8.286 Class 16
November 2, 2022

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 3

Review from the previous lecture

Summary: Complete Friedmann Equations and Energy Conservation

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}$$

$$\ddot a = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)a$$

$$\dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) .$$

The items in red are new.

Review from the previous lecture

Black-Body Radiation

Energy density: $u = g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3}$

Pressure: $p = \frac{1}{3}u$

Number density: $n = g^*\frac{\zeta(3)}{\pi^2}\frac{(kT)^3}{(\hbar c)^3}$

Entropy density: $s = g\frac{2\pi^2}{45}\frac{k^4T^3}{(\hbar c)^3}$,

where for photons

$$g = g^* = 2 ,$$

But g and g^* will have different values for different particles, to be discussed today.

Review from the previous lecture

Neutrinos — A Brief History

- In 1930, Wolfgang Pauli proposed the existence of the neutrino — an unseen particle that he theorized to explain how beta decay ($n \longrightarrow p + e^-$, inside a nucleus) could be consistent with energy conservation. (Niels Bohr, by contrast, proposed that energy conservation was only valid statistically.) Pauli called it a neutron, while the particle that we know as a neutron was not discovered until 1932, by James Chadwick.
- In 1934 Enrico Fermi developed a full theory of beta decay, and gave the neutrino its current name ("little neutral one").
- The neutrino was not seen observationally until 1956 by Clyde Cowan and Frederick Reines at the Savannah River nuclear reactor.
- Cowan died in 1974 at the age of 54, and Reines was awarded the Nobel Prize for this work in 1995, at the age of 77.

Neutrino Mass, Take 1

- During the 20th century, neutrinos were thought to be massless (rest mass = 0). We now know that they have a very small but nonzero mass, but for the period that we will be discussing now (the radiation-dominated era), the masses are negligible. As long as $mc^2 \ll kT$, the particle will act as if it is massless.
- So, for now (Take 1), we will pretend neutrinos are massless.

Photons are Bosons, Neutrinos are Fermions

- All particles can be divided into these two classes.
- For bosons, any number of particles can exist in the same quantum state. This is what allows photons to build up a classical electromagnetic field, which involves a very large number of photons. A laser in particular concentrates a huge number of photons in a single quantum state.
- For fermions, by contrast, there can be no more than one particle in a given quantum state. Electrons are also fermions — the one-electron-per-quantum-state rule is called the *Pauli Exclusion Principle*, and is responsible for essentially all of chemistry.
- In relativistic quantum field theory, one can prove the *spin-statistics theorem*: all particles with integer spin (in units of $\hbar$) are bosons, and all particles with half-integer spin ($\frac{1}{2}$, $\frac{3}{2}$, etc.) are fermions. (And those are the only possibilities.)

Consequences of Fermi Statistics

Reminder:

$$u = g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3}\ , \qquad n = g^*\frac{\zeta(3)}{\pi^2}\frac{(kT)^3}{(\hbar c)^3}\ .$$

- Because there are fewer states that fermions can occupy, the number density, energy density, pressure, and entropy density for fermions are all reduced.
- For fermions,

  g is reduced by a factor of 7/8.

  g^* is reduced by a factor of 3/4.

Neutrino Flavors

Neutrinos come in 3 different species, or *flavors*:

Electron neutrino ν_e:	$e^- + p \longrightarrow n + \nu_e$
Muon neutrino ν_μ:	$\mu^- + p \longrightarrow n + \nu_\mu$
Tau neutrino ν_τ:	$\tau^- + p \longrightarrow n + \nu_\tau$.

A muon is essentially a heavy electron, with $m_\mu c^2 = 105.7$ MeV, compared to $m_e c^2 = 0.511$ MeV. A tau is a still heavier version of the electron, with $m_\tau c^2 = 1776.9$ MeV.

Neutrino States

- 3 flavors implies a factor of 3 in g and g^*.
- Neutrinos exist as particles and antiparticles, unlike photons, which are their own antiparticles. The particle/antiparticle option leads to a factor of 2 in g and g^*
- While photons can be left or right circularly polarized, neutrinos are always seen to be *left-handed*: the spin is opposite the direction of the momentum. Antineutrinos are always right-handed.

An Aside on Discrete Symmetries

- Before the left-handed property of neutrinos was discovered, it was thought that that the laws of physics were invariant under *parity transformations* ($x \to -x, y \to -y, z \to -z$). But the parity transform of a left-handed neutrino would be a right-handed neutrino, which has never been seen, so the laws of physics are **NOT** parity-invariant.
- The handedness of neutrinos is consistent with CP symmetry, charge conjugation time parity. The CP transform of a left-handed neutrino is a right-handed antineutrino — both exist and, as far as we know, behave identically. However, CP symmetry is known to be violated by neutral kaons.
- However, CPT symmetry — charge conjugation times parity times time-reversal — is required by relativistic quantum field theory and is believed to be a symmetry of nature.

g and g^* for Neutrinos

$$g_\nu = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{21}{4} .$$

$$g^*_\nu = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{3}_{\substack{\text{3 species} \\ \nu_e,\nu_\mu,\nu_\tau}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{1}_{\text{Spin states}} = \frac{9}{2} .$$

Hotter Still

If we follow the universe further back in time, we will find that at some point kT becomes large compared to $m_e c^2 = 0.511$ MeV, the rest energy of an electron. Then electron-positron pairs start to behave as massless particles, and contribute to the black-body radiation.

$$g_{e^+e^-} = \underbrace{\frac{7}{8}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = \frac{7}{2} .$$

$$g^*_{e^+e^-} = \underbrace{\frac{3}{4}}_{\text{Fermion factor}} \times \underbrace{1}_{\text{Species}} \times \underbrace{2}_{\text{Particle/antiparticle}} \times \underbrace{2}_{\text{Spin states}} = 3 .$$

For 0.511 MeV $\ll kT \ll$ 106 MeV

For electrons, $m_e c^2 = 0.511$ MeV.

For muons, $m_\mu c^2 = 106$ MeV.

For 0.511 MeV $\ll kT \ll$ 106 MeV, electrons and positrons act like massless particles, and only a negligible number of muons would be produced.

The energy density can therefore be calculated from

$$g_{\rm tot} = \underbrace{2}_{\rm photons} + \underbrace{\frac{21}{4}}_{\rm neutrinos} + \underbrace{\frac{7}{2}}_{e^+e^-} = 10\,\frac{3}{4}$$

Energy Density of Radiation Today

- Temperature of the cosmic microwave background (CMB) today: $T_\gamma = 2.7255 \pm 0.0006$ K.* This gives $kT_\gamma = 2.35 \times 10^{-4}$ eV.
- Continuing our "Take 1" pretense that neutrinos are massless, the radiation that exists in the universe today includes photons and neutrinos. **But $T_\nu \neq T_\gamma$.**
- The complication occurs when the e^+e^- pairs "freeze out," (i.e., disappear), as kT falls below 0.511 MeV. This happens around $t = 1$ second. Neutrino interactions become weaker as the temperature falls, and by this time they have become so weak that the neutrinos absorb only a negligible amount of the e^+e^- energy. It essentially all goes into heating the photons, which then become hotter than the neutrinos.

*D.J. Fixsen, Ap. J. **707,** 916 (2009). Based mainly on the COBE (Cosmic Background Explorer) data, 1989 – 1993.

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- You will calculate this on Problem Set 7. The key is to use *entropy*, not energy, since entropy is simply conserved. Energy density, by contrast, obeys

$$\dot\rho = -3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) ,$$

so one needs to calculate the pressure p as the e^+e^- pairs freeze out. That's complicated.

- The result (that you will find) is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma .$$

- This ratio is maintained to the present day, so the total radiation energy density today is

$$u_{\rm rad,0} = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right]\frac{\pi^2}{30}\frac{(kT_\gamma)^4}{(\hbar c)^3}$$
$$= 7.01 \times 10^{-14}\ {\rm J/m^3} ,$$

which is what we used when we estimated $t_{\rm eq}$, the time of matter-radiation equality.

- We (crudely) found $\sim 75,000$ years. Ryden gives 47,000 years. The Particle Data Group (2020) gives $51,100 \pm 800$ years.

The Real Story of Neutrino Masses

- We have not yet measured the mass of a neutrino, but we have seen neutrinos "oscillate" from one flavor to another:
  - Electron neutrinos from the Sun arrive at Earth as a mixture of all three flavors.
  - Neutrinos produced by cosmic rays in the upper atmosphere have been found to undergo oscillations on their way to ground level.
  - Neutrinos produced by reactors and accelerators have been seen to oscillate.
- Oscillations require a nonzero mass: essentially because a massless particle experiences an infinite time dilation, so time stops.
- The oscillations measure the differences of the squares of the masses.

Neutrino Masses and Quantum Superpositions

- Quantum theory allows for states that are superpositions of other states.
- Neutrinos are produced in states of definite flavor, called ν_e, ν_μ, and ν_τ. (I.e., $e^- + p \longrightarrow n + \nu_e$, while $\mu^- + p$ leads to ν_μ, and $\tau^- + p$ leads to ν_τ.) But these are not states of definite mass!
- The states of definite mass are called ν_1, ν_2, and ν_3.
- Each flavor state is a superposition of all three states of definite mass, and each state of definite mass is a superposition of all three flavor states.

Differences of Squares of Neutrino Masses

As of 2020, the Particle Data Group reports:

$$\Delta m_{21}^2\, c^4 = (7.53 \pm 0.18) \times 10^{-5}\ \text{eV}^2\ ,$$
$$\Delta m_{32}^2\, c^4 = \left(2.546^{+0.034}_{-0.040}\right) \times 10^{-3}\,\text{eV}^2\ ,$$

or

$$\Delta m_{32}^2\, c^4 = (2.453 \pm 0.034) \times 10^{-3}\,\text{eV}^2\ ,$$

where the two options for Δm_{32}^2 depend on assumptions about the ordering of the masses. Note that $\sqrt{\Delta m_{21}^2\, c^4} = 8.68 \times 10^{-3}$ eV, and $\sqrt{\Delta m_{32}^2\, c^4} = 0.0505$ eV or 0.0495 eV. Recall that today $kT_\gamma = 2.35 \times 10^{-4}$ eV, which is much smaller.

Does Neutrino Mass Affect Our Calculation of t_{eq}?

No!

We wrote

$$u_{\text{rad},0} = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3}$$
$$= 7.01 \times 10^{-14}\ \text{J/m}^3\ ,$$

but what we really used was

$$u_{\text{rad}}(t) = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3} \left(\frac{a(t_0)}{a(t)}\right)^4\ ,$$

which is valid for t anywhere near the time t_{eq}.

Cosmological Bound on the Sum of ν Masses

- From cosmology of large-scale structure, we know that*

$$(m_1 + m_2 + m_3)c^2 \leq 0.17 \text{ eV}.$$

- Why? Because neutrinos "free-stream" easily from one place to another. If they carried too much mass, they would even out the mass density and suppress large-scale structure.

*S. R. Choudhury and S. Hannestad, JCAP 2020, No. 7, 037 (2020), arXiv:1907.12598.

Neutrino Mass and Spin States

- The measurements of the mass differences imply that at least 2 of the 3 neutrino masses must be nonzero.
- If the mass of a neutrino is nonzero, then it **cannot** always be left-handed.
- To see this, consider a left-handed neutrino moving in the z direction, with spin in the $-z$ direction. With $m > 0$, it must move slower than c. So an observer can move along the z-axis faster than the neutrino. To such an observer, the momentum of the ν will be in the $-z$ direction, the spin will be in the $-z$ direction, and the ν will appear right-handed.
- How could this right-handed neutrino fit into our theory?

Majorana and Dirac Masses

There are two possibilities for neutrino mass:

Dirac Mass: Right-handed neutrino would be a new as-yet unseen type of particle. But it would interact so weakly that it would not have been produced in significant numbers during the big bang.

Majorana Mass: If *lepton number* is not conserved (which seems plausible), so the neutrino is absolutely neutral, then the right-handed neutrino could be the particle that we have called the anti-neutrino.

Neutrino Masses and Neutrinoless Double Beta Decay

- Key experiment to distinguish Majorana from Dirac mass: neutrinoless double beta decay. Standard double beta decay looks like

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e \ .$$

If the ν has a Majorana mass, and therefore it is its own antiparticle, then the reaction could happen without the two final $\bar{\nu}_e$'s, which can essentially annihilate each other. (The annihilation could happen as part of the interaction, so the energy is given to the $(A, Z + 2)$ and $2e^-$ particles, with no other particles emitted.)