8.286 Class 21
November 18, 2020

PROBLEMS OF THE CONVENTIONAL (NON-INFLATIONARY) HOT BIG BANG MODEL

(Modified 12/27/20 to fix two typos on p. 10, two typos on p. 16, and one on p. 19. There are also small clarifications on pp. 21 and 30, and the end of the slides reached in class is marked.)

Calendar for the Home Stretch:

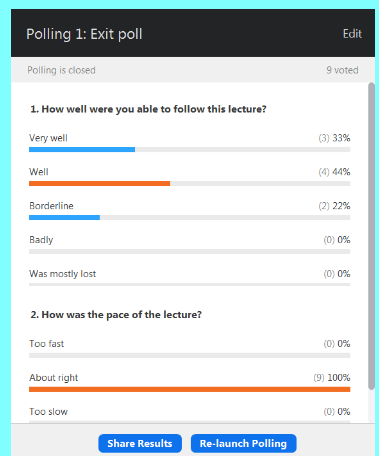
NOVEMBER/DECEMBER				
MON	TUES	WED	THURS	FRI
16 Class 20	17	18 Class 21	19	20 **PS 8 due**
23 **Thanksgiving Week**	24 —	25 —	26 —	27 —
30 Class 22	December 1	2 Class 23 **Quiz 3**	3	4
7 Class 24	8	9 Class 25 **PS 9 due** Last Class	10	11

Announcements

- For today only, due to an MIT faculty meeting, I am postponing my office hour by one hour, so it will be 5:05-6:00 pm.
- Problem Set 8 is due this Friday, November 20.
- Quiz 3 will be on Wednesday, December 2, the Wednesday after the Thanksgiving break.

  It will follow the pattern of the two previous quizzes: Review Problems, a Review Session, and modified office hours the week of the quiz. Details to be announced.
- Lecture Notes 8, on the subject of today's class, will soon be posted.
- I have posted *Notes on Thermal Equilibrium* on the Lecture Notes web page. These are intended as background and clarification for Ryden's sections on hydrogen recombination and deuterium synthesis. It will not be covered by Quiz 3, but there will be one or two problems about it on the last problem set.
- There will be one last problem set, Problem Set 9, due the last day of classes, Wednesday December 9. **No final exam!**

Exit Poll, Last Class



Summary of Last Lecture

☆ Age of the universe with matter, radiation, vacuum energy, and curvature:

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{x\,dx}{\sqrt{\Omega_{m,0}x + \Omega_{\text{rad},0} + \Omega_{\text{vac},0}x^4 + \Omega_{k,0}x^2}} \ .$$

☆ Look-Back time:

Change variable of integration from x to z, with $1 + z = a(t_0)/a(t) = 1/x$. Then integrate over z from 0 to z_S, the redshift of the source:

$$t_{\text{look-back}}(z_S) = \frac{1}{H_0}\int_0^{z_S} \frac{dz'}{(1+z')\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{\text{rad},0}(1+z')^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1+z')^2}} \ .$$

Review from last class:

Ryden Benchmark and Planck 2018 Best Fit

Parameters	Ryden Benchmark	Planck 2018 Best Fit
$\boldsymbol{H_0}$	68	67.7 ± 0.4 km·s^{-1}·Mpc^{-1}
Baryonic matter $\boldsymbol{\Omega_b}$	0.048	$0.0490 \pm 0.0007^*$
Dark matter $\boldsymbol{\Omega_{\text{dm}}}$	0.262	$0.261 \pm 0.004^*$
Total matter $\boldsymbol{\Omega_m}$	0.31	0.311 ± 0.006
Vacuum energy $\boldsymbol{\Omega_{\text{vac}}}$	0.69	0.689 ± 0.006

Review from last class:

Controversy in Parameters: "Hubble Tension"

☆ From the CMB, the best number is from

Planck 2018: $\boldsymbol{H_0 = 67.66 \pm 0.42}$ **km sec^{-1} Mpc^{-1}**

☆ From standard candles and Cepheid variables,

SH0ES (Supernovae, H0, for the Equation of State of dark energy, group led by Adam Riess):

$\boldsymbol{H_0 = 74.03 \pm 1.42}$ **km sec^{-1} Mpc^{-1}.**

☆ The difference is about 4.3 σ. If the discrepancy is random and the normal probability distribution applies, the probability of such a large deviation is about 1 in 50,000.

☆ From the "tip of the red giant branch",

Wendy Freedman's group:

$\boldsymbol{H_0 = 69.6 \pm 0.8\text{(stat)} \pm 1.7\text{(sys)}}$ **km sec^{-1} Mpc^{-1}**

References: A. Riess et al., Astrophys. J. 876 (2019) 85 [arXiv:1903.07603].
W. Freedman et al., arXiv:2002.01550 (2020).

Review from last class:

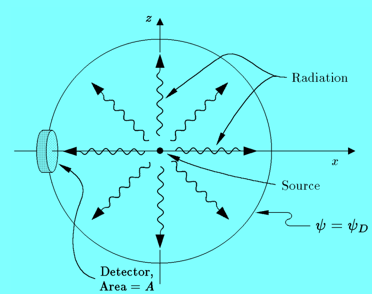
The Hubble Diagram: Radiation Flux vs. Redshift

☆ For a closed universe, write the metric:

$$ds^2 = -c^2\,dt^2 + \tilde{a}^2(t)\left\{d\psi^2 + \sin^2\psi\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right\} \ ,$$

where $\sin\psi \equiv \sqrt{k}\,r$.

☆ Consider a sphere centered at the source, at the same radius as us. The fraction of photons hitting the sphere that hit the detector is just the ratio of the areas.



Review from last class:

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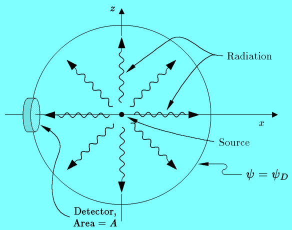


☆ The power hitting the sphere is the power of the source, reduced by two factors of $(1+z_S)$: one for the redshift of each photon, one for the redshift of the arrival rate of photons.

☆ Need $\psi(z_S)$ to evaluate the area of the sphere. $ds^2 = 0$ gives expression for $d\psi/dt$. Integration over t relates $\psi(z_S)$ to time of emission, and hence redshift, since $1 + z = a(t_0)/a(t_S)$. Changing variable of integration from t to z, the integral can be expressed in terms of $H(z)$, which is determined by the first-order Friedmann equation.

Review from last class:

☆ Final answer (flux J from source of power P at redshift z_S):

$$J = \frac{P H_0^2 |\Omega_{k,0}|}{4\pi(1+z_S)^2 c^2 \sin^2\psi(z_S)}\ ,$$

where

$$\psi(z_S) = \sqrt{|\Omega_{k,0}|} \times \int_0^{z_S} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\text{rad},0}(1+z)^4 + \Omega_{\text{vac},0} + \Omega_{k,0}(1+z)^2}}\ .$$

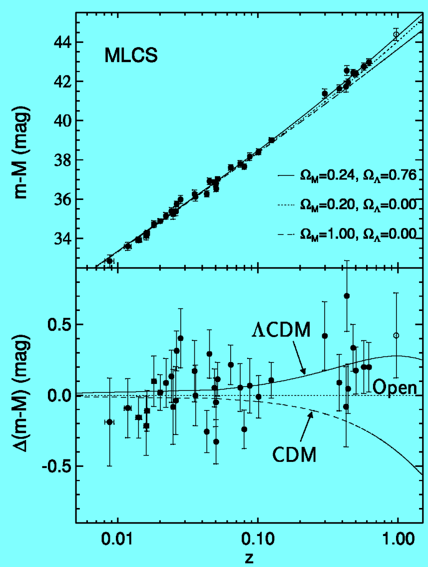
Supernovae Type Ia as Standard Candles

Supernovae Type Ia are believed to be the result of a binary system containing a white dwarf — a stellar remnant that has burned its nuclear fuel, and is supported by electron degeneracy pressure. As the white dwarf accretes gas from its companion star, its mass builds up to 1.4 $M_\odot$, the Chandrasekhar limit, the maximum mass that can be supported by electron degeneracy pressure. The star then collapses, leading to a supernova explosion. Because the Chandrasekhar limit is fixed by physics, all SN Ia are very similar in power output.

There are still some known variations in power output, but they are found to be correlated with the shape of the light curve: if the light curve rises and falls slowly, the supernova is brighter than average.

The properties of SN Ia are known best from observation — theory lags behind.

IF you would like to learn more about this, see Ryden, Section 6.5 [First edition: 7.5] (which we skipped — you should not feel obligated to read this).



Hubble diagram from Riess *et al.*, *Astronomical Journal* **116**, No. 3, 1009 (1998) [http://arXiv.org/abs/astro-ph/9805201].

(High-z Supernova Search Team)

Dimmer Supernovae Imply Acceleration

- ☆ The acceleration of the universe is deduced from the fact that distant supernovae appear to be 20-30% dimmer than expected.
- ☆ Why does dimness imply acceleration?
  - Consider a supernova of specified apparent brightness.
  - "Dimmer" implies data point is to the left of where expected — at lower z.
  - Lower z implies slower recession, which implies that the universe was expanding slower than expected in the past — hence, acceleration!

Other Possible Explanations for Dimness

- ☆ Absorption by dust.
  - **But absorption usually reddens** the spectrum. This would have to be **"gray" dust,** absorbing uniformly at all observed wavelengths. Such dust is possible, but not known to exist anywhere.
  - Dust would most likely be in the host galaxy, which would cause **variable absorption,** depending on SN location in galaxy. Such variability is not seen.
- ☆ Chemical evolution of heavy element abundance.
  - But nearby and distant SN Ia look essentially identical.
  - For nearby SN Ia, heavy element abundance varies, and does not appear to affect brightness.
- ☆ Additional evidence against dust or chemical evolution: A SN Ia has been found at $z = 1.7$, which is early enough to be in the decelerating era of the vacuum energy density model. It is consistent with deceleration, but not consistent with either models of absorption or chemical evolution.

Evidence for the Accelerating Universe

1) Supernova Data: distant SN Ia are dimmer than expected by about 20–30%.
2) Cosmic Microwave Background (CMB) anisotropies: gives $\Omega_{\rm vac}$ close to SN value. Also gives $\Omega_{\rm tot} = 1$ to 1/2% accuracy, which cannot be accounted for without dark energy.
3) Inclusion of $\Omega_{\rm vac} \approx 0.70$ makes the age of the universe consistent with the age of the oldest stars.

- ☆ With the 3 arguments together, the case for the accelerating universe and $\Omega_{\rm dark\ energy} \approx 0.70$ has persuaded almost everyone.
- ☆ The simplest explanation for dark energy is vacuum energy, but "quintessence" is also possible.

Particle Physics of a Cosmological Constant

- ☆ $$u_{\rm vac} = \rho_{\rm vac} c^2 = \frac{\Lambda c^4}{8\pi G}$$
- ☆ Contributions to vacuum energy density:
  1) Quantum fluctuations of the photon and other bosonic fields: positive and divergent.
  2) Quantum fluctuations of the electron and other fermionic fields: negative and divergent.
  3) Fields with nonzero values in the vacuum, like the Higgs field.

- If infinities are cut off at the Planck scale (quantum gravity scale), then infinities become finite, but

> 120 orders of magnitude too large!

- For lack of a better explanation, many cosmologists (including Steve Weinberg and yours truly) seriously discuss the possibility that the vacuum energy density is determined by "anthropic" selection effects: that is, maybe there are many types of vacuum (as predicted by string theory), with different vacuum energy densities, with most vacuum energy densities roughly 120 orders of magnitude larger than ours. Maybe we live in a very low energy density vacuum because that is where almost all living beings reside. A large vacuum energy density would cause the universe to rapidly fly apart (if positive) or implode (if negative), so life could not form.

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The Horizon/Homogeneity Problem

- General question: how can we explain the large-scale uniformity of the universe?
- Possible answer: maybe the universe just started out uniform.
  - There is no argument that excludes this possibility, since we don't know how the universe came into being.
  - However, if possible, it seems better to explain the properties of the universe in terms of things that we can understand, rather than to attribute them to things that we don't understand.

The Horizon in Cosmology

- The concept of a horizon was first introduced into cosmology by Wolfgang Rindler in 1956.
- The "horizon problem" was discussed (not by that name) in at least two early textbooks in general relativity and cosmology: Weinberg's *Gravitation and Cosmology* (1972), and Misner, Thorne, and Wheeler's (MTW's) *Gravitation* (1973).

The Cosmic Microwave Background

- The strongest evidence for the uniformity of the universe comes from the CMB, since it has been measured so precisely.
- The radiation appears slightly hotter in one direction than in the opposite direction, by about one part in a thousand — but this nonuniformity can be attributed to our motion through the background radiation.
- Once this effect is subtracted out, using best-fit parameters for the velocity, it is found that the residual temperature pattern is uniform to a few parts in 10^5.
- Could this be simply the phenomenon of thermal equilibrium? If you put an ice cube on the sidewalk on a hot summer day, it melts and come sto the same temperature as the sidewalk.
- **BUT: in the conventional model of the universe, it did not have enough time for thermal equilibrium to explain the uniformity, if we assume that it did not start out uniform. If no matter, energy, or information can travel faster than light, then it is simply not possible.**

Basic History of the CMB

- In conventional cosmological model, the universe at the earliest times was radiation-dominated. It started to be matter-dominated at $t_{\rm eq} \approx 50,000$ years, the time of matter-radiation equality.
- At the time of *decoupling* $t_d \approx 380,000$ years, the universe cooled to about 3000 K, by which time the hydrogen (and some helium) combined so thoroughly that free electrons were very rare. At earlier times, the universe was in a mainly plasma phase, with many free electrons, and photons were essentially frozen with the matter. At later times, the universe was transparent, so photons have traveled on straight lines. We can say that the CMB was released at about 380,000 years.
- Since the photons have been mainly traveling on straight lines since $t = t_d$, they have all traveled the same distance. Therefore the locations from which they were released form a sphere centered on us. This sphere is called the *surface of last scattering*, since the photons that we receive now in the CMB was mostly scattered for the last time on or very near this surface.

- As we learned in Lecture Notes 4, the horizon distance is defined as the present distance of the furthest particles from which light has had time to reach us, since the beginning of the universe.
- For a matter-dominated flat universe, the horizon distance at time t is $3ct$, while for a radiation-dominated universe, it is $2ct$.
- At $t = t_d$ the universe was well into the matter-dominated phase, so we can approximate the horizon distance as

$$\ell_h(t_d) \approx 3ct_d \approx 1,100,000 \text{ light-years.}$$

For comparison, we would like to calculate the radius of the surface of last scattering at time t_d, since this region is the origin of the photons that we are now receiving in the CMB. I will denote the physical radius of the surface of last scattering, at time t, as $\ell_p(t)$.

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- To calculate $\ell_p(t_d)$, I will make the crude approximation that the universe has been matter-dominated at all times. (We will find that this *horizon problem* is very severe, so even if our calculation is wrong by a factor of 2, it won't matter.)
- Strategy: find $\ell_p(t_0)$, and scale to find $\ell_p(t_d)$. Under the assumption of a flat matter-dominated universe, we learned that the physical distance today to an object at redshift z is

$$\ell_p(t_0) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] .$$

$$\ell_p(t_0) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] .$$

- The redshift of the surface of last scattering is about

$$1 + z = \frac{a(t_0)}{a(t_d)} = \frac{3000\text{ K}}{2.7\text{ K}} \approx 1100 .$$

- If we take $H_0 = 67.7$ km-s^{-1}-Mpc^{-1}, one finds that $H_0^{-1} \approx 14.4 \times 10^9$ yr and $\ell_p(t_0) \approx 28.0 \times 10^9$ light-yr. (Note that $\ell_p(t_0)$ is equal to 0.970 times the current horizon distance — very close.)
- To find $\ell_p(t_d)$, just use the fact that the redshift is related to the scale factor:

$$\ell_p(t_d) = \frac{a(t_d)}{a(t_0)}\ell_p(t_0) \approx \frac{1}{1100} \times 28.0 \times 10^9 \text{ lt-yr} \approx 2.55 \times 10^7 \text{ lt-yr} .$$

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- Comparison: At the time of decoupling, the ratio of the radius of the surface of last scattering to the horizon distance was

$$\frac{\ell_p(t_d)}{\ell_h(t_d)} \approx \frac{2.55 \times 10^7 \text{ lt-yr}}{1.1 \times 10^6 \text{ lt-yr}} \approx 23 .$$

Summary of the Horizon Problem

Suppose that one detects the cosmic microwave background in a certain direction in the sky, and suppose that one also detects the radiation from precisely the opposite direction. At the time of emission, the sources of these two signals were separated from each other by about 46 horizon distances. Thus it is absolutely impossible, within the context of this model, for these two sources to have come into thermal equilibrium by any physical process.

Although our calculation ignored the dark energy phase, we have found in previous examples that such calculations are wrong by some tens of a percent. (For example we found $t_{\text{eq}} \approx 75,000$ years, when it should have been about 50,000 years.) Since $46 \gg 1$, there is no way that a more accurate calculation could cause this problem to go away.

This and the following slides were not reached, but will be discussed in the next class.

The Flatness Problem

- A second problem of the conventional cosmological model is the *flatness problem*: why was the value of Ω in the early universe so extraordinarily close to 1?
- Today we know, according to the Planck satellite team analysis (2018), that

$$\Omega_0 = 0.9993 \pm 0.0037$$

at 95% confidence. I.e., $\Omega = 1$ to better than 1/2 of 1%.

- As we will see, this implies that Ω in the early universe was extaordinarily close to 1. For example, at $t = 1$ second,

$$|\Omega - 1|_{t=1\text{ sec}} < 10^{-18} .$$

- ☆ The underlying fact is that the value $\Omega = 1$ is a point of unstable equilibrium, something like a pencil balancing on its point. If Ω is ever ***exactly*** equal to one, it will remain equal to one forever — that is, a flat ($k = 0$) universe remains flat. However, if Ω is ever slightly larger than one, it will rapidly grow toward infinity; if Ω is ever slightly smaller than one, it will rapidly fall toward zero. For Ω to be anywhere near 1 today, Ω in the early universe must have been extraordinarily close to one.
- ☆ Like the horizon problem, the flatness problem could in principle be solved by the initial conditions of the universe: maybe the universe began with $\Omega \equiv 1$.
  - But, like the horizon problem, it seems better to explain the properties of the universe, if we can, in terms of things that we can understand, rather than to attribute them to things that we don't understand.

History of the Flatness Problem

The mathematics behind the flatness problem was undoubtedly known to almost anyone who has worked on the big bang theory from the 1920's onward, but apparently the first people to consider it a problem in the sense described here were Robert Dicke and P.J.E. Peebles, who published a discussion in 1979.*

*R.H. Dicke and P.J.E. Peebles, "The big bang cosmology — enigmas and nostrums," in **General Relativity: An Einstein Centenary Survey**, eds: S.W. Hawking and W. Israel, Cambridge University Press (1979).

The Mathematics of the Flatness Problem

Start with the first-order Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} .$$

Remembering that $\Omega = \rho/\rho_c$ and that $\rho_c = 3H^2/(8\pi G)$, one can divide both sides of the equation by H^2 to find

$$1 = \frac{\rho}{\rho_c} - \frac{kc^2}{a^2H^2} \quad \Longrightarrow \quad \boxed{\Omega - 1 = \frac{kc^2}{a^2H^2} .}$$

Evolution of $\Omega - 1$ During the Radiation-Dominated Phase

$$\boxed{\Omega - 1 = \frac{kc^2}{a^2H^2} .}$$

For a (nearly) flat radiation-dominated universe, $a(t) \propto t^{1/2}$, so $H = \dot{a}/a = 1/(2t)$. So

$$\boxed{\Omega - 1 \propto \left(\frac{1}{t^{1/2}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t \quad \text{(radiation dominated).}}$$

Evolution of $\Omega - 1$ During the Matter-Dominated Phase

$$\Omega - 1 = \frac{kc^2}{a^2 H^2} .$$

For a (nearly) flat matter-dominated universe, $a(t) \propto t^{2/3}$, so $H = \dot{a}/a = 2/(3t)$. So

$$\Omega - 1 \propto \left(\frac{1}{t^{2/3}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t^{2/3} \quad \text{(matter-dominated)}.$$

Tracing $\Omega - 1$ from Now to 1 Second

Today,

$$|\Omega_0 - 1| < .01 .$$

I will do a crude calculation, treating the universe as matter dominated from 50,000 years to the present, and as radiation-dominated from 1 second to 50,000 years.

During the matter-dominated phase,

$$(\Omega - 1)_{t=50{,}000 \text{ yr}} \approx \left(\frac{50{,}000}{13.8 \times 10^9}\right)^{2/3} (\Omega_0 - 1) \approx 2.36 \times 10^{-4} (\Omega_0 - 1) .$$

$$|\Omega_0 - 1| < .01 .$$

$$(\Omega - 1)_{t=50{,}000 \text{ yr}} \approx \left(\frac{50{,}000}{13.8 \times 10^9}\right)^{2/3} (\Omega_0 - 1) \approx 2.36 \times 10^{-4} (\Omega_0 - 1) .$$

During the radiation-dominated phase,

$$(\Omega - 1)_{t=1 \text{ sec}} \approx \left(\frac{1 \text{ sec}}{50{,}000 \text{ yr}}\right) (\Omega - 1)_{t=50{,}000 \text{ yr}}$$
$$\approx 1.49 \times 10^{-16} (\Omega_0 - 1) .$$

The conclusion is therefore

$$|\Omega - 1|_{t=1 \text{ sec}} < 10^{-18} .$$

The conclusion is therefore

$$|\Omega - 1|_{t=1 \text{ sec}} < 10^{-18} .$$

Even if we put ourselves mentally back into 1979, we would have said that $0.1 < \Omega_0 < 2$, so $|\Omega_0 - 1| < 1$, and would have concluded that

$$|\Omega - 1|_{t=1 \text{ sec}} < 10^{-16} .$$

The Dicke & Peebles paper, that first pointed out this problem, also considered $t = 1$ second, but concluded (without showing the details) that

$$|\Omega - 1|_{t=1 \text{ sec}} < 10^{-14} .$$

They were perhaps more conservative, but concluded nonetheless that this extreme fine-tuning cried out for an explanation.