8.286 Class 17
November 7, 2022

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE, PART 4

Announcements: Quiz on Wednesday

- See web page for full information.
- Office hours:
  - Alan: today, 5:15-6:15 pm, Room 8-320.
  - Marianne: tomorrow, 4-5 pm in Room 6C-442 (also called the Cosman Room).
- Time and Place: The quiz will be during our regular class time on Wednesday, 11:05 am - 12:25 pm, in Room 50-340 (Walker Memorial).

Review from the previous lecture

The Real Story of Neutrino Masses

- We have not yet measured the mass of a neutrino, but we have seen neutrinos "oscillate" from one flavor to another:
  - Electron neutrinos from the Sun arrive at Earth as a mixture of all three flavors.
  - Neutrinos produced by cosmic rays in the upper atmosphere have been found to undergo oscillations on their way to ground level.
  - Neutrinos produced by reactors and accelerators have been seen to oscillate.
- Oscillations require a nonzero mass: essentially because a massless particle experiences an infinite time dilation, so time stops.
- The oscillations measure the differences of the squares of the masses.

Review from the previous lecture

Neutrino Masses and Quantum Superpositions

- Quantum theory allows for states that are superpositions of other states.
- Neutrinos are produced in states of definite flavor, called ν_e, ν_μ, and ν_τ. (I.e., $e^- + p \longrightarrow n + \nu_e$, while $\mu^- + p$ leads to ν_μ, and $\tau^- + p$ leads to ν_τ.) But these are not states of definite mass!
- The states of definite mass are called ν_1, ν_2, and ν_3.
- Each flavor state is a superposition of all three states of definite mass, and each state of definite mass is a superposition of all three flavor states.

Review from the previous lecture

Differences of Squares of Neutrino Masses

As of 2020, the Particle Data Group reports:

$$\Delta m_{21}^2\, c^4 = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2\ ,$$
$$\Delta m_{32}^2\, c^4 = \left(2.546^{+0.034}_{-0.040}\right) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

or

$$\Delta m_{32}^2\, c^4 = (2.453 \pm 0.034) \times 10^{-3}\,\mathrm{eV}^2\ ,$$

where the two options for Δm_{32}^2 depend on assumptions about the ordering of the masses. Note that $\sqrt{\Delta m_{21}^2\, c^4} = 8.68 \times 10^{-3}$ eV, and $\sqrt{\Delta m_{32}^2\, c^4} = 0.0505$ eV or 0.0495 eV. Recall that today $kT_\gamma = 2.35 \times 10^{-4}$ eV, which is much smaller.

Review from the previous lecture

Does Neutrino Mass Affect Our Calculation of t_{eq}?

No!

We wrote

$$u_{\mathrm{rad},0} = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3}$$
$$= 7.01 \times 10^{-14}\ \mathrm{J/m}^3\ ,$$

but what we really used was

$$u_{\mathrm{rad}}(t) = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} \frac{(kT_\gamma)^4}{(\hbar c)^3} \left(\frac{a(t_0)}{a(t)}\right)^4\ ,$$

which is valid for t anywhere near the time t_{eq}.

Review from the previous lecture

Cosmological Bound on the Sum of ν Masses

- From cosmology of large-scale structure, we know that*

$$(m_1 + m_2 + m_3)c^2 \leq 0.17\ \mathrm{eV}.$$

- Why? Because neutrinos "free-stream" easily from one place to another. If they carried too much mass, they would even out the mass density and suppress large-scale structure.

*S. R. Choudhury and S. Hannestad, JCAP 2020, No. 7, 037 (2020), arXiv:1907.12598.

Review from the previous lecture

Neutrino Mass and Spin States

- The measurements of the mass differences imply that at least 2 of the 3 neutrino masses must be nonzero.
- If the mass of a neutrino is nonzero, then it cannot always be left-handed.
- To see this, consider a left-handed neutrino moving in the z direction, with spin in the $-z$ direction. With $m > 0$, it must move slower than c. So an observer can move along the z-axis faster than the neutrino. To such an observer, the momentum of the ν will be in the $-z$ direction, the spin will be in the $-z$ direction, and the ν will appear right-handed.
- How could this right-handed neutrino fit into our theory?

Review from the previous lecture

Majorana and Dirac Masses

There are two possibilities for neutrino mass:

Dirac Mass: Right-handed neutrino would be a new as-yet unseen type of particle. But it would interact so weakly that it would not have been produced in significant numbers during the big bang.

Majorana Mass: If *lepton number* is not conserved (which seems plausible), so the neutrino is absolutely neutral, then the right-handed neutrino could be the particle that we have called the anti-neutrino.

Review from the previous lecture

Neutrino Masses and Neutrinoless Double Beta Decay

☆ Key experiment to distinguish Majorana from Dirac mass: neutrinoless double beta decay. Standard double beta decay looks like

$$(A, Z) \to (A, Z + 2) + 2e^- + 2\bar{\nu}_e \ .$$

If the ν has a Majorana mass, and therefore it is its own antiparticle, then the reaction could happen without the two final $\bar{\nu}_e$'s, which can essentially annihilate each other. (The annihilation could happen as part of the interaction, so the energy is given to the $(A, Z + 2)$ and $2e^-$ particles, with no other particles emitted.)

Thermal History of the Universe

Assuming that the early universe can be described as radiation-dominated and flat (excellent approximations), then

$$H^2 = \frac{8\pi}{3} G\rho \ , \quad a(t) \propto t^{1/2} \ , \quad H = \frac{\dot{a}}{a} = \frac{1}{2t} \ ,$$

which implies

$$\rho = \frac{3}{32\pi G t^2} \ .$$

We also know

$$u = g \frac{\pi^2}{30} \frac{(kT)^4}{(\hbar c)^3} \ , \quad \text{and} \quad \rho = u/c^2 \ ,$$

so

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G} \right)^{1/4} \frac{1}{\sqrt{t}} \ .$$

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Assuming 0.511 MeV $\ll kT \ll$ 106 MeV (i.e., assuming kT is between mc^2 for the electron and muon),

$$g_{\rm tot} = \underbrace{2}_{\text{photons}} + \underbrace{\frac{21}{4}}_{\text{neutrinos}} + \underbrace{\frac{7}{2}}_{e^+e^-} = 10\frac{3}{4} \ .$$

For $t = 1$ second, this gives $kT = 0.860$ MeV.

Assuming 0.511 MeV $\ll kT \ll$ 106 MeV (i.e., assuming kT is between mc^2 for the electron and muon), we find that at $t = 1$ second, $kT = 0.860$ MeV.

Since $T \propto 1/\sqrt{t}$, we can write

$$kT = \frac{0.860\text{ MeV}}{\sqrt{t\text{ (in sec)}}} ,$$

or equivalently

$$T = \frac{9.98 \times 10^9\text{ K}}{\sqrt{t\text{ (in sec)}}} .$$

Relation Between a and T

- Conservation of entropy implies that $s \propto 1/a^3(t)$, but we also know that $s \propto gT^3$. It follows that

$$g^{1/3}T \propto \frac{1}{a(t)} .$$

Recombination

- “Baryonic” matter is matter made of protons, neutrons, and electrons. I.e., it is ordinary matter, as opposed to dark matter or dark energy.
- About 80% of baryonic matter is hydrogen. Most of the rest is helium. Elements heavier then helium make up a very small fraction. So we mostly have hydrogen.
- At high T, hydrogen atoms ionize, become free protons and electrons. The ionization temperature depends on density, but for the density of the early universe, it is about 4,000 K. (Ryden calculates it on p. 154 as 3760 K.)
- When T falls below 4,000 K, the protons and electrons combine to form neutral H. This is called “recombination,” but “combination” would be more accurate.

Decoupling

- Photons interact strongly with free electrons.
- The reason can be understood classically: when an electromagnetic wave hits a free electron, the electron experiences the $\vec{F} = e\vec{E}$ force of the electric field. Since its mass is very small, it oscillates rapidly, and sends electromatic radiation in all directions, using energy that it removes from the incoming wave. Thus, the incoming wave is scattered.
- The result is that the universe was opaque to photons in the ionized phase (plasma phase), but became transparent when the ionized gas became neutral atoms.
- The transition to a transparent universe is called “decoupling” (i.e., the photons “decouple” from the matter of the universe).

Time of Decoupling t_d

- The result is that the universe was opaque to photons in the ionized phase (plasma phase), but became transparent when the ionized gas became neutral atoms.
- The transition to a transparent universe is called "decoupling" (i.e., the photons "decouple" from the matter of the universe).

- At $T_{\rm rec} = 4{,}000$ K, about half of the hydrogen is ionized.
- Note that $kT_{\rm rec} \approx 0.34$ eV, while the ionization energy of H is 13.6 eV. The reason for the big difference is that $n_{\rm baryon}/n_\gamma \approx 10^{-9}$, so it is rare for electrons and protons to find each other.
- Since even a very small density of free electrons is enough to make the universe opaque, photon decoupling does not occur until T falls to $T_{\rm dec} \approx 3{,}000$ K.

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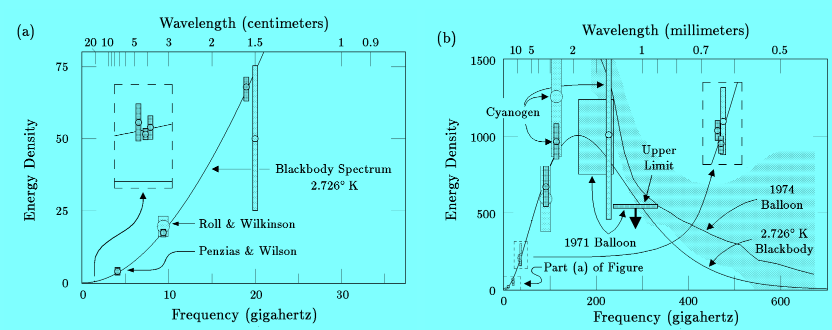
- Approximating the universe as flat and matter-dominated from $T_{\rm dec}$ to today, we can estimate the time of decoupling by

$$t_d = \left(\frac{T_0}{T_d}\right)^{3/2} t_0$$
$$\approx \left(\frac{2.7\,\mathrm{K}}{3000\,\mathrm{K}}\right)^{3/2} \times \left(13.7\times 10^9\ \mathrm{yr}\right) \approx 370{,}000\ \mathrm{yr}\ .$$

Spectrum of the Cosmic Microwave Background

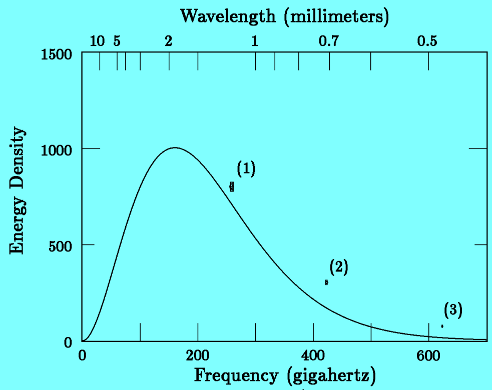
$$\rho_\nu(\nu)\,d\nu = \frac{16\pi^2\hbar\nu^3}{c^3}\frac{1}{e^{2\pi\hbar\nu/kT}-1}\,d\nu\ ,$$

where $\rho_\nu(\nu)\,d\nu$ is the energy density of photons in the frequency range from ν to $\nu + d\nu$.



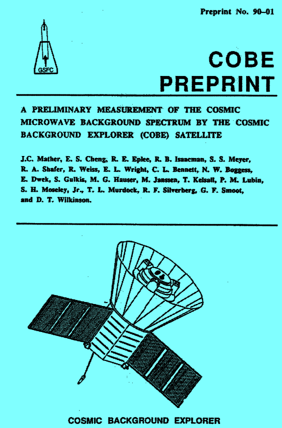
CMB Data in 1975

The situation got worse before it got better:



Data from Berkeley-Nagoya Rocket Flight, 1987

Points 2 and 3 differ from the blackbody curve by 12 and 16 standard deviations, respectively!



Preprint No. 90–01

GSFC

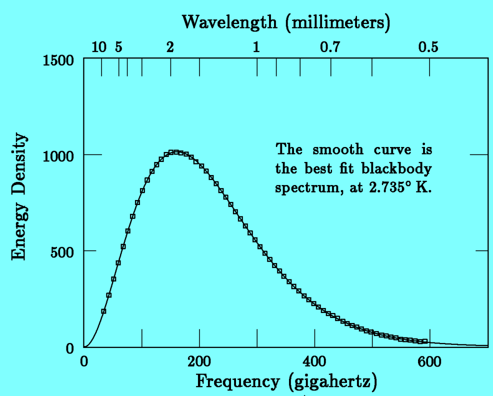
COBE PREPRINT

A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE COSMIC BACKGROUND EXPLORER (COBE) SATELLITE

J.C. Mather, E. S. Cheng, R. E. Eplee, R. B. Isaacman, S. S. Meyer, R. A. Shafer, R. Weiss, E. L. Wright, C. L. Bennett, N. W. Boggess, E. Dwek, S. Gulkis, M. G. Hauser, M. Janssen, T. Kelsall, P. M. Lubin, S. H. Moseley, Jr., T. L. Murdock, R. F. Silverberg, G. F. Smoot, and D. T. Wilkinson.

COSMIC BACKGROUND EXPLORER

The Cosmic Background Explorer (COBE) satellite was launched in the fall of 1989. In January 1990, at meeting of the American Astronomical Society in Washington, D.C., the first data on the spectrum of the cosmic microwave background was announced. Shown is the cover page of the original preprint.



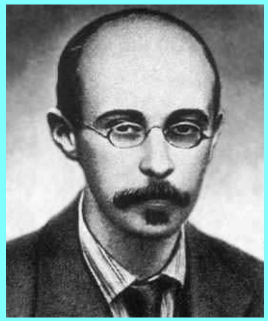
This is the original COBE measurement of the CMB spectrum, Jan 1990. The Energy density is in units of electron volts per cubic meter per gigahertz. The error bars are shown as 1% of the peak intensity. This graph was based on 9 minutes of data. Later data analysis reduced the error bars by a factor of 100, with still a perfect fit to the blackbody spectrum.

Historical Interlude:

Albert Einstein and Alexander Friedmann

Albert Einstein and the Friedmann Equations

Albert Einstein



Alexander A. Friedmann

Publication of the Friedmann Equations

On the Curvature of Space

A. Friedmann
Petersburg
Received June 29, 1922
Zeitschrift für Physik

Einstein's Reaction

REMARK ON THE WORK OF A. FRIEDMANN (FRIEDMANN 1922)
"ON THE CURVATURE OF SPACE"
A. Einstein, Berlin
Received September 18, 1922
Zeitschrift für Physik

The work cited contains a result concerning a non-stationary world which seems suspect to me. Indeed, those solutions do not appear compatible with the field equations (A). From the field equations it follows necessarily that the divergence of the matter tensor T_{ik} vanishes. This along with the anzatzes (C) and (D) leads to the condition

$$\partial\rho/\partial x_4 = 0$$

which together with (8) implies that the world-radius R is constant in time. The significance of the work therefore is to demonstrate this constancy.

REFERENCES: Friedmann, A. 1922, *Zs. f. Phys.*, 10, 377.

Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

Sequence of Events

June 29, 1922: Friedmann's paper received at *Zeitschrift für Physik*.

September 18, 1922: Einstein's refutation received at *Zeitschrift für Physik*.

December 6, 1922: Friedmann learns about Einstein's objection from his friend, Yuri A. Krutkov, who is visiting in Berlin. Friedmann writes a detailed letter to Einstein. Einstein is traveling and does not read it.

May, 1923: Einstein meets Krutkov in Leiden, both attending the farewell lecture by Lorentz, who was retiring.

Krutkov's letters to his sister: "On Monday, May 7, 1923, I was reading, together with Einstein, Friedmann's article in the *Zeitschrift für Physik*." May 18: "I defeated Einstein in the argument about Friedmann. Petrograd's honor is saved!"*

May 31, 1923: Einstein's retraction of his refutation is received at *Zeitschrift für Physik*.

* Quoted in *Alexander A. Friedmann: the Man who Made the Universe Expand*, by E.A. Tropp, V. Ya. Frenkel, & A.D. Chernin.

Einstein's Retraction

A NOTE ON THE WORK OF A. FRIEDMANN
"ON THE CURVATURE OF SPACE"
A. Einstein, Berlin
Received May 31, 1923
Zeitschrift für Physik

I have in an earlier note (Einstein 1922) criticized the cited work (Friedmann 1922). My objection rested however — as Mr. Krutkoff in person and a letter from Mr. Friedmann convinced me — on a calculational error. I am convinced that Mr. Friedmann's results are both correct and clarifying. They show that in addition to the static solutions to the field equations there are time varying solutions with a spatially symmetric structure.

REFERENCES:

Einstein, A. 1922, *Zs. f. Phys.*, 11, 326.
Friedmann, A. 1922, *Ebenda,* 10, 377.

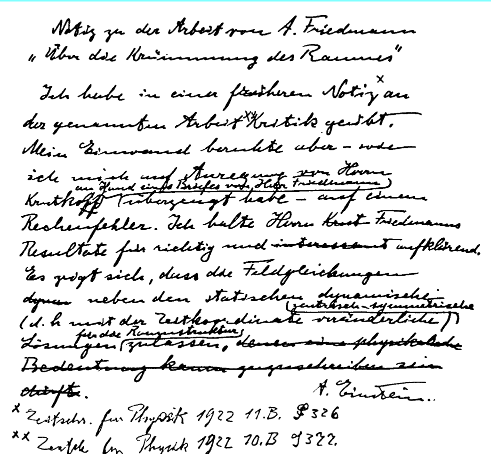
Translation: *Cosmological Constants*, edited by Jeremy Bernstein and Gerald Feinberg

Einstein and Krutkov

Albert Einstein
Barcelona, 1923

Yuri A. Krutkov.

Einstein's Draft



Notiz zu der Arbeit von A. Friedmann „Über die Krümmung des Raumes"

Ich habe in einer früheren Notiz* an der genannten Arbeit** Kritik geübt. Mein Einwand beruhte aber – wie ich mich auf Anregung von Herrn Krutkoff [an Hand eines Briefes von Herrn Friedmann] überzeugt habe – auf einem Rechenfehler. Ich halte Herrn Friedmanns Resultate für richtig und aufklärend. Es zeigt sich, dass die Feldgleichungen neben den statischen dynamische (zentrisch-symmetrische) (d. h. mit der Zeitkoordinate veränderliche) Lösungen für die Raumstruktur zulassen, denen eine physikalische Bedeutung kaum zuzuschreiben sein dürfte.

A. Einstein.

* Zeitschr. f. Physik 1922 11.B. S.326
** Zeitschr. f. Physik 1922 10.B S.377.

Einstein's draft of 1923 in which he withdrew his earlier objection to Friedmann's dynamic solutions to the field equations. The last bit of the last sentence was: "a physical significance can hardly be ascribed to them". He crossed this out before sending the note to print.

"a physical significance can hardly be ascribed to them."

* From *The Invented Universe,* by Pierre Kerszberg

A Brief History of the Cosmological Constant

- In 1917, Einstein applied his new GR to the universe, and discovered that a static universe would collapse.
- Convinced that the universe was static, Einstein introduced the *cosmological constant* Λ into his field equations — the equations that describe how matter affects the metric — to create a gravitational repulsion to oppose the collapse.
- From a modern point of view, Λ represents a *vacuum energy density* $u_{\rm vac}$, with

$$u_{\rm vac} = \rho_{\rm vac} c^2 = \frac{\Lambda c^4}{8\pi G} \, ,$$

because $u_{\rm vac}$ appears in the field equations exactly as a vacuum energy density would. To Einstein, however, it was simply a new term in the field equations. Before quantum theory, the vacuum was viewed as completely empty, so it was inconceivable that it could have a nonzero energy density.

- Once the expansion of the universe was discovered by Hubble in 1929, Einstein abandoned Λ as being no longer needed or wanted.
- In 1998, however, two (large) groups of astronomers, both using measurements of Type Ia supernovae at redshifts $z \lesssim 1$, discovered evidence that the expansion of the universe is currently accelerating. At the time, it was shocking! *Science* magazine proclaimed it (correctly!) as the "Breakthough of the Year".
- In 2011 the Nobel Prize in Physics was awarded to Saul Permutter, Brian Schmidt, and Adam Riess for this discovery. In 2015 the Breakthrough Prize in Fundamental Physics was awarded to these three, and also the two entire teams.