein, A. 1922, *Zs. f. Phys.*, 11, 326.
Friedmann, A. 1922, *Ebenda,* 10, 377.

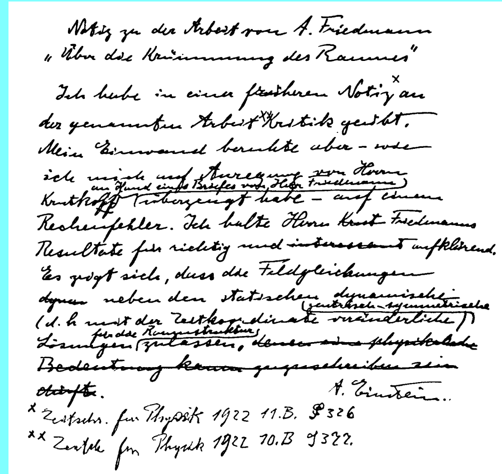
Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

Einstein and Krutkov

Albert Einstein
Barcelona, 1923

Yuri A. Krutkov.

Einstein's Draft



Notiz zu der Arbeit von A. Friedmann
„Über die Krümmung des Raumes"

Ich habe in einer früheren Notiz[x] an der genannten Arbeit[xx] Kritik geübt. Mein Einwand beruhte aber – wie ich mich ~~auf Anregung von Herrn Krutkoff~~ (an Hand eines Briefes von Herrn Friedmann) überzeugt habe – auf einem Rechenfehler. Ich halte Herrn ~~Krut~~ Friedmanns Resultate für richtig und ~~interessant~~ aufklärend. Es zeigt sich, dass die Feldgleichungen ~~dyn~~ neben den statischen ~~dynamische~~ (zentrisch-symmetrische) (d. h. mit der Zeitkoordinate veränderliche) Lösungen (für die Raumstruktur) zulassen, ~~denen eine physikalische Bedeutung kaum zugeschrieben sein dürfte~~.

A. Einstein.

[x] Zeitschr. für Physik 1922 11.B. S. 326
[xx] Zeitschr. für Physik 1922 10.B S. 322.

Einstein's draft of 1923 in which he withdrew his earlier objection to Friedmann's dynamic solutions to the field equations. The last bit of the last sentence was: "a physical significance can hardly be ascribed to them". He crossed this out before sending the note to print.

"a physical significance can hardly be ascribed to them."

* From *The Invented Universe,* by Pierre Kerszberg

A Brief History of the Cosmological Constant

- ☆ In 1917, Einstein applied his new GR to the universe, and discovered that a static universe would collapse.
- ☆ Convinced that the universe was static, Einstein introduced the *cosmological constant* Λ into his field equations — the equations that describe how matter affects the metric — to create a gravitational repulsion to oppose the collapse.
- ☆ From a modern point of view, Λ represents a *vacuum energy density* u_{vac}, with

$$u_{\text{vac}} = \rho_{\text{vac}} c^2 = \frac{\Lambda c^4}{8\pi G} \; ,$$

because u_{vac} appears in the field equations exactly as a vacuum energy density would. To Einstein, however, it was simply a new term in the field equations. Before quantum theory, the vacuum was viewed as completely empty, so it was inconceivable that it could have a nonzero energy density.

- ☆ Once the expansion of the universe was discovered by Hubble in 1929, Einstein abandoned Λ as being no longer needed or wanted.
- ☆ In 1998, however, two (large) groups of astronomers, both using measurements of Type Ia supernovae at redshifts $z \lesssim 1$, discovered evidence that the expansion of the universe is currently accelerating. At the time, it was shocking! *Science* magazine proclaimed it (correctly!) as the "Breakthough of the Year".
- ☆ In 2011 the Nobel Prize in Physics was awarded to Saul Permutter, Brian Schmidt, and Adam Riess for this discovery. In 2015 the Breakthrough Prize in Fundamental Physics was awarded to these three, and also the two entire teams.

Gravitational Effect of Pressure

$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)a \; .$$

Vacuum Energy and the Cosmological Constant:

$$u_{\rm vac} = \rho_{\rm vac}c^2 = \frac{\Lambda c^4}{8\pi G} \; .$$

Recall that

$$\dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) \; ,$$

where the overdot indicates a time derivative. So

$$\dot\rho_{\rm vac} = 0 \quad \Longrightarrow \quad p_{\rm vac} = -\rho_{\rm vac}c^2 = -\frac{\Lambda c^4}{8\pi G} \; .$$

Defining $\rho = \rho_n + \rho_{\rm vac}$ and $p = p_n + p_{\rm vac}$,

$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}G\left(\rho_n + \frac{3p_n}{c^2} - 2\rho_{\rm vac}\right)a \; .$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G(\rho_n + \rho_{\rm vac}) - \frac{kc^2}{a^2} \; .$$

Dominance of vacuum energy at late time implies

$$a(t) \propto e^{H_{\rm vac}t} \; ,$$

$$H \to H_{\rm vac} = \sqrt{\frac{8\pi}{3}G\rho_{\rm vac}} \; .$$

Age of the Universe with Λ

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\big(\underbrace{\rho_m}_{\propto\frac{1}{a^3(t)}} + \underbrace{\rho_{\rm rad}}_{\propto\frac{1}{a^4(t)}} + \rho_{\rm vac}\big) - \frac{kc^2}{a^2} \; .$$

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left(\frac{\Omega_{m,0}}{x^3} + \frac{\Omega_{\rm rad,0}}{x^4} + \Omega_{\rm vac}\right) - \frac{kc^2}{a^2} \; ,$$

where $x \equiv a(t)/a(t_0)$.

Define

$$\Omega_{k,0} \equiv -\frac{kc^2}{a^2(t_0)H_0^2} \; .$$

$$\left(\frac{\dot a}{a}\right)^2 = \left(\frac{\dot x}{x}\right)^2 = \frac{H_0^2}{x^4}\left(\Omega_{m,0}x + \Omega_{\rm rad,0} + \Omega_{\rm vac,0}x^4 + \Omega_{k,0}x^2\right) \; .$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\rm rad,0} - \Omega_{\rm vac,0} \; .$$

Finally,

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0}x + \Omega_{\rm rad,0} + \Omega_{\rm vac,0}x^4 + \Omega_{k,0}x^2}} \; .$$

Numerical Integration with Mathematica

IN: t0[H0_,Ωm0_,Ωrad0_,Ωvac0_,Ωk0_] := (1/H0) *

NIntegrate[x/Sqrt[Ωm0 x + Ωrad0 + Ωvac0 x^4 + Ωk0 x^2], {x,0,1}]

IN: PlanckH0 := Quantity[67.66,"km/sec/Mpc"]

IN: PlanckΩm0 := 0.311

IN: PlanckΩvac0 := 0.689

IN: UnitConvert[t0[PlanckH0,PlanckΩm0,0,PlanckΩvac0,0],"Years"]

OUT: 1.38022×10^{10} years