8.286 Class 22
November 23, 2022

PROBLEMS OF THE CONVENTIONAL (NON-INFLATIONARY) HOT BIG BANG MODEL

Review from the previous lecture

Particle Physics of a Cosmological Constant

- $\boxed{u_{\rm vac} = \rho_{\rm vac} c^2 = \frac{\Lambda c^4}{8\pi G}}$
- Contributions to vacuum energy density:
  1) Quantum fluctuations of the photon and other bosonic fields: positive and divergent.
  2) Quantum fluctuations of the electron and other fermionic fields: negative and divergent.
  3) Fields with nonzero values in the vacuum, like the Higgs field.

Review from the previous lecture

- If infinities are cut off at the Planck scale (quantum gravity scale), then infinities become finite, but

  $>$ 120 orders of magnitude too large!

- For lack of a better explanation, many cosmologists (including Steve Weinberg and yours truly) seriously discuss the possibility that the vacuum energy density is determined by "anthropic" selection effects: that is, maybe there are many types of vacuum (as predicted by string theory), with different vacuum energy densities, with most vacuum energy densities roughly 120 orders of magnitude larger than ours. Maybe we live in a very low energy density vacuum because that is where almost all living beings reside. A large vacuum energy density would cause the universe to rapidly fly apart (if positive) or implode (if negative), so life could not form.

Anthropic Selection Effects and the String Theory Landscape

- Since the inception of string theory, theorists have sought to find the vacuum of string theory — with no success.
- Since about 2000, most string theorists have come to believe that there is no unique vacuum.
- Instead, there are perhaps 10^{500} or more long-lived metastable states, any one of which could serve as a substrate for a pocket universe. This is the landscape!
- Eternal inflation, which we will talk about later, can lead to an infinite number of "pocket universes," of which one would be the universe in which we live. The pocket universes are filled with different types of vacuum, very likely providing examples of every type of vacuum in the string theory landscape.
- Although string theory would govern everywhere, each type of vacuum would have its own low-energy physics — its own "standard model," its own "constants" of nature, and its own vacuum energy density.

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vacua with low energy densities like ours.

- But how could we explain why we are living in such a fantastically unusual type of vacuum?
- Possible answer: maybe it is a selection effect. I.e., maybe life only forms where the vacuum energy density is unusually small.

- As early as 1987, Steve Weinberg pointed out that the vacuum energy density might be explained as a selection effect.
- Maybe the vacuum energy density ***IS*** huge in most pocket universes. Nonetheless, we need to remember that vacuum energy causes the expansion of the universe to accelerate. If large and negative, the universe quickly implodes. If large and positive, the universe flies apart before galaxies can form. It is plausible, therefore, that life can arise only if the vacuum energy density is very near zero.
- In 1998 Martel, Shapiro, and Weinberg made a serious calculation of the effect of the vacuum energy density on galaxy formation. They found that to within a factor of order 5, they could "explain" why the vacuum energy density is as small as what we measure.

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- But I would advocate that anthropic explanations be thought of as the explanation of last resort — the best evidence for an anthropic explanation is the absence of any other.

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- Possible answer: maybe the universe just started out uniform.
  - There is no argument that excludes this possibility, since we don't know how the universe came into being.
  - However, if possible, it seems better to explain the properties of the universe in terms of things that we can understand, rather than to attribute them to things that we don't understand.

The Horizon in Cosmology

- The concept of a horizon was first introduced into cosmology by Wolfgang Rindler in 1956.
- The "horizon problem" was discussed (not by that name) in at least two early textbooks in general relativity and cosmology: Weinberg's *Gravitation and Cosmology* (1972), and Misner, Thorne, and Wheeler's (MTW's) *Gravitation* (1973).

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- Could this be simply the phenomenon of thermal equilibrium? If you put an ice cube on the sidewalk on a hot summer day, it melts and comes to the same temperature as the sidewalk.
- **BUT: in the conventional model of the universe, it did not have enough time for thermal equilibrium to explain the uniformity, if we assume that it did not start out uniform. If no matter, energy, or information can travel faster than light, then it is simply not possible.**

Basic History of the CMB

- In conventional cosmological model, the universe at the earliest times was radiation-dominated. It started to be matter-dominated at $t_{\rm eq} \approx 50{,}000$ years, the time of matter-radiation equality.
- At the time of *decoupling* $t_d \approx 380{,}000$ years, the universe cooled to about 3000 K, by which time the hydrogen (and some helium) combined so thoroughly that free electrons were very rare. At earlier times, the universe was in a mainly plasma phase, with many free electrons, and photons were essentially frozen with the matter. At later times, the universe was transparent, so photons have traveled on straight lines. We can say that the CMB was released at about 380,000 years.
- Since the photons have been mainly traveling on straight lines since $t = t_d$, they have all traveled the same distance. Therefore the locations from which they were released form a sphere centered on us. This sphere is called the *surface of last scattering*, since the photons that we receive now in the CMB was mostly scattered for the last time on or very near this surface.

- As we learned in Lecture Notes 4, the horizon distance is defined as the present distance of the furthest particles from which light has had time to reach us, since the beginning of the universe.
- For a matter-dominated flat universe, the horizon distance at time t is $3ct$, while for a radiation-dominated universe, it is $2ct$.
- At $t = t_d$ the universe was well into the matter-dominated phase, so we can approximate the horizon distance as

$$\boxed{\ell_h(t_d) \approx 3ct_d \approx 1,100,000 \text{ light-years.}}$$

For comparison, we would like to calculate the radius of the surface of last scattering at time t_d, since this region is the origin of the photons that we are now receiving in the CMB. I will denote the physical radius of the surface of last scattering, at time t, as $\ell_p(t)$.

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- To calculate $\ell_p(t_d)$, I will make the crude approximation that the universe has been matter-dominated at all times. (We will find that this *horizon problem* is very severe, so even if our calculation is wrong by a factor of 2, it won't matter.)
- Strategy: find $\ell_p(t_0)$, and scale to find $\ell_p(t_d)$. Under the assumption of a flat matter-dominated universe, we learned that the physical distance today to an object at redshift z is

$$\ell_p(t_0) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] .$$

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- The redshift of the surface of last scattering is about

$$1 + z = \frac{a(t_0)}{a(t_d)} = \frac{3000 \text{ K}}{2.7 \text{ K}} \approx 1100 .$$

- If we take $H_0 = 67.7$ km-s^{-1}-Mpc^{-1}, one finds that $H_0^{-1} \approx 14.4 \times 10^9$ yr and $\ell_p(t_0) \approx 28.0 \times 10^9$ light-yr. (Note that $\ell_p(t_0)$ is equal to 0.970 times the current horizon distance — very close.)
- To find $\ell_p(t_d)$, just use the fact that the redshift is related to the scale factor:

$$\boxed{\begin{aligned}\ell_p(t_d) &= \frac{a(t_d)}{a(t_0)}\ell_p(t_0) \\ &\approx \frac{1}{1100} \times 28.0 \times 10^9 \text{ lt-yr} \approx 2.55 \times 10^7 \text{ lt-yr} \ .\end{aligned}}$$

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- Comparison: At the time of decoupling, the ratio of the radius of the surface of last scattering to the horizon distance was

$$\boxed{\frac{\ell_p(t_d)}{\ell_h(t_d)} \approx \frac{2.55 \times 10^7 \text{ lt-yr}}{1.1 \times 10^6 \text{ lt-yr}} \approx 23 .}$$

Summary of the Horizon Problem

Suppose that one detects the cosmic microwave background in a certain direction in the sky, and suppose that one also detects the radiation from precisely the opposite direction. At the time of emission, the sources of these two signals were separated from each other by about 46 horizon distances. Thus it is absolutely impossible, within the context of this model, for these two sources to have come into thermal equilibrium by any physical process.

Although our calculation ignored the dark energy phase, we have found in previous examples that such calculations are wrong by some tens of a percent. (For example we found $t_{\rm eq} \approx 75,000$ years, when it should have been about 50,000 years.) Since $46 \gg 1$, there is no way that a more accurate calculation could cause this problem to go away.

The Flatness Problem

- A second problem of the conventional cosmological model is the *flatness problem*: why was the value of Ω in the early universe so extraordinarily close to 1?
- Today we know, according to the Planck satellite team analysis (2018), that

$$\Omega_0 = 0.9993 \pm 0.0037$$

at 95% confidence. I.e., $\Omega = 1$ to better than 1/2 of 1%.

- As we will see, this implies that Ω in the early universe was extaordinarily close to 1. For example, at $t = 1$ second,

$$|\Omega - 1|_{t=1\text{ sec}} < 10^{-18} .$$

- The underlying fact is that the value $\Omega = 1$ is a point of unstable equilibrium, something like a pencil balancing on its point. If Ω is ever ***exactly*** equal to one, it will remain equal to one forever — that is, a flat ($k = 0$) universe remains flat. However, if Ω is ever slightly larger than one, it will rapidly grow toward infinity; if Ω is ever slightly smaller than one, it will rapidly fall toward zero. For Ω to be anywhere near 1 today, Ω in the early universe must have been extraordinarily close to one.
- Like the horizon problem, the flatness problem could in principle be solved by the initial conditions of the universe: maybe the universe began with $\Omega \equiv 1$.
  - But, like the horizon problem, it seems better to explain the properties of the universe, if we can, in terms of things that we can understand, rather than to attribute them to things that we don't understand.

History of the Flatness Problem

The mathematics behind the flatness problem was undoubtedly known to almost anyone who has worked on the big bang theory from the 1920's onward, but apparently the first people to consider it a problem in the sense described here were Robert Dicke and P.J.E. Peebles, who published a discussion in 1979.*

*R.H. Dicke and P.J.E. Peebles, "The big bang cosmology — enigmas and nostrums," in **General Relativity: An Einstein Centenary Survey**, eds: S.W. Hawking and W. Israel, Cambridge University Press (1979).

The Mathematics of the Flatness Problem

Start with the first-order Friedmann equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2} .$$

Remembering that $\Omega = \rho/\rho_c$ and that $\rho_c = 3H^2/(8\pi G)$, one can divide both sides of the equation by H^2 to find

$$1 = \frac{\rho}{\rho_c} - \frac{kc^2}{a^2H^2} \quad \Longrightarrow \quad \boxed{\Omega - 1 = \frac{kc^2}{a^2H^2} .}$$

Evolution of $\Omega - 1$ During the Radiation-Dominated Phase

$$\boxed{\Omega - 1 = \frac{kc^2}{a^2H^2} .}$$

For a (nearly) flat radiation-dominated universe, $a(t) \propto t^{1/2}$, so $H = \dot{a}/a = 1/(2t)$. So

$$\boxed{\Omega - 1 \propto \left(\frac{1}{t^{1/2}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t \quad \text{(radiation dominated).}}$$

Evolution of $\Omega - 1$ During the Matter-Dominated Phase

$$\boxed{\Omega - 1 = \frac{kc^2}{a^2H^2} .}$$

For a (nearly) flat matter-dominated universe, $a(t) \propto t^{2/3}$, so $H = \dot{a}/a = 2/(3t)$. So

$$\boxed{\Omega - 1 \propto \left(\frac{1}{t^{2/3}}\right)^2 \left(\frac{1}{t^{-1}}\right)^2 \propto t^{2/3} \quad \text{(matter-dominated).}}$$

Tracing $\Omega - 1$ from Now to 1 Second

Today,

$$\boxed{|\Omega_0 - 1| < .01 .}$$

I will do a crude calculation, treating the universe as matter dominated from 50,000 years to the present, and as radiation-dominated from 1 second to 50,000 years.

During the matter-dominated phase,

$$\boxed{(\Omega - 1)_{t=50{,}000\text{ yr}} \approx \left(\frac{50{,}000}{13.8 \times 10^9}\right)^{2/3} (\Omega_0 - 1) \approx 2.36 \times 10^{-4}\,(\Omega_0 - 1) .}$$

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During the radiation-dominated phase,

$$\boxed{\begin{aligned}(\Omega - 1)_{t=1\ \mathrm{sec}} &\approx \left(\frac{1\ \mathrm{sec}}{50{,}000\ \mathrm{yr}}\right) (\Omega - 1)_{t=50{,}000\ \mathrm{yr}} \\ &\approx\ 1.49 \times 10^{-16}\,(\Omega_0 - 1)\ .\end{aligned}}$$

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Even if we put ourselves mentally back into 1979, we would have said that $0.1 < \Omega_0 < 2$, so $|\Omega_0 - 1| < 1$, and would have concluded that

$$\boxed{|\Omega - 1|_{t=1\ \mathrm{sec}} < 10^{-16}\ .}$$

The Dicke & Peebles paper, that first pointed out this problem, also considered $t = 1$ second, but concluded (without showing the details) that

$$\boxed{|\Omega - 1|_{t=1\ \mathrm{sec}} < 10^{-14}\ .}$$

They were perhaps more conservative, but concluded nonetheless that this extreme fine-tuning cried out for an explanation.