**8.286 Class 28 (The Last!)**
**December 14, 2022**

# COSMIC INFLATION PART 3

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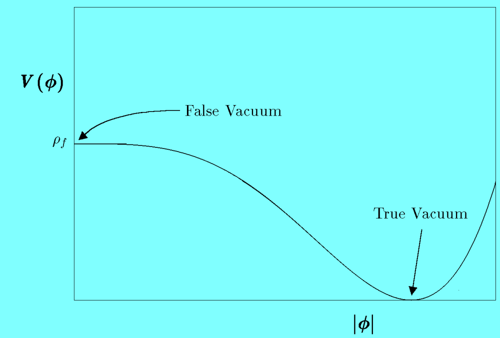
*Review from the previous lecture*

The easiest version of inflation to explain is called "hilltop" inflation, or "new" inflation. It assumes an inflaton potential energy density resembling that of the standard model Higgs field:




More general potential energy functions are possible, as we will discuss in a few minutes.
— well, on the next slide.



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*Not from the last lecture, but it could have been.*

## Chaotic Inflation

In 1983 Andrei Linde pointed out that inflation can occur in a much more general class of potentials, even one as simple as

$$V(\phi) = \frac{1}{2} m \phi^2 \ .$$

Linde assumed that before inflation the inflaton field $\phi$ was "chaotic," taking on very different values in different parts of space, so that there will be places where before inflation $\phi$ was very large.

Linde showed that if the initial value of $\phi$ was sufficiently large, there would be enough inflation as $\phi$ rolled down the potential energy hill to accomplish successful inflation.



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*Review from the previous lecture*

## Summary of Inflationary Scenario

- Initial conditions: We can only speculate about the state of the universe before inflation. But as long as, somehow, the inflaton field in some regions of space was in the range of values that can start inflation, then inflation will happen. Once inflation starts, the inflating region rapidly comes to dominate the volume of the universe.
- Evolution: if $\rho$ is fixed at $\rho_f$, then $p = -\rho c^2$, and

$$\frac{\ddot{a}}{a} = \frac{8\pi}{3} G \rho_f \ .$$

*Review from the previous lecture*

☆ Evolution: if $\rho$ is fixed at $\rho_f$, then $p = -\rho c^2$, and

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☆ A simple, homogeneous and isotropic solution is the de Sitter solution:

$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2 \ ,$$

where

$$a(t) \propto e^{\chi t} \ , \ \chi = \sqrt{\frac{8\pi}{3} G\rho_f} \ .$$



*Review from the previous lecture*

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☆ Cosmological "No-Hair" Conjecture: For "reasonable" initial conditions, even if far from homogeneous and isotropic, $\rho = \rho_f$ implies that the region will approach de Sitter space.

☆ De Sitter Event Horizon: In de Sitter space, if two objects have a physical separation larger than $c\chi^{-1}$, the Hubble length, at any time $t_1$, light from the first will never reach the second. This is called an *event horizon*.



*Review from the previous lecture*

## The Ending of Inflation

A standard scalar field in a flat FRW universe obeys the equation of motion:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2}\nabla_i^2 \phi = -\frac{\partial V}{\partial \phi} \ ,$$

where $\nabla_i^2$ is the Laplacian operator in comoving coordinates $x^i$, and $V(\phi)$ is the potential energy function (i.e., the potential energy per volume).

The spatial derivative piece soon becomes negligible, due to the $(1/a^2)$ suppression, which reflects the fact that the stretching of space causes $\phi$ to become nearly uniform over huge regions. The equation is then identical to that of a ball sliding on a hill described by $V(\phi)$, but with a viscous damping (i.e., friction) described by the term $3(\dot{a}/a)\dot{\phi}$.

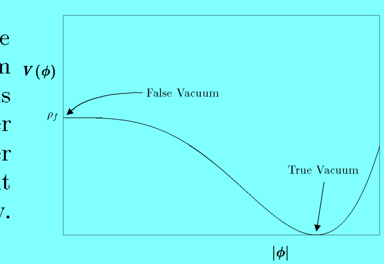


*Review from the previous lecture*

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -\frac{\partial V}{\partial \phi} \ .$$

Fluctuations in $\phi$ due to thermal and/or quantum effects will cause the field to start to slide down the hill. This will not happen globally, but in regions, typically of size $c\chi^{-1}$.

Within a region, $\phi$ will start to oscillate about the true vacuum value, at the bottom of the hill. Interactions with other fields will allow $\phi$ to give its energy to the other fields, producing a "hot soup" of other particles, which is exactly the starting point of the conventional hot big bang theory. This is called *reheating*.




The standard hot big bang scenario begins. Inflation has played the role of a prequel, setting the initial conditions for conventional cosmology.

Review from the previous lecture

# Solutions to the Cosmological Problems

1) **Horizon Problem:** In inflationary models, uniformity is achieved in a tiny region BEFORE inflation starts. Without inflation, such regions would be far too small to matter. But inflation can stretch a tiny region of uniformity to become large enough to include the entire visible universe and more. For inflation at the GUT scale, $10^{16}$ GeV, we need expansion by about $10^{28}$, which is about 65 time constants of the exponential expansion.



2) **Flatness Problem:** Just look at Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}\ .$$

"Flatness" is the statement that the final term in this equation is negligible. But during inflation, $\rho \approx \rho_v =$ const, while $a(t)$ grows exponentially. If $a(t)$ grows by at least $10^{28}$ during inflation, the final term is suppressed by a factor of $\left(10^{28}\right)^2 = 10^{56}$.



3) **Monopole Problem:** Solved by dilution, as long as the inflation occurs during or after the process of monopole production. For inflation at the GUT scale, the volume of any comoving region increases during inflation by a factor of about $\left(10^{28}\right)^3 = 10^{84}$ or more! That is plenty enough to make monopoles impossible to find.

Some small number of monopoles could be produced during reheating, so it makes sense to look for them. But, except for an irreproducible single event seen by Blas Cabrera at Stanford in 1982, magnetic monopoles have not been seen.



# Ripples in the Cosmic Microwave Background

The CMB is uniform in all directions to an accuracy of a few parts in 100,000. Nonetheless, at the level of a few parts in 100,000 there ARE anisotropies, and they have now been measured to high precision. Since the CMB is essentially a snapshot of the universe at $t \approx 380{,}000$ yr, these ripples are interpreted as perturbations in the cosmic mass density at this time.

In the early days of inflation, such density perturbations were a cause for worry. (The ripples had not yet been seen, but cosmologists knew that the early universe must have had density perturbations, or else galaxies and stars could never have formed.) Inflation smooths out the universe so effectively, that it looked like no density perturbations could survive.

## Quantum Mechanics to the Rescue (Again)

Why again? We spoke earlier about how quantum mechanics was necessary to save us from freezing to death. If classical mechanics ruled, all thermal energy would gradually disappear into shorter and shorter wavelength electromagnetic radiation.

If inflation happened with classical physics, it would smooth the universe so perfectly that stars and galaxies could never form.

But quantum mechanics is intrinsically probabilistic. While the classical version of inflation predicts an almost exactly uniform mass density, the intrinsic randomness of the quantum version implies that the mass density will be a little higher in some places, and a little lower in others.



In 1965, Andrei Sakharov, the Russian nuclear physicist and political activist, proposed in a rather wildly speculative paper that quantum fluctuations might account for the structure of the universe.

In 1981, Mukhanov and Chibisov tried to calculate the density fluctuations in pre-inflationary/inflationary model invented by Alexei Starobinsky in 1980.

In summer 1982, Gary Gibbons and Stephen Hawking organized the Nuffield Workshop on the Very Early Universe in Cambridge UK, where a number of physicists worked feverishly and argued through the night about how to calculate these perturbations in inflation. In the end, all agreed. Four papers emerged: Hawking, Starobinsky, Guth & Pi, and Bardeen, Steinhardt, & Turner.

Basic conclusion: the amplitude of the density perturbations is very "model-dependent," meaning that it depends on the unknown details of $V(\phi)$. But: the spectrum — the way in which the intensity of the ripples depends on the wavelength of the ripples — is the same for a wide range of "simple" inflationary models. Simple = "Single field / slow-roll models," i.e. models with a single inflaton field, and with small values for $dV/d\phi$ and $d^2V/d\phi^2$.



## Observations of the Ripples in the CMB

In 1982, it seemed (at least to me) out of the question that these ripples would ever be seen.

There have now been 3 satellite experiments to measure the CMB, plus many many ground-based experiments. The three satellites were:

COBE: Cosmic Background Explorer, launched by NASA in 1989, after 15 years of planning. In 1992 it announced its first measurements of CMB anisotropies. The angular resolution was crude, about 7°, but the results agreed with inflation.

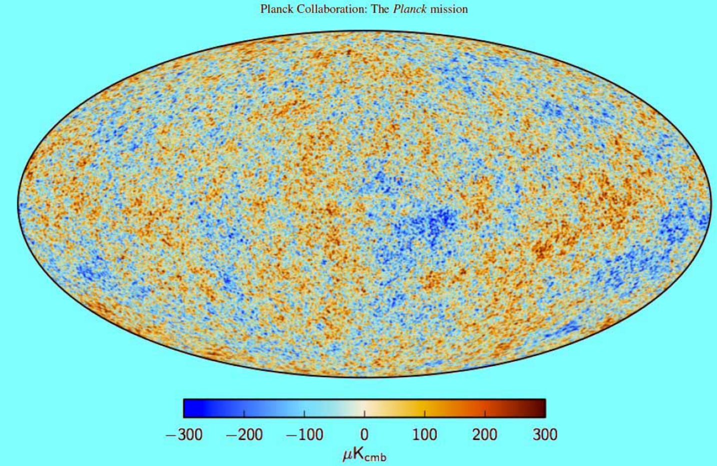
WMAP: The Wilkinson Microwave Anisotropy Probe, launched by NASA in 2001. 45 times more sensitive, with 33 times better angular resolution than COBE. Still consistent with inflation.

Planck: Launched in 2009 by ESA. Resolution about 2.5 times better than WMAP. Results still consistent with inflation.

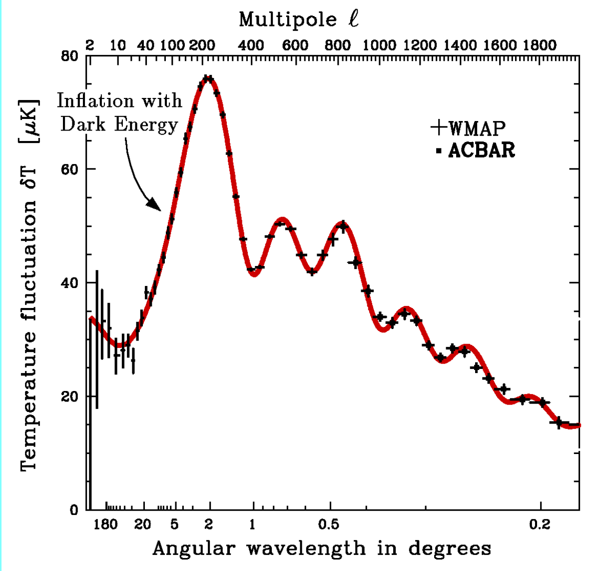


## Ripples in the Cosmic Microwave Background

Planck Collaboration: The *Planck* mission



# CMB: Comparison of Theory and Experiment

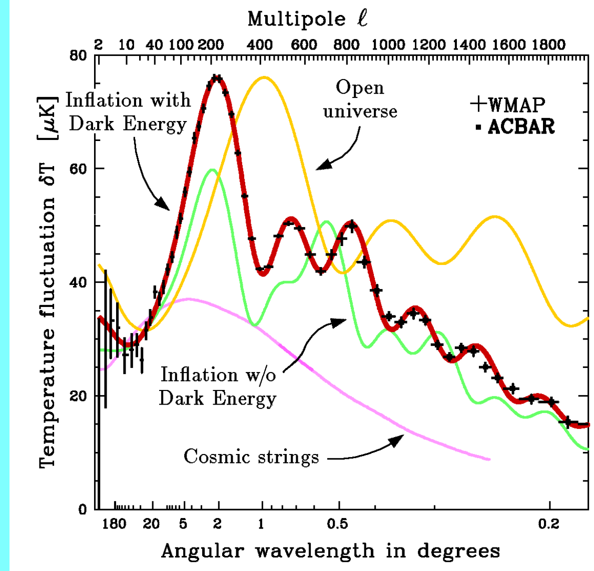




Graph by Max Tegmark, for A. Guth & D. Kaiser, *Science* **307, 884** (Feb 11, 2005), updated to include WMAP 7-year data (Jan 2010).



# CMB: Comparison of Theory and Experiment





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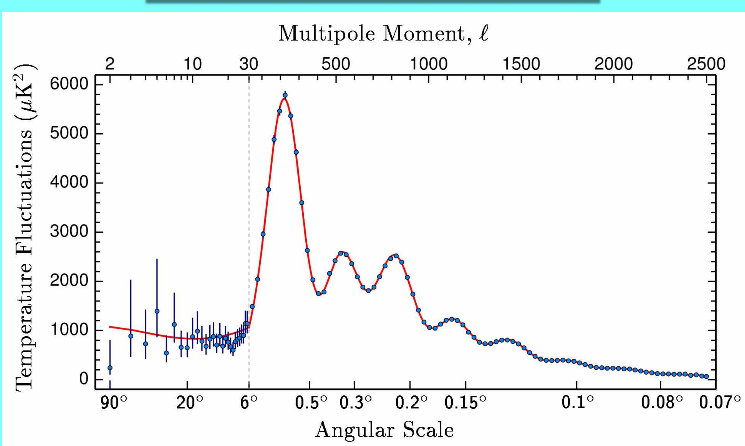
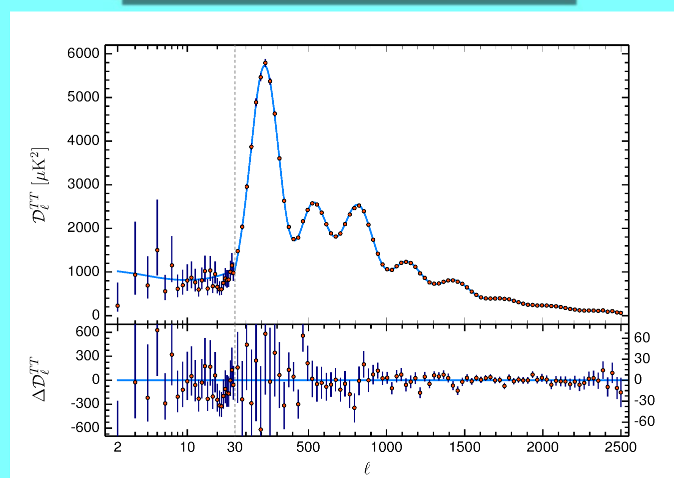
# Planck 2015 Spectrum




Planck Collaboration, 2015



# Planck 2018 Spectrum




Lower panel shows difference between data and model.

*This and the following slides were not reached in class.*

## Eternal Inflation

In hilltop inflation, while the scalar field rolls down the hill in the potential energy diagram, there is always some small quantum mechanical probability that the field remains at the top.

Approximate calculations show that the probability of remaining at the top falls off exponentially with time. That is, the false vacuum has an exponential decay law, like a radioactive substance.

In any successful model of inflation, the half-life of the false vacuum is much longer than the doubling time of the exponential expansion of $a(t)$.

So, in one half-life of the decay, half of the region in false vacuum stops inflating, but the region remaining in the false vacuum state becomes much larger than the original size of the full region! Thus, the volume of false vacuum region grows exponentially in time.



The ending of inflation happens in localized patches, where in each patch there is a local big bang, forming what we call a "pocket universe". The theory seems to lead to the production of pocket universes ad infinitum. The collection of pocket universes is called a "multiverse".

Is this relevant to physics?

Maybe. It offers a possible explanation of the very small vacuum energy density of our universe. If there is an infinite set of pocket universes, with each one filled with a different vacuum-like state (string theory, for example, gives a huge number of vacuum-like states), then there will be pocket universes with very small vacuum energies. Only those with small vacuum energies will develop life, since the others will implode of fly apart before life could form. All of this is speculative and controversial, however.



## Bubble Nucleation in an Eternally Inflating Universe