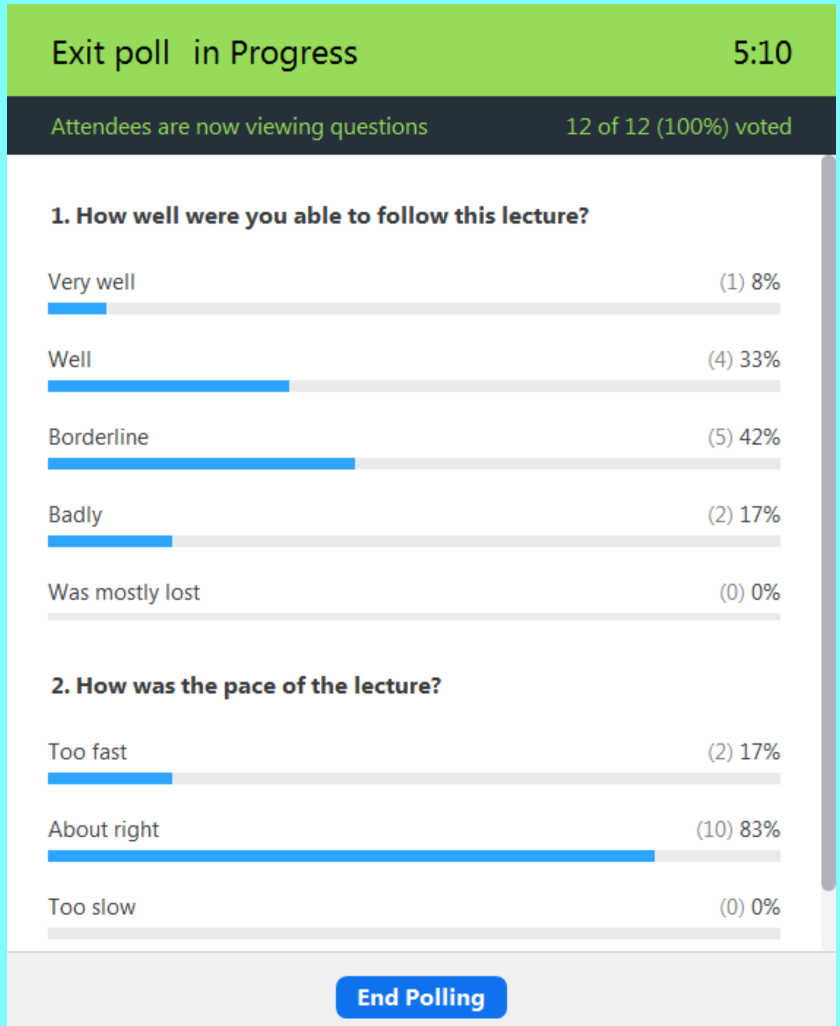
8.286 Lecture 25 (Last!)
December 9, 2020

THE INFLATIONARY UNIVERSE

Announcements

Problem Set 9 is due today, 12:30 pm.

Exit Poll, Class 24 (Previous Class)



Alan Guth
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Skip during class, but include for later reference

Summary of Last Class: Grand Unified Theories and the Magnetic Monopole Problem

- ☆ Standard Model of Particle Physics: gauge theory with symmetry group $SU(3) \times SU(2) \times U(1)$.
- ☆ Gauge symmetry: a symmetry described by $u(x)$, where u is an element of the symmetry group, and $x \equiv (\vec{x}, t)$ is the spacetime coordinate. A gauge transformation changes the fields, but not the physics.
- ☆ $SU(3)$ describes the strong interactions, carried by 8 types of gluons. $SU(2) \times U(1)$ describes the weak and electromagnetic interactions, carried by the photon, W^+, W^-, and Z.
- ☆ Higgs fields: actually a complex doublet, but we mainly talked about a toy model with a real triplet of Higgs fields, $\vec{\phi}$, transforming like an ordinary 3D vector under ordinary 3D rotations.
- ☆ Spontaneous symmetry breaking: the minimum energy state is when $|\vec{\phi}| = \phi_v \neq 0$, so it must randomly pick out a direction and break the symmetry down to rotations in 1D, or $U(1)$.

Skip during class, but include for later reference

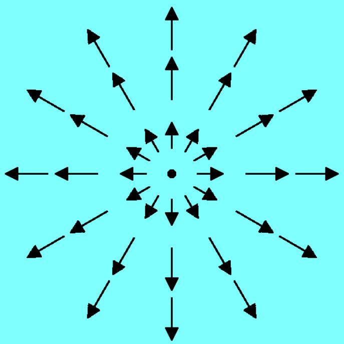
- Masses: the nonzero Higgs field values produce restoring forces for some of the other fields, giving them masses. In particular, the force-carrying gauge bosons associated with broken symmetries acquire a mass, while others remain massless.

- **Grand Unified Theories:** Combine $SU(3)$, $SU(2)$, and $U(1)$ of standard model into one group, the simplest being $SU(5)$. The $SU(5)$ is broken by GUT Higgs fields to $SU(3) \times SU(2) \times U(1)$.

- Predictions of GUTs: proton decay, magnetic monopoles. Magnetic monopoles have not be seen, and $\tau_{\text{proton}} \gtrsim 10^{34}$ years.

- We described a magnetic monopole in the toy theory with vector Higgs $\vec{\phi}$:

  They are topologically stable.



- Monopoles have mass $m_M c^2 \sim 10^{16}$ GeV, with an expected number density of order $1/\xi^3$, where ξ is the correlation length, which must be less than the horizon length.

- **PROBLEM:** If this many monopoles were produced, today they would outweigh everything else in the universe by a factor $> 10^{20}$.

The Inflationary Universe Scenario

- Inflationary cosmology attempts to describe the behavior of the universe at ridiculously early times — perhaps as early as 10^{-37} seconds.
- Surprisingly, it can still make predictions that can be tested today.
- Inflation can provide a solution to the horizon problem, the flatness problem, and the magnetic monopole problem.
- If correct, inflation can even explain the origin of essentially all the matter in the universe. (One has to start with a bit of matter: a few grams!)

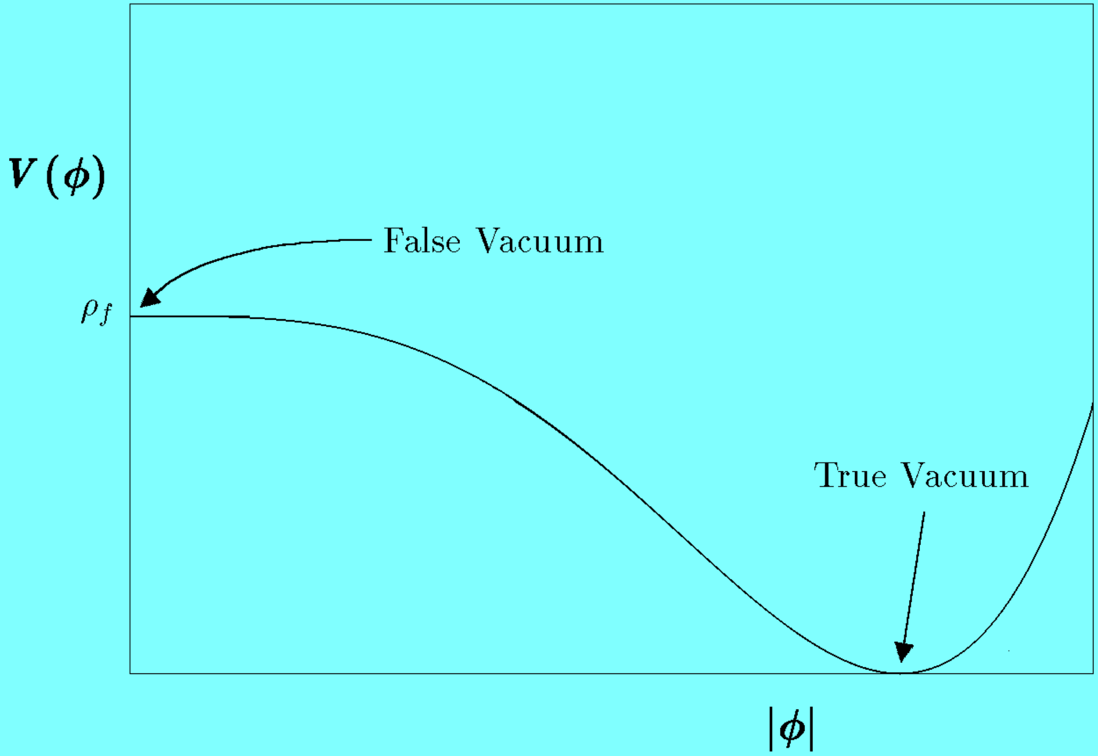
The inflationary scenario assumes the existence of a scalar field ϕ that resembles the Higgs field of the standard model. It is usually assumed to be some beyond-the-standard-model field, associated with a particle of mass $m_\phi c^2$ much higher than anything that we can currently produce in particle accelerators. Any theory with supersymmetry (a symmetry between bosons and fermions), including string theory, leads to many such fields.

Whatever the scalar field that drives inflation is, it is called the "inflaton".

Inflation is not really a theory, but rather a class of theories, since there are many options for how the inflaton field might behave.

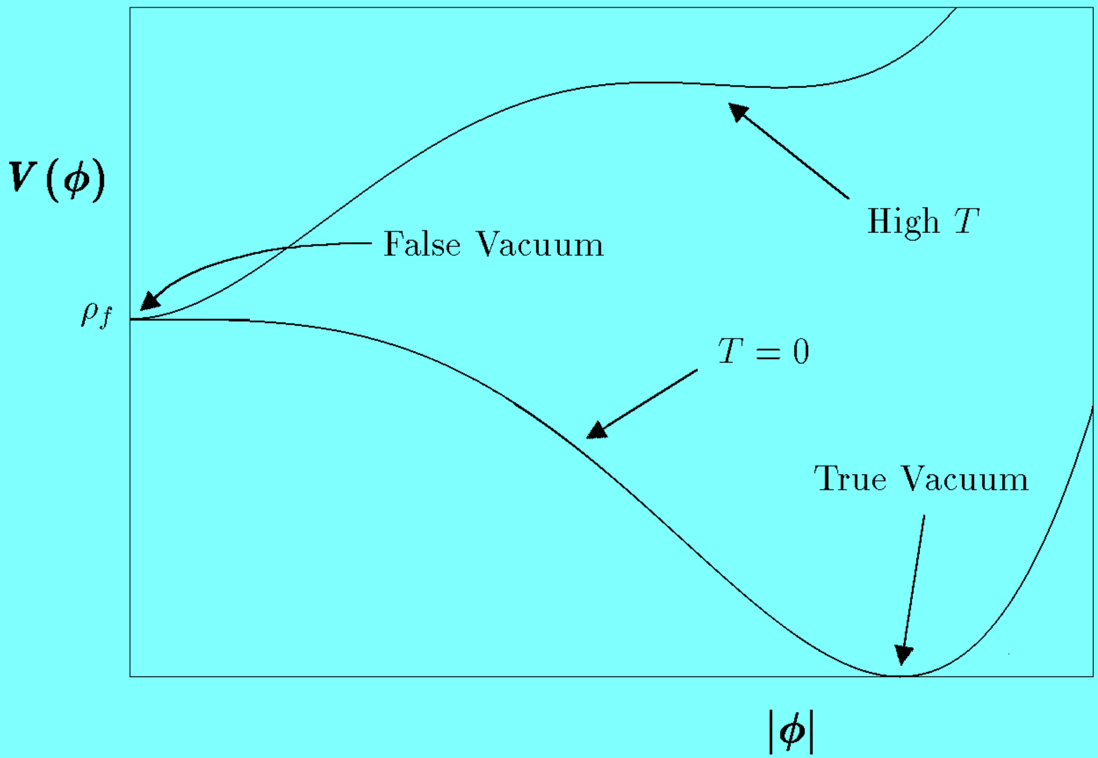
It is conceivable that the inflaton might be the Higgs field of the standard model, but that can work only if the Higgs field interacts with gravity in a particular way, which can be tested only at energies well beyond what we have access to.

The easiest version of inflation to explain is called "hilltop" inflation, or "new" inflation. It assumes an inflaton potential energy density resembling that of the standard model Higgs field:



More general potential energy functions are possible, as we will discuss in a few minutes.

One can also calculate the “finite temperature effective potential” for this theory:



It is the finite temperature effective potential that would be minimized in thermal equilibrium.

Start of Inflation

There is no accepted (or even persuasive) theory of the origin of the universe, so the starting point is uncertain. Inflation starts when the scalar field is at the top of the hill, no matter how it got there.

The scalar field can reach the top of the hill by:

1) Cooling from high temperature ("new" inflation: Linde 1982, Albrecht & Steinhardt, 1982). But: there is not enough time for thermal equilibrium to be reached, so it must be *assumed.*

2) With spatially dependent "chaotic" initial conditions, it will happen somewhere (Linde, 1983). This is probably the dominant point of view today.

3) Creation of the universe by "tunneling from nothing" (Vilenkin, 1983, Linde 1984).

4) Initial conditions for the "wave function of the universe" (Hartle & Hawking, 1983).

5) Who knows?

The good news is that the predictions of inflation do not depend on how it started. This is also bad news, since it means that it is very hard to learn anything about how it started.

The Inflationary Era

Once the inflaton is at the top of the hill, the mass/energy density is fixed, leading to a large negative pressure and gravitational repulsion:

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \; ; \quad \dot{\rho} = 0 \quad \Longrightarrow \quad p = -\rho c^2 \; .$$

Assuming approximate Friedmann-Robertson-Walker evolution,

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right) = \frac{8\pi}{3}G\rho_f ,$$

where ρ_f = mass density of the false vacuum. Thus, ρ_f produces gravitational repulsion.

The de Sitter Solution

The homogeneous isotropic solution can be described as a Robertson-Walker flat universe:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2 \ ,$$

where

$$a(t) \propto e^{\chi t} \ , \ \chi = \sqrt{\frac{8\pi}{3} G\rho_{\mathrm{f}}} \ .$$

This is called de Sitter spacetime.

By a change of coordinates, de Sitter spacetime can, surprisingly, be described as an open universe, a closed universe, or a static universe!

Cosmological "No-Hair" Conjecture

Conjecture: For "reasonable" initial conditions, even if far from homogeneous and isotropic, $\rho = \rho_f$ implies that the region will approach de Sitter space.

Conjectured by Hawking & Moss (1982). Can be proven for linearized perturbations about de Sitter spacetime (Jensen & Stein-Schabes, 1986, 1987). Was shown by Wald (1983) to hold for a class of very large (but spatially homogeneous) perturbations.

Analogous to the Black Hole No-Hair Theorem, which implies that gravitationally collapsing matter approaches a stationary black hole state that depends only on the mass, angular momentum, and charge.

Qualitative behavior: any distortion of the metric is stretched by the expansion to look smooth and flat. Any initial matter distribution is diluted away by the expansion.

De Sitter Event Horizon

In the de Sitter metric, with $a(t) = be^{\chi t}$, the coordinate distance that light can travel between times t_1 and t_2 is

$$\Delta r(t_1, t_2) = \int_{t_1}^{t_2} \frac{c}{a(t)}\,\mathrm{d}t = \frac{c}{b}\int_{t_1}^{t_2} e^{-\chi t}\,\mathrm{d}t = \frac{c}{b\,\chi}\left[e^{-\chi t_1} - e^{-\chi t_2}\right] \;,$$

which is bounded as $t_2 \to \infty$. If we multiply by $a(t_1)$ and take the limit,

$$\lim_{t_2\to\infty} a(t_1)\,\Delta r(t_1, t_2) = c\chi^{-1} \;,$$

which means that if two objects have a physical separation larger than $c\chi^{-1}$, the Hubble length, at any time, light from the first will never reach the second. This is called an event horizon. Event horizons protect an inflating patch from the rest of the universe: once the patch is large compared to $c\chi^{-1}$, nothing from outside can penetrate further than $c\chi^{-1}$.

Event Horizon in the Universe Today

Our universe today is entering a de Sitter phase, in which the dark energy dominates.

In the Review Problems for Quiz 3, Problem 17, the present event horizon was calculated, finding $z = 1.87$.

That means that events that are happening now (i.e., at the same value of the cosmic time) will NEVER be visible to us or our descendents.

The Ending of Inflation

A standard scalar field in a flat FRW universe obeys the equation of motion:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla_i^2\phi = -\frac{\partial V}{\partial \phi} \ ,$$

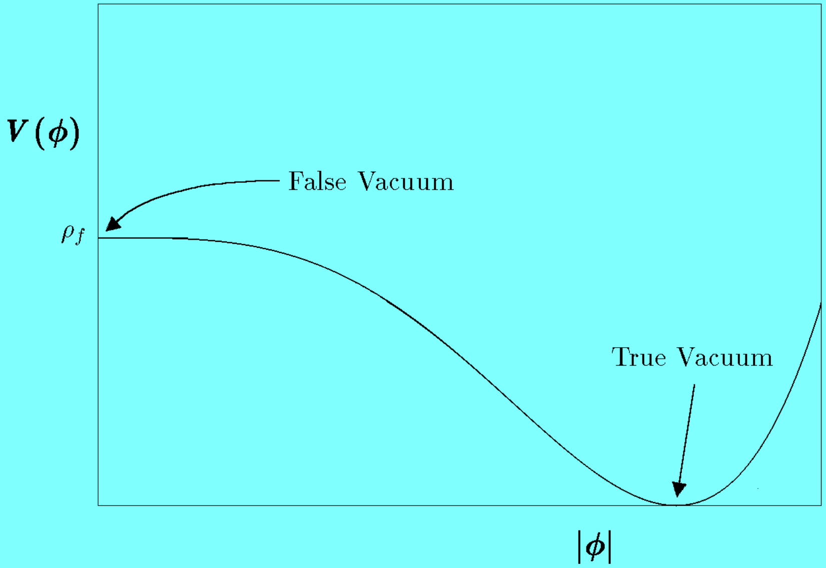
where ∇_i^2 is the Laplacian operator in comoving coordinates x^i, and $V(\phi)$ is the potential energy function (i.e., the potential energy per volume).

The spatial derivative piece soon becomes negligible, due to the $(1/a^2)$ suppression, which reflects the fact that the stretching of space causes ϕ to become nearly uniform over huge regions. The equation is then identical to that of a ball sliding on a hill described by $V(\phi)$, but with a viscous damping (i.e., friction) described by the term $-3(\dot{a}/a)\dot{\phi}$.

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \ .$$

Fluctuations in ϕ due to thermal and/or quantum effects will cause the field to start to slide down the hill. This will not happen globally, but in regions, typically of size $c\chi^{-1}$.

Within a region, ϕ will start to oscillate about the true vacuum value, at the bottom of the hill. Interactions with other fields will allow ϕ to give its energy to the other fields, producing a "hot soup" of other particles, which is exactly the starting point of the conventional hot big bang theory. This is called *reheating*.



The standard hot big bang scenario begins. Inflation has played the role of a prequel, setting the initial conditions for conventional cosmology.

Numerical Estimates

The energy scale at which inflation happened is not known. One plausible guess is the GUT scale, $E_{\rm GUT} \approx 10^{16}$ GeV. It cannot be higher (too much gravitational radiation), but can be as low as about 10^3 GeV.

For $E_{\rm GUT}$, we can estimate

$$\rho_f \approx \frac{E_{\rm GUT}^4}{\hbar^3 c^5} = 2.3 \times 10^{81} {\rm g/cm}^3 \ .$$

Then

$$\chi^{-1} \approx 2.8 \times 10^{-38} \ {\rm s} \ , \quad c\chi^{-1} = 8.3 \times 10^{-28} \ {\rm cm} \ ,$$

and the mass of a minimal region of inflation would be about

$$M \approx \frac{4\pi}{3}(c\chi^{-1})^3 \rho_f \approx 5.6 \ {\rm gram.}$$

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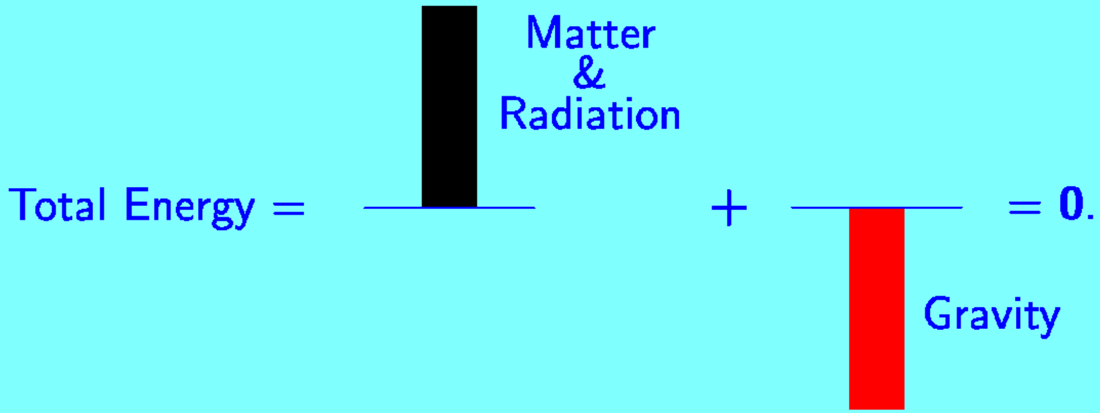

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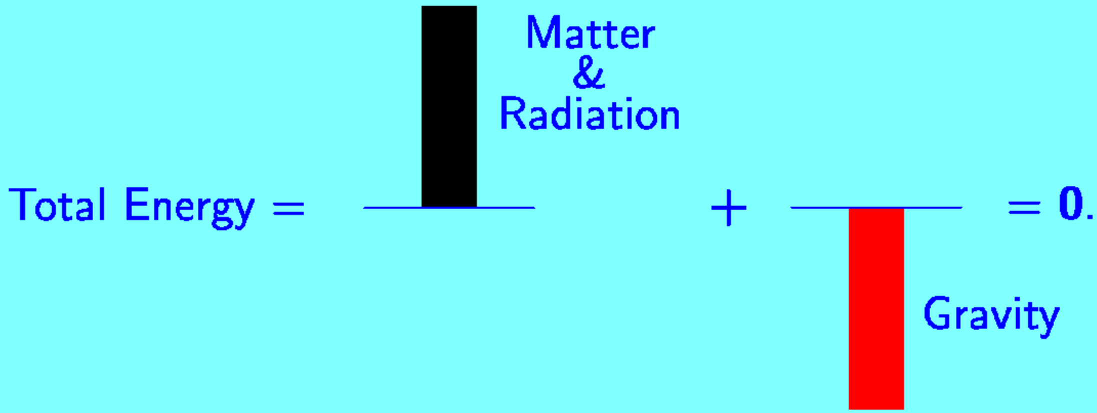


- ☆ Warning: the concept of total energy in GR is controversial. Some authors would just say that total energy is not defined.

Solutions to the Cosmological Problems

1) **Horizon Problem:** In inflationary models, uniformity is achieved in a tiny region BEFORE inflation starts. Without inflation, such regions would be far too small to matter. But inflation can stretch a tiny region of uniformity to become large enough to include the entire visible universe and more. For inflation at the GUT scale, 10^{16} GeV, we need expansion by about 10^{28}, which is about 65 time constants of the exponential expansion.

2) **Flatness Problem:** Just look at Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} \ .$$

"Flatness" is the statement that the final term in this equation is negligible. But during inflation, $\rho \approx \rho_v = \text{const}$, while $a(t)$ grows exponentially. If $a(t)$ grows by at least 10^{28} during inflation, the final term is suppressed by a factor of $\left(10^{28}\right)^2 = 10^{56}$.

3) **Monopole Problem:** Solved by dilution, as long as the inflation occurs during or after the process of monopole production. For inflation at the GUT scale, the volume of any comoving region increases during inflation by a factor of about $\left(10^{28}\right)^3 = 10^{84}$ or more! That is plenty enough to make monopoles impossible to find.

Some small number of monopoles could be produced during reheating, so it makes sense to look for them. But, except for an irreproducible single event seen by Blas Cabrera at Stanford in 1982, magnetic monopoles have not been seen.

Ripples in the Cosmic Microwave Background

The CMB is uniform in all directions to an accuracy of a few parts in 100,000. Nonetheless, at the level of a few parts in 100,000 there ARE anisotropies, and they have now been measured to high precision. Since the CMB is essentially a snapshot of the universe at $t \approx 380,000$ yr, these ripples are interpreted as perturbations in the cosmic mass density at this time.

In the early days of inflation, such density perturbations were a cause for worry. (The ripples had not yet been seen, but cosmologists knew that the early universe must have had density perturbations, or else galaxies and stars could never have formed.) Inflation smooths out the universe so effectively, that it looked like no density perturbations could survive.

Quantum Mechanics to the Rescue (Again)

Why again? We spoke earlier about how quantum mechanics was necessary to save us from freezing to death. If classical mechanics ruled, all thermal energy would gradually disappear into shorter and shorter wavelength electromagnetic radiation.

If inflation happened with classical physics, it would smooth the universe so perfectly that stars and galaxies could never form.

But quantum mechanics is intrinsically probabilistic. While the classical version of inflation predicts an almost exactly uniform mass density, the intrinsic randomness of the quantum version implies that the mass density will be a little higher in some places, and a little lower in others.

In 1965, Andrei Sakharov, the Russian nuclear physicist and political activist, proposed in a rather wildly speculative paper that quantum fluctuations might account for the structure of the universe.

In 1981, Mukhanov and Chibisov tried to calculate the density fluctuations in pre-inflationary/inflationary model invented by Alexei Starobinsky in 1980.

In summer 1982, Gary Gibbons and Stephen Hawking organized the Nuffield Workshop on the Very Early Universe in Cambridge UK, where a number of physicists worked feverishly and argued through the night about how to calculate these perturbations in inflation. In the end, all agreed. Four papers emerged: Hawking, Starobinsky, Guth & Pi, and Bardeen, Steinhardt, & Turner.

Basic conclusion: the amplitude of the density perturbations is very "model-dependent," meaning that it depends on the unknown details of $V(\phi)$. But: the spectrum — the way in which the intensity of the ripples depends on the wavelength of the ripples — is the same for a wide range of "simple" inflationary models. Simple = "Single field / slow-roll models," i.e. models with a single inflaton field, and with small values for $\mathrm{d}V/\mathrm{d}\phi$ and $\mathrm{d}^2V/\mathrm{d}\phi^2$.

Observations of the Ripples in the CMB

In 1982, it seemed (at least to me) out of the question that these ripples would ever be seen.

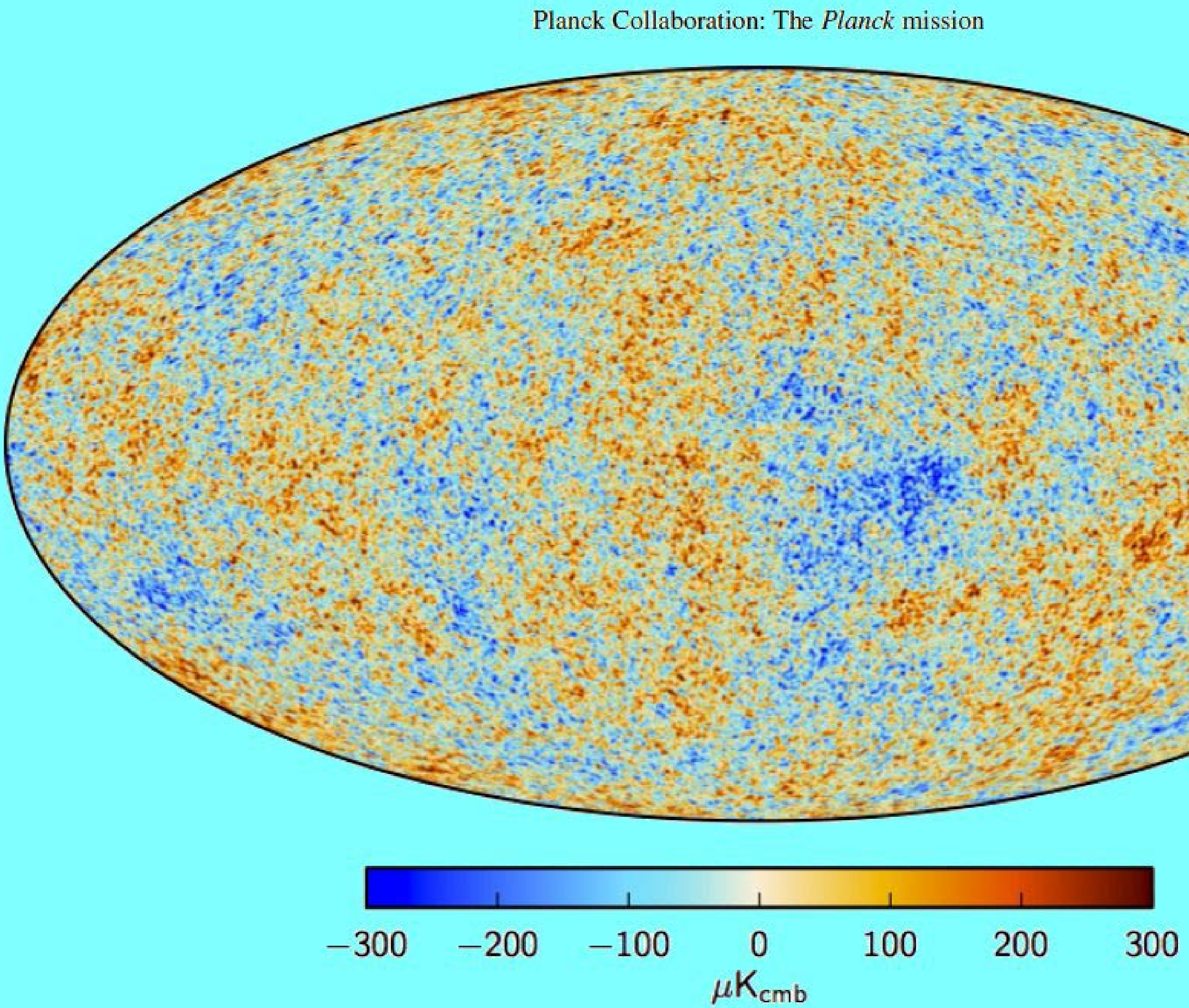
There have now been 3 satellite experiments to measure the CMB, plus many many ground-based experiments. The three satellites were:

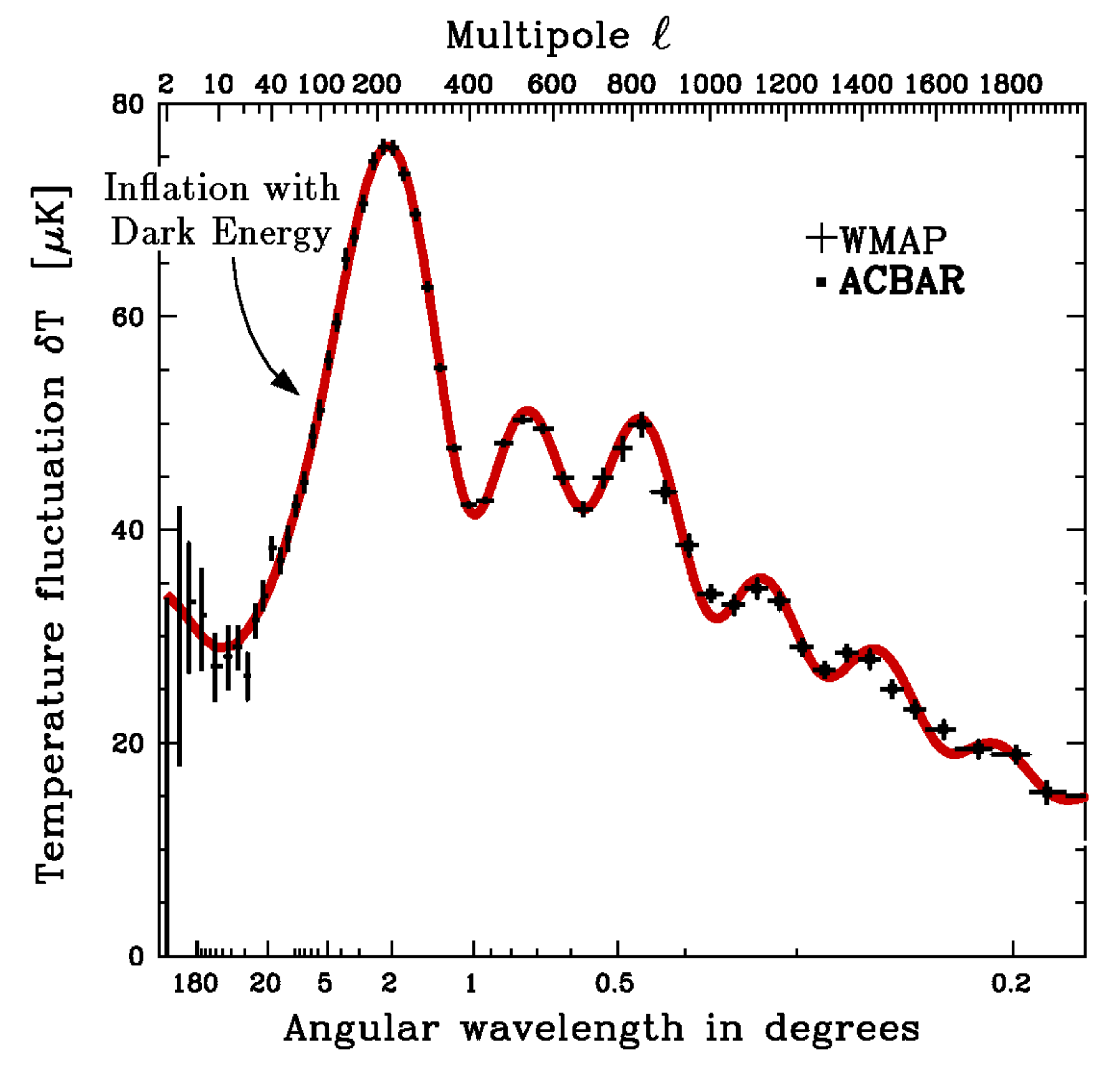
COBE: Cosmic Background Explorer, launched by NASA in 1989, after 15 years of planning. In 1992 it announced its first measurements of CMB anisotropies. The angular resolution was crude, about 7°, but the results agreed with inflation.

WMAP: The Wilkinson Microwave Anisotropy Probe, launched by NASA in 2001. 45 times more sensitive, with 33 times better angular resolution than COBE. Still consistent with inflation.

Planck: Launched in 2009 by ESA. Resolution about 2.5 times better than WMAP.

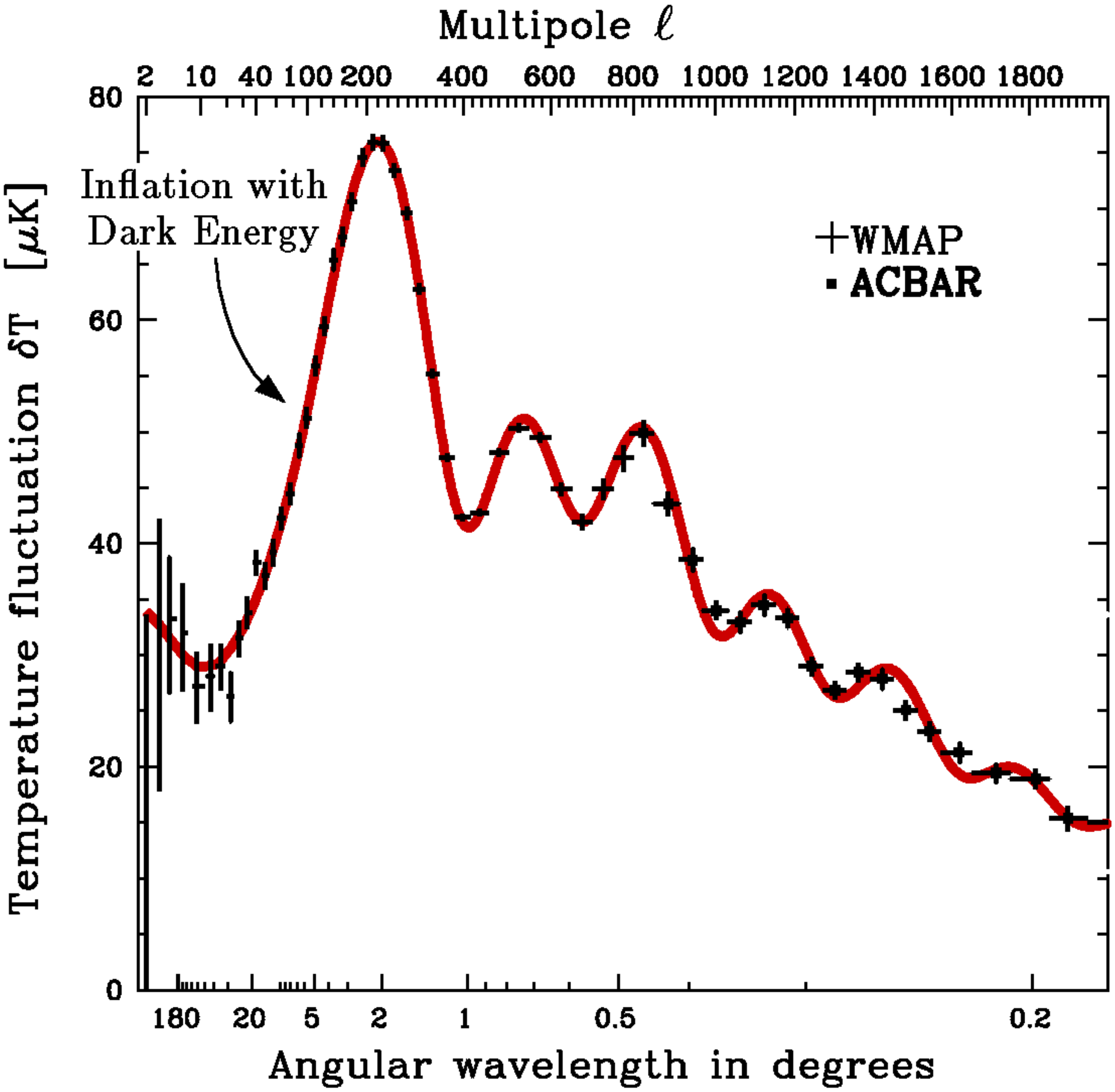
Ripples in the Cosmic Microwave Background





CMB: Comparison of Theory and Experiment

Graph by Max Tegmark, for A. Guth & D. Kaiser, *Science* **307, 884** (Feb 11, 2005), updated to include WMAP 7-year data (Jan 2010).



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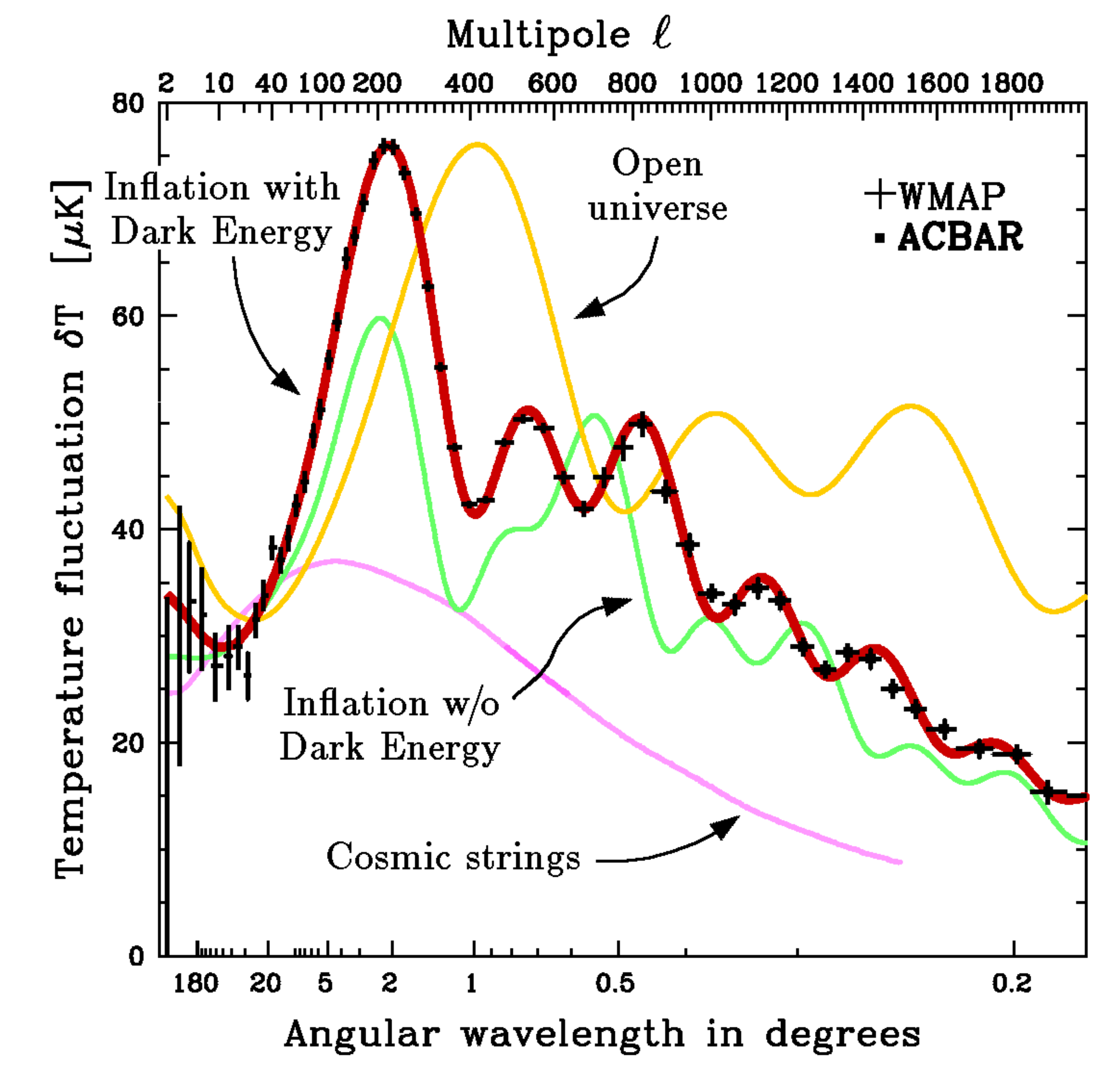

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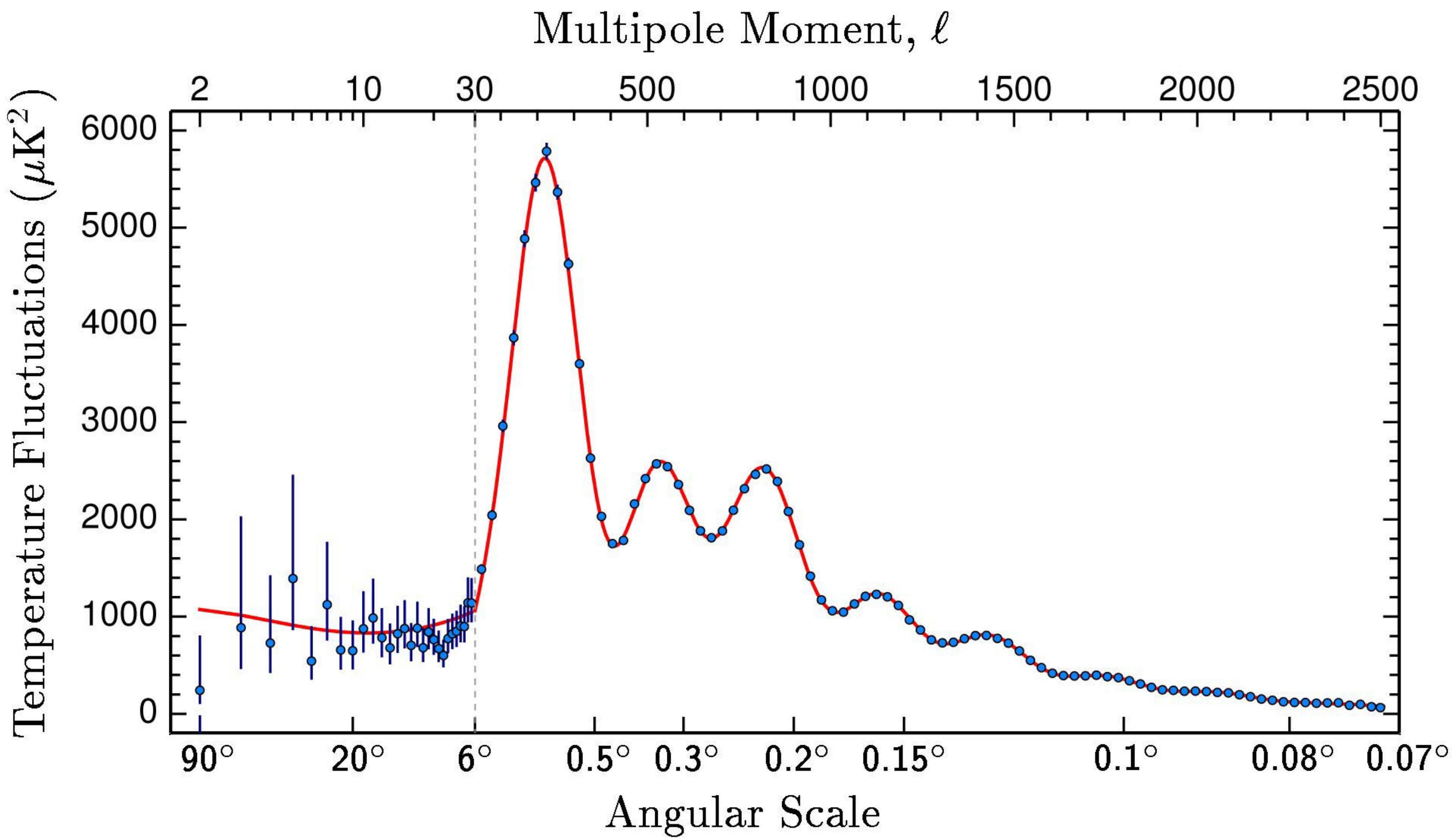
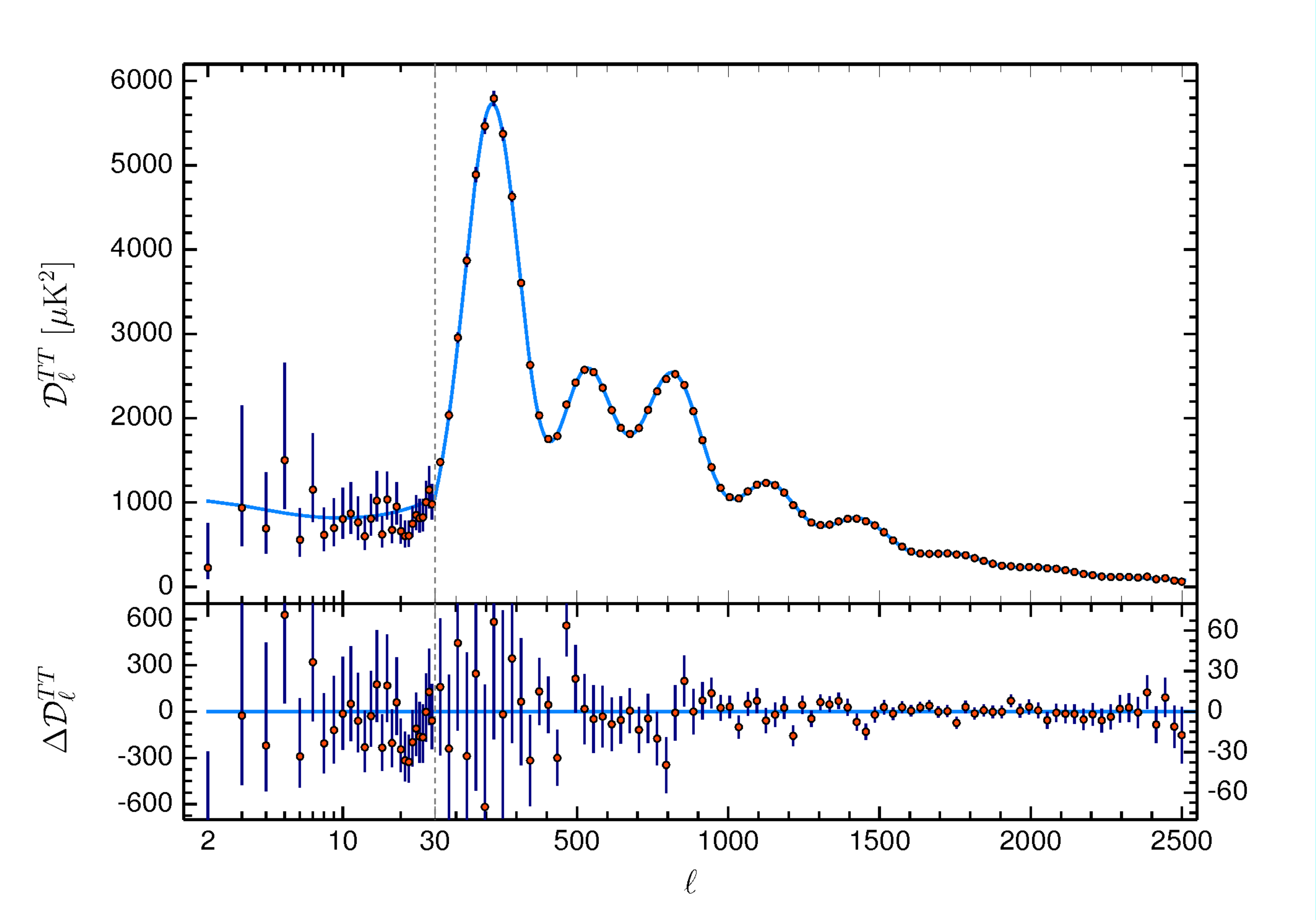
Planck 2015 Spectrum



Planck Collaboration, 2015

Planck 2018 Spectrum



Lower panel shows difference between data and model.

Eternal Inflation

In hilltop inflation, while the scalar field rolls down the hill in the potential energy diagram, there is always some small quantum mechanical probability that the field remains at the top.

Approximate calculations show that the probability of remaining at the top falls off exponentially with time. That is, the false vacuum has an exponential decay law, like a radioactive substance.

In any successful model of inflation, the half-life of the false vacuum is much longer than the doubling time of the exponential expansion of $a(t)$.

So, in one half-life of the decay, half of the region in false vacuum stops inflating, but the region remaining in the false vacuum state becomes much larger than the original size of the full region! Thus, the volume of false vacuum region grows exponentially in time.

The ending of inflation happens in localized patches, where in each patch there is a local big bang, forming what we call a “pocket universe”. The theory seems to lead to the production of pocket universes ad infinitum. The collection of pocket universes is called a “multiverse”.

Is this relevant to physics?

Maybe. It offers a possible explanation of the very small vacuum energy density of our universe. If there is an infinite set of pocket universes, with each one filled with a different vacuum-like state (string theory, for example, gives a huge number of vacuum-like states), then there will be pocket universes with very small vacuum energies. Only those with small vacuum energies will develop life, since the others will implode of fly apart before life could form. All of this is speculative and controversial, however.