8.286 Class 27
December 12, 2022

COSMIC INFLATION
PART 2

The Inflationary Universe Scenario

- Inflationary cosmology attempts to describe the behavior of the universe at ridiculously early times — perhaps as early as 10^{-37} seconds.
- Surprisingly, it can still make predictions that can be tested today.
- Inflation can provide a solution to the horizon problem, the flatness problem, and the magnetic monopole problem.
- If correct, inflation can even explain the origin of essentially all the matter in the universe. (One has to start with a bit of matter: a few grams!)

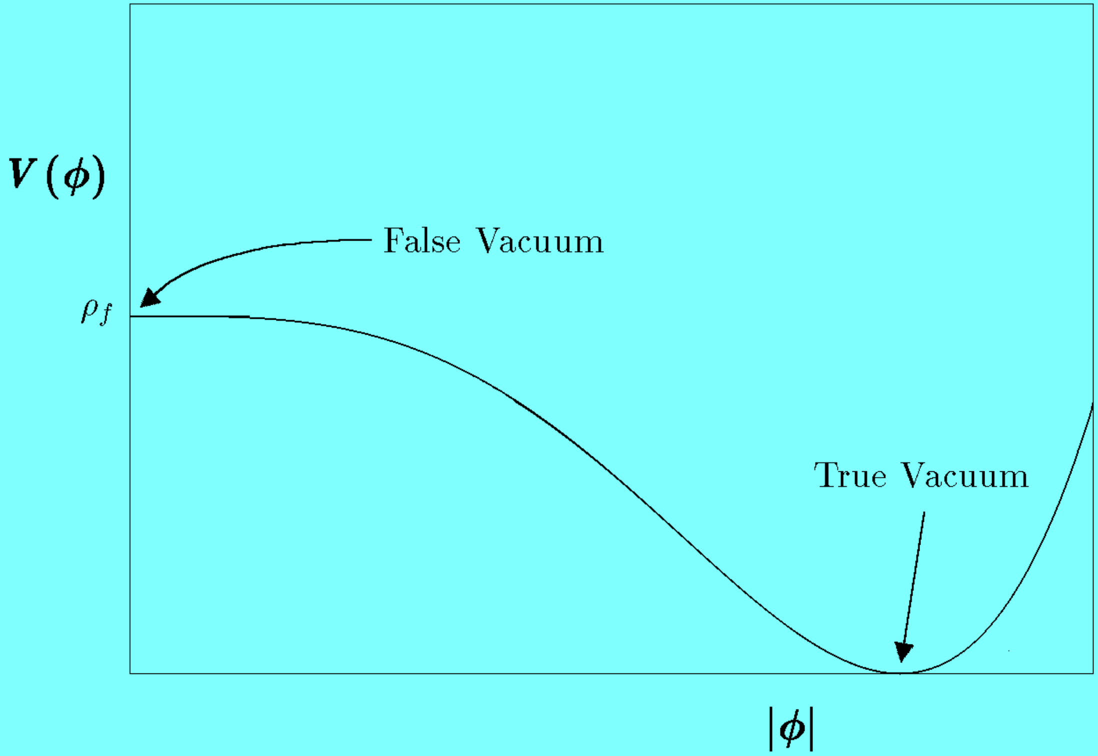
The inflationary scenario assumes the existence of a scalar field ϕ that resembles the Higgs field of the standard model. It is usually assumed to be some beyond-the-standard-model field, very likely at the GUT scale.

Whatever the scalar field that drives inflation is, it is called the "inflaton".

Inflation is not really a theory, but rather a class of theories, since there are many options for how the inflaton field might behave.

It is conceivable that the inflaton might be the Higgs field of the standard model, but that can work only if the Higgs field interacts with gravity in a particular way, which can be tested only at energies well beyond what we have access to.

The easiest version of inflation to explain is called "hilltop" inflation, or "new" inflation. It assumes an inflaton potential energy density resembling that of the standard model Higgs field:



More general potential energy functions are possible, as we will discuss in a few minutes.

Start of Inflation (Summary)

There is no accepted (or even persuasive) theory of the origin of the universe, so the starting point is uncertain. Inflation starts when the scalar field is at the top of the hill, no matter how it got there.

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The good news is that the predictions of inflation do not depend on how it started. This is also bad news, since it means that it is very hard to learn anything about how it started.

The Inflationary Era

Once the inflaton is at the top of the hill, the mass/energy density is fixed, leading to a large negative pressure and gravitational repulsion:

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \;\; ; \;\; \dot{\rho} = 0 \quad \Longrightarrow \quad p = -\rho c^2 \; .$$

Assuming approximate Friedmann-Robertson-Walker evolution,

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right) = \frac{8\pi}{3}G\rho_f ,$$

where ρ_f = mass density of the false vacuum. Thus, ρ_f produces gravitational repulsion.

The de Sitter Solution

The homogeneous isotropic solution can be described as a Robertson-Walker flat universe:

$$\boxed{\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2\ ,}$$

where

$$\boxed{a(t) \propto e^{\chi t}\ ,\ \ \chi = \sqrt{\frac{8\pi}{3} G\rho_{\mathrm{f}}}\ .}$$

This is called de Sitter spacetime.

By a change of coordinates, de Sitter spacetime can, surprisingly, be described as an open universe, a closed universe, or a static universe!

Cosmological "No-Hair" Conjecture

Conjecture: For "reasonable" initial conditions, even if far from homogeneous and isotropic, $\rho = \rho_f$ implies that the region will approach de Sitter space.

Conjectured by Hawking & Moss (1982). Can be proven for linearized perturbations about de Sitter spacetime (Jensen & Stein-Schabes, 1986, 1987). Was shown by Wald (1983) to hold for a class of very large (but spatially homogeneous) perturbations.

Analogous to the Black Hole No-Hair Theorem, which implies that gravitationally collapsing matter approaches a stationary black hole state that depends only on the mass, angular momentum, and charge.

Qualitative behavior: any distortion of the metric is stretched by the expansion to look smooth and flat. Any initial matter distribution is diluted away by the expansion.

De Sitter Event Horizon

In the de Sitter metric, with $a(t) = be^{\chi t}$, the coordinate distance that light can travel between times t_1 and t_2 is

$$\Delta r(t_1, t_2) = \int_{t_1}^{t_2} \frac{c}{a(t)}\,\mathrm{d}t = \frac{c}{b}\int_{t_1}^{t_2} e^{-\chi t}\,\mathrm{d}t = \frac{c}{b\,\chi}\left[e^{-\chi t_1} - e^{-\chi t_2}\right] \;,$$

which is bounded as $t_2 \to \infty$. If we multiply by $a(t_1)$ and take the limit,

$$\lim_{t_2 \to \infty} a(t_1)\,\Delta r(t_1, t_2) = c\chi^{-1} \;,$$

which means that if two objects have a physical separation larger than $c\chi^{-1}$, the Hubble length, at any time t_1, light from the first will never reach the second. This is called an event horizon. Event horizons protect an inflating patch from the rest of the universe: once the patch is large compared to $c\chi^{-1}$, nothing from outside can penetrate further than $c\chi^{-1}$.

Event Horizon in the Universe Today

Our universe today is entering a de Sitter phase, in which the dark energy dominates.

In the Review Problems for Quiz 3, Problem 17, the present event horizon was calculated, finding $z = 1.87$.

That means that events that are happening now (i.e., at the same value of the cosmic time), at distances for which the redshift is larger than 1.87, will NEVER be visible to us or our descendents.

The Ending of Inflation

A standard scalar field in a flat FRW universe obeys the equation of motion:

$$\boxed{\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla_i^2\phi = -\frac{\partial V}{\partial \phi}\ ,}$$

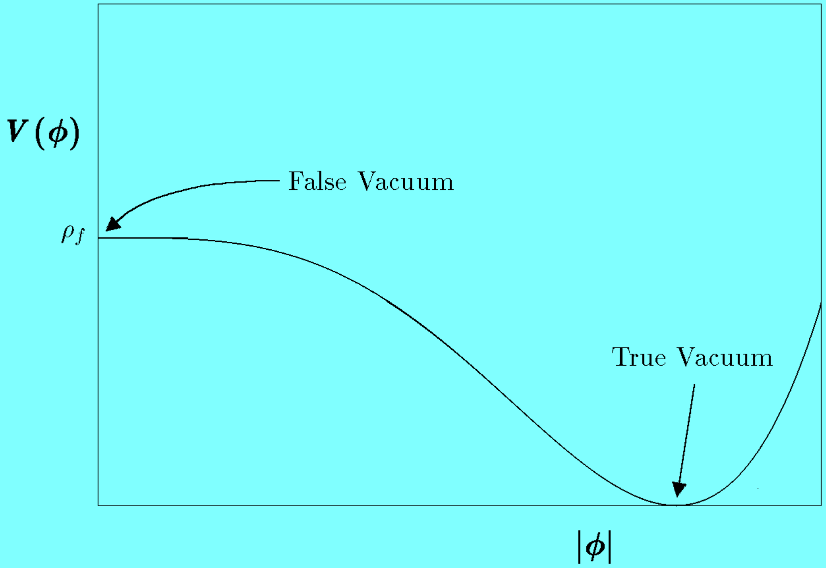
where ∇_i^2 is the Laplacian operator in comoving coordinates x^i, and $V(\phi)$ is the potential energy function (i.e., the potential energy per volume).

The spatial derivative piece soon becomes negligible, due to the $(1/a^2)$ suppression, which reflects the fact that the stretching of space causes ϕ to become nearly uniform over huge regions. The equation is then identical to that of a ball sliding on a hill described by $V(\phi)$, but with a viscous damping (i.e., friction) described by the term $3(\dot{a}/a)\dot{\phi}$.

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \ .$$

Fluctuations in ϕ due to thermal and/or quantum effects will cause the field to start to slide down the hill. This will not happen globally, but in regions, typically of size $c\chi^{-1}$.

Within a region, ϕ will start to oscillate about the true vacuum value, at the bottom of the hill. Interactions with other fields will allow ϕ to give its energy to the other fields, producing a "hot soup" of other particles, which is exactly the starting point of the conventional hot big bang theory. This is called *reheating*.



The standard hot big bang scenario begins. Inflation has played the role of a prequel, setting the initial conditions for conventional cosmology.

Numerical Estimates

The energy scale at which inflation happened is not known. One plausible guess is the GUT scale, $E_{\rm GUT} \approx 10^{16}$ GeV. It cannot be higher (too much gravitational radiation), but can be as low as about 10^3 GeV.

For $E_{\rm GUT}$, we can estimate

$$\boxed{\rho_f \approx \frac{E_{\rm GUT}^4}{\hbar^3 c^5} = 2.3 \times 10^{81} {\rm g/cm}^3 \ .}$$

Then

$$\boxed{\chi^{-1} \approx 2.8 \times 10^{-38} \text{ s} \ , \quad c\chi^{-1} = 8.3 \times 10^{-28} \text{ cm} \ ,}$$

and the mass of a minimal region of inflation would be about

$$\boxed{M \approx \frac{4\pi}{3} (c\chi^{-1})^3 \rho_f \approx 5.6 \text{ gram.}}$$

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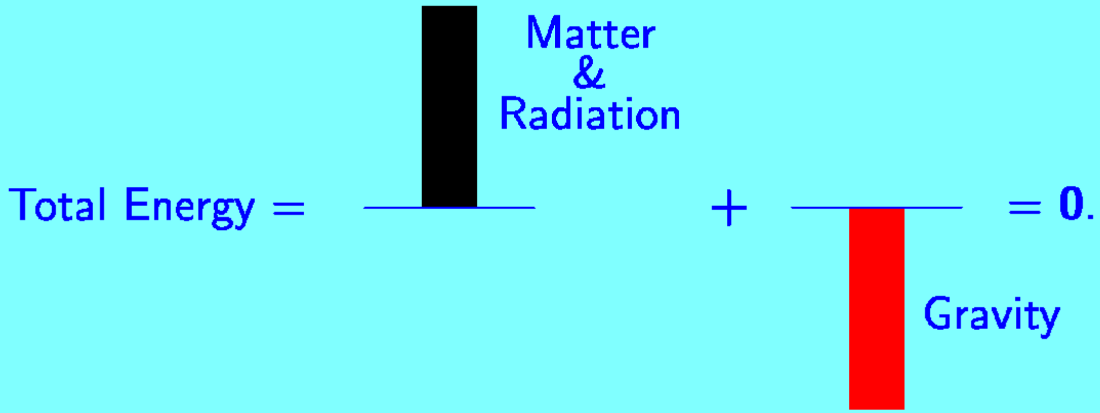

Alan Guth
Massachusetts Institute of Technology

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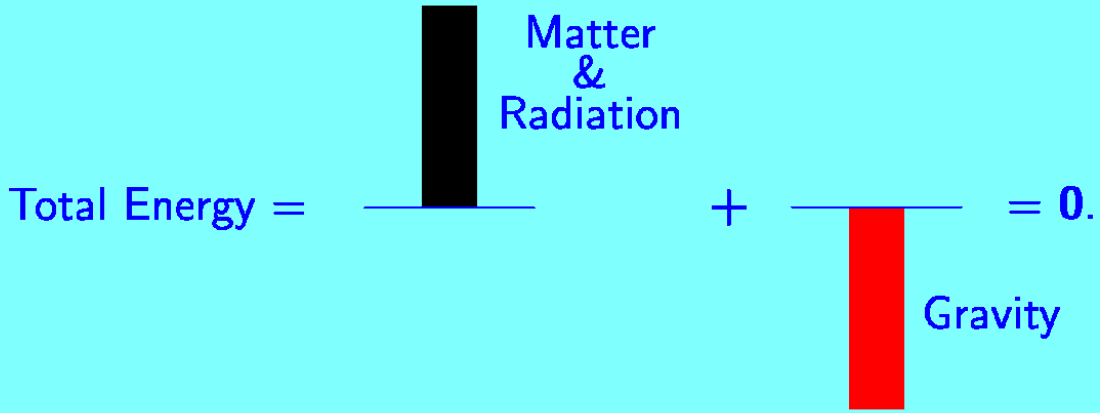

☆ Warning: the concept of total energy in GR is controversial. Some authors would just say that total energy is not defined.

Solutions to the Cosmological Problems

1) **Horizon Problem:** In inflationary models, uniformity is achieved in a tiny region BEFORE inflation starts. Without inflation, such regions would be far too small to matter. But inflation can stretch a tiny region of uniformity to become large enough to include the entire visible universe and more. For inflation at the GUT scale, 10^{16} GeV, we need expansion by about 10^{28}, which is about 65 time constants of the exponential expansion.