Phase transition in evolutionary prisoner's dilemma game.

Abstract

Prisoner's game is studied on different lattice types when players interacting with their neighbors can follow two strategies: to cooperate or to defect unconditionally. The players updated in random sequence have chance to adopt one of the neighboring strategies with a probability depending on the payoff difference. Using Monte Carlo simulations I study the frequency of cooperators in the stationary state. I calculate critical exponents in the system and show that they are the same for performed lattice types with error 10-15%. I also discuss possibility of analytical solutions to the problem.

Text

The evolution of cooperation is a fundamental problem in biology because unselfish behavior apparently contradicts Darwinian selection. To study it game theory usually considers two strategies – a player can either cooperate or defect. When two players meet each other they have to chose one of the strategies. If any player cooperates, it pays cost of cooperation – r, if its opponent cooperates, it gets 1+r benefit. If player defects, it pays 0 and if its opponent defects, it gets 0.

Thus, in case of mutual cooperation both players get 1, in case of mutual defection, both get 0. In case of opposite strategies, cheater gets 1+r, while cooperator gets -r. If the game is not iterated and/or players meet randomly, there is no reason why any player should cooperate[1]. However, if the same players meet each other in the iterated game and cost of cooperation is small enough, cooperation may be favorable[2-4].

Spatial organization of population corresponds to the condition “the same players meet each other”. When cooperators form clusters, it may be still beneficial to cooperate on the cooperator/defector border.

As reported in [4] there is a phase transition in the system when parameter r changes. There is a critical number r_c, bellow which there is some fraction of cooperators after long evolution, and above which no cooperators can exist.

There is a critical exponent connected to this transition – average fraction of cooperators vanishes as $(r_c-r)^b$.

Spatially structured populations are modeled by confining players to lattice sites. The performance P_x of a player at site x is determined by payoffs accumulated in its interactions with its neighbors.

Player at a site x can change his strategy with some probability. This probability is determined by relative payoff to the neighbor chosen at random. Update rule that I used in this project is the same as in [4]:

$$W(x,y)=[1+\exp(-(P_x-P_y)/k)]$$

where k denotes the amount of noise (it is set to 0.1 in all simulations presented here). This update rule states that the strategy of a better performing player is readily adopted, whereas it is unlikely (but not impossible) to adopt the strategies of worse performing players.

Exponents in update are important as they help to fill relative weight of cost and benefit. Simulations for update rule “change to the neighbor strategy of neighbor does better” fail to get phase transition as there is no difference for small r, where real action is found in the system.

I consider evolution of fraction of cooperators on different types of lattices – square, triangle and hexagonal. Lattices with N^2 sites are studied. As a cooperator surrounded by cheaters will unlikely survive, simulations usually start with large (80%) initial fraction of cooperators. Starting from a random initial configuration, the population is updated with probability W(x,y). The stationary state is characterized by the frequency of cooperators after large muber Monte Carlo Steps per Site (MCS).

Fig 1 presents a rough picture of phase transition(N=100, MCS=4000). Data from fig 1 is used to estimate r_c. Thus, for square lattice r_c=,0.021 for triangular r_c=0.16 and for hexagonal r_c=0.015.

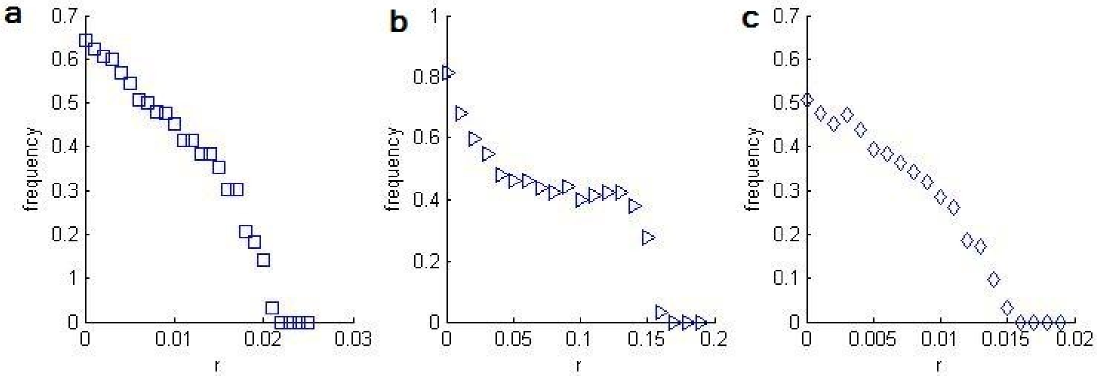

Fig.1 Phase transitions in different lattice types: a-square, b-triangular, c-hexagonal

To find critical exponents simulations with N=400, MCS=5000 are performed (averaged for 3 runs). As close to transition point fluctuations become larger, simulations far enough from r=0 but not so close to critical point are performed.

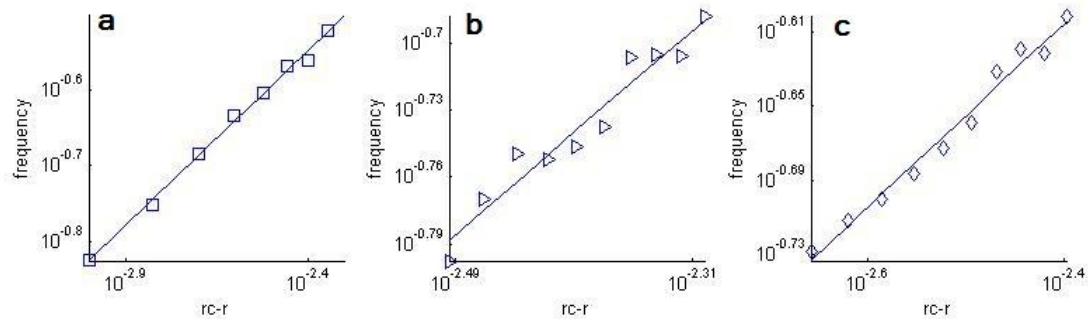

Fig.2 Lines for critical exponents verification in log-log scale. Line corresponds to power-low dependence with following exponents: a-square (b=0.46), b-triangular (b=0.51), c-hexagonal (b=0.50)

To demonstrate that numbers obtained for critical exponent verification are the result of stable situation, Fig 3 (left) shows that frequency is stable after 5000 MCS. Fig 3 (right) demonstrates corresponding clustering organization of cooperators in the system.

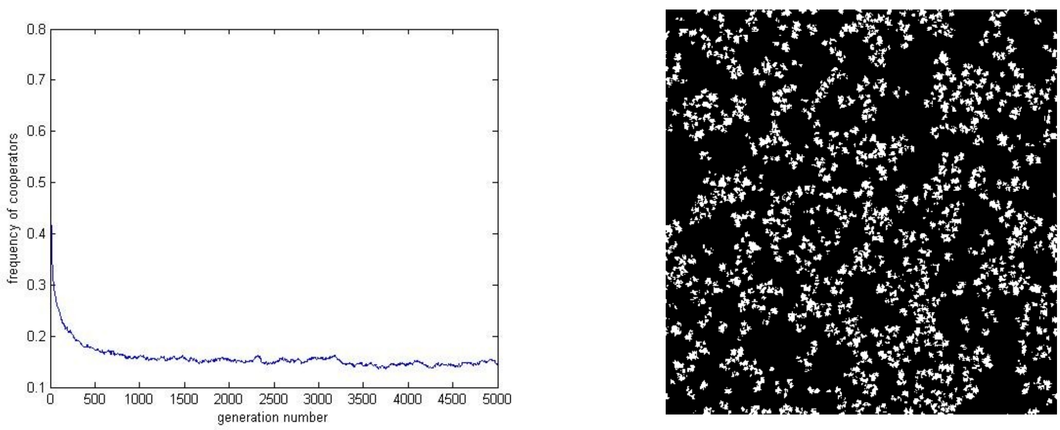

Fig 3 left – frequency of cooperators dependence on generation, right – clustering in the end (white-cooperators, black-cheaters). Square lattice, 400*400, 5000MCS, r=0.02

Analytical solution discussion.

One way to describe the system is to weight each link as $\exp(-|P_x-P_y|/k)$ thus simulating that nodes with very different payoffs are unlikely to exist next to each other. This corresponds to some kind of clock model. According to the above discussion the important role plays coexistence of different scales payoff differences. Thus, Potts model as one possible realization of clock model will not work for the system (as the transition does depend not on the parameter k, but on the parameter r, which regulates differences in payoffs).

Conclusions.

The evolution of cooperation among players who can follow two strategies and are placed on different types of lattices has been studied and critical exponents for phase transition obtained. All of them are a little smaller than percolation critical exponent (b=0.57) probably because were taken not so close to transition point and because transition point was not determined with high precision (the same 10%).

References.

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