Fracture Strength of Disordered Brittle Media

Sina Moeini*
Concrete Sustainability Hub, Department of Civil and Environmental Engineering,
Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, United State
(Dated: May 18, 2012)

Using random fuse model, fracture of disordered brittle material is studied. By aid of numerical simulations and renormalization group the validity of weakest link hypothesis and Duxbury-Leath-Beale (DLB) distribution for honeycomb lattice are investigated. It has been found that weakest link hypothesis is not valid for the considered system sizes. It has also been showed that DLB is valid.

Since early 1920's the fact that ultimate strength of brittle material is significantly lower than the theoretical one predicted from atomic bonds strength caught the attention of the scientists. Griffith [1] came up with the theory of fracture (rupture) mechanics to explain such phenomenon; criteria such as fracture toughness and critical energy release rate was introduced as a material property to estimate the ultimate strength of pre existing flaws inside (cracks) the media. It was shown experimentlly rupture stress has an inverse relationship with square root of crack length.

At the onset of fracture theory it was believed that the energy required to increase crack face by unit area, due to the exerted force on the body was equal to the surface energy. However, later it was discovered that this energy for ductile materials is much higher than surface energy. Later it was found out that in ductile media energy dissipation not only occurs by elongation of the crack, but also by generation of dislocations and micro cracks inside the bulk.

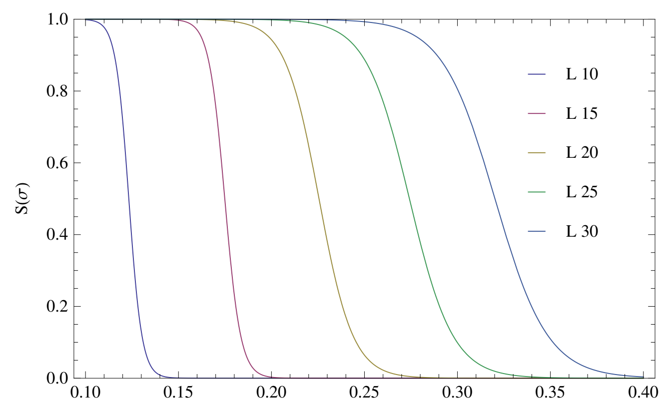

FIG. 1. Surviving probability versus stress for $1-p=0.01$

Weakest Link Hypothesis (WLH) is based on the notion of noninteracting sub-volumes. It states that if a body is subdivided to smaller regions, the total strength of the media is determined by weakest region. This theory is often used to explain why the bigger brittle bodies exhibit lower strength rather than smaller ones.

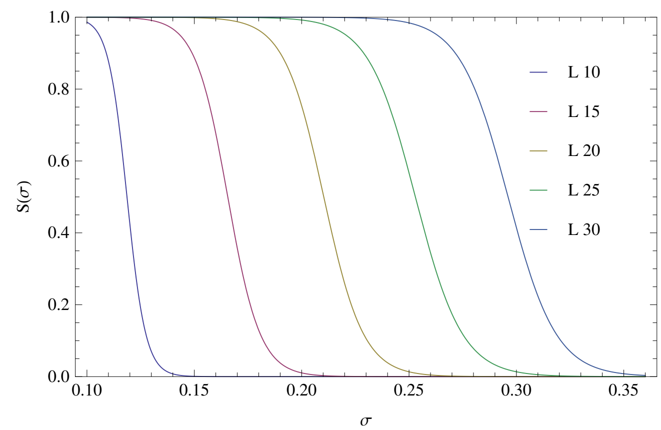

FIG. 2. Surviving probability versus stress for $1-p=0.02$

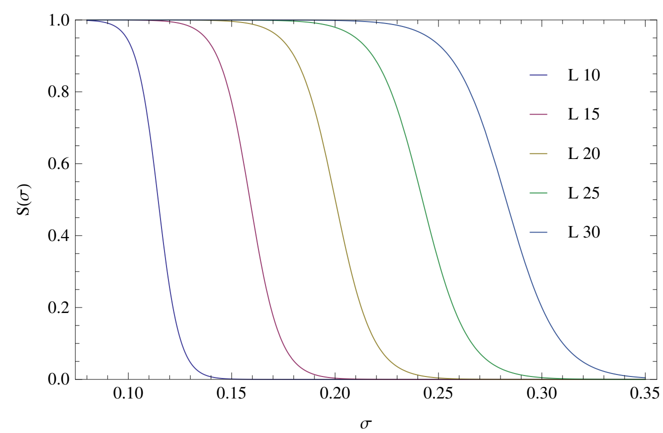

FIG. 3. Surviving probability versus stress for $1-p=0.03$

As it was mentioned before the failure often occurs by accumulation of flaws, which questions the non-interacting sub-volumes premise. Fracture network models have provided an insight to address such interactions. Random Fuse Model (RFM) is one of the simple models that have been proposed by Arcangelis et al. [2] for such a purpose; network of fuses are placed on a lattice as bonds, some of the fuses are removed randomly. An external voltage is place across the lattice to mimic the external loading on the body.

Recently, Manzato et al. [4] examined RFM on 2-d diamond lattice in order to study fracture of a disordered

* Corresponding author: sin am@mit.edu

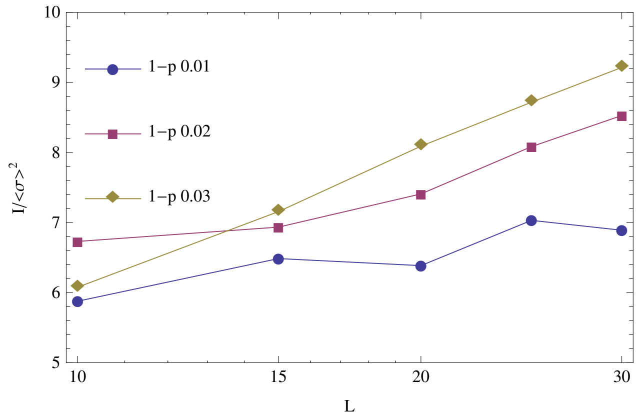

FIG. 4. Averaged failure stress as a function of system size

media , by combination of Renormalization Group (RG), extreme value theory and numerical simulations. Using numerical simulations they concluded WLH is valid for diamond lattice; they also showed the fracture strength can be described by Duxbury-Leath-Beal (DLB) distribution. In this letter we examine the results of Manzato et al. against honeycomb lattice. It would be concluded that: (i) WLH is not valid for honeycomb lattice, and (ii) fracture strength obeys DLB distribution.

The effect of interactions on the overall behavior of the system can be investigated using RG. In this context we use RG to relate system size to strength of the material. The RG procedure is followed as illustrated in ref. [4]. The coarse graining is equivalent to the weakest link hypothesis, in two dimensions:

$$S_L(\sigma) = [S_{L/2}(\sigma)]^2. \tag{1}$$

where L, σ and S denote the system size, the external stress and survival probability, respectively. The fixed point can be found throughout the following equation

$$S^*(\sigma) = [S^*(a\sigma + b)]^4. \tag{2}$$

The above equation is known to have three solutions: Gumbel, Weibull and Frechet distributions. Only Gumbel and Weibull distribution are known to be relevant to fracture.

At this point we will try to examine the weakest link hypothesis by numerical simulations. For this purpose we replace bonds by fuses on a $L \times L$ honeycomb lattice (which has a percolation threshold of $1 - \sin\pi/18$). $1 - p$ fraction of the fuses are removed, randomly. Periodic boundary condition is imposed on the horizontal direction. A constant difference voltage V is place between top and bottom of the lattice. Nodal matrix analysis is employed to determine the current in each fuse. If the current inside any of the fuses exceeds the prescribed threshold, the fuse with the maximum current is removed from the network, and the nodal analysis is repeated. We examined for WLH for 5 different system sizes ($L = 10$, 15, 20, 25 and 30) and three different bond probability of 0.01, 0.02 and 0.03. At each step the results are averaged over 10^4 trials. The analogy of strain and stress are V/L and I/L, respectivley.

The results are depicted in Fig. 1 it can be shown that the WLH is not valid for these system sizes and percolation probabilities. This might be because of small system sizes (due to limited computation power). Next we checked to see if DLB distribution is valid for this type of lattice. Following ref. [4] it can be shown that $\langle\sigma\rangle = \sigma_0/\sqrt{2\log L}$. It can be seen $I/\langle\sigma\rangle^2$ has a linear relationship with $\log L$ and therefore is in a good agreement with the theory.

With the aid of the numerical solutions the weakest link hypothesis was examined for honeycomb lattices, using RFM. It was observed that for the chosen system lengths the weakest link hypothesis is not valid. Later the validity of the DLB distribution was tested using the same model. It was confirmed the numerical distribution is in a good agreement with the theory. It should be mentioned that these results might not be valid due to the limited size of the systems.

[1] A.A. Griffith,Phil. Trans. Roy. Soc. Lon. A **221**, 163 (1921).

[2] L. de Arcangelis, S. Render, and H.J. Hermann, J. Phys. Lett. **46**, 585 (1985).

[3] P.M. Duxbury, P.L. Leath, and P.D. Beale, Phys. Rev. B **36**, 367 (1987).

[4] C. Manzato, A. Shekhawat, P.K.V.V Nukala, M.J. Alava, J.P. Sethna, and S. Zapperi, Phys. Rev. Lett. **108**, 065504 (2012).