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8.334 Final Project Website

**Aims**

I started the project with the following aims:

* Convey basic results from statistical field theory, in particular regarding phase transitions, in a visual, intuitive fashion. Lattice models, graphs and diagrams preferred over exact mathematical statements and excessive natural language descriptions.
* Build JavaScript classes to implement interactive lattice models in real-time, for arbitrary lattice geometry and Hamiltonian.
* Produce a website outlining concepts leading up to interactive applets about Ising models, free energy, and 1D position-space renormalization. Use simple, engaging explanations and diagrams appropriate for a high school-level reader.
* Develop an analogy for physical fields based on many individuals with political leanings that evolve in time, influenced by their neighbors and external fields. For instance, model a 2D Ising spin glass as a convention hall filled with individuals who are pro-left or pro-right.

**Outline of Website**

* Introduction to phases of matter, assuming minimal reader knowledge. Use phases of water to connect microscopic and macroscopic pictures.
* Thermodynamics and phase diagrams, introduced via the carrot-and-puddle model.
* Universality
* Interactive 2D Ising model, following political convention analogy.

**Code Developed**

* spinView.m: A quick proof-of-principle in MATLAB for a 2D Ising model on a square lattice, where user can adjust parameters in real time.
* spyn.js: JavaScript library for fast object-oriented implementation of spin crystal. Since the spectrum is discrete, decides which spin to flip next based on multiplicity of spins at each energy. If a spin is flipped, the spectrum entries for it and its neighbors are updated.

The following classes are defined:

* + Spyn: an abstract lattice node
    - .value: internal value
    - .attachNeighbor(spyn): attaches neighboring spin
    - .updateNeighbors():
    - .makeKey(): provides list describing state of spin and neighbors
    - …
  + Crystal: a system of Spyn objects, can be evolved according to Hamiltonian
    - .spyns: list of spins in crystal
    - .params: parameters for Hamiltonian
    - .energy(): calculate average internal energy
    - .flip(): flips spin, following Boltzmann ensemble
    - …
  + Spectrum:
    - .keys: list of all possible states for a spin and its direct neighbors
    - .addState(spyn): increments the total number of this spin
    - …
  + Applet:
    - .activate(state): turns the main loop running the crystal on and off
    - .drawCrystal(): draws the crystal, can be overwritten with page-specific code
    - …
* convention.js: JavaScript to initialize and run 2D Ising model of political convention. Sets up user controls and events.
* global.js: extensions of the JavaScript array type to do vector math, etc.

**What I Would Implement Given The Time**

**Percolation Applet**

* 2D lattice with bonds continuously created and destroyed following user-adjustable percolation probability. Light up top and bottom edge when connected.

**Stadium Crowd Applet**

* Triangular lattice of sports fans in a stadium. Each fan can sit or stand. Standing gives zero energy, sitting carries an energy penalty for each downward-facing bond connected to another standing fan.

**Math Track (to supplement applets)**

* Continuous and discontinuous phase transitions from saddle-point approximation, explained graphically.
* Description of canonical ensemble. Why exponential functions are used (want combination of independent systems to have independent probability, hence need F[x+y] = F[x]\*F[y]).
* Definition of critical exponents from leading exponent.

**Free Energy Applet**

* Starting with snapshot of 2D Ising model, highlight bonds. Each bond contributes “friction” in the political analogy.
* Combine
* For a small set of fixed parameters in the Hamiltonian and a 4x4 crystal, display a stacked bar graph. Position along the x-axis indicates energy. Each slice of a given bar represents a collection of states with net magnetization, and has a thickness corresponding to multiplicity of states with that magnetization and energy. Hovering the mouse over a slice displays a typical configuration.
* Pose the question: how many states do we need to keep track of for a crystal of length N? On a graph, show how 2N eclipses other large numbers. Motivates alternate approaches to describing essential aspects of large interacting systems.