Phase Instabilities in Active-Passive Mixtures with Nematic Alignment

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We examine the effect of nematic torques on instabilities in mixtures of active and passive Brownian particles. We derive the dynamics of the density fields of such mixtures and introduce approximations to close these dynamic equations. Using linear stability analysis, we use our theory to determine conditions under which active-passive mixtures with nematic alignment should form moving clusters. Our results suggest that it is possible for alignment interactions to lower the threshold for such mixtures to destabilize.

I. INTRODUCTION

Active matter denotes a class of systems that are driven out of equilibrium by non-conservative forces at the level of the systems' individual constituents [1–3]. Because of this, the theory of equilibrium statistical mechanics cannot be readily used to understand emergent phenomena in active matter. Over the last 10-20 years, much work has been done to develop theories of phase transitions in non-equilibrium systems. One notable example is the theory of motility-induced phase separation [4], which refers to phase separation of self-propelled Brownian particles that interact repulsively. More recently, work has been done to characterize phase separation in more complex active systems, such as active particles that experience alignment interactions and mixtures of active and passive particles [5, 6]. In Ref. [5], it is shown that nematic torques can induce phase separation in active suspensions by effectively increasing the persistence of the active particles.

In this paper, we develop a field theory to characterize how nematic alignment affects phase instabilities in active-passive mixtures. In Section II, we follow the methodology in Ref. [6] to construct the dynamics of the one-particle distribution functions from the active and passive particles' equations of motion. In Section III, we introduce several approximations to close the dynamics of the distribution functions and account for the alignment interactions, following Ref. [5] for the latter. Finally, in Section IV, we obtain quantitative criteria for the active-passive mixture to become unstable.

II. DYNAMIC EQUATIONS FOR MIXTURES

We start from microscopic equations of motion for the active and passive Brownian particles in 2 dimensions. In the overdamped limit, the positions $\mathbf{r}_i^a(t)$ and $\mathbf{r}_i^p(t)$ of the active and passive particles and the angular orientations $\theta_i(t)$ of the active particles evolve according to

$$\dot{\mathbf{r}}_i^a = v_0\hat{\mathbf{u}}(\theta_i) + \mu\sum_j \mathbf{F}(\mathbf{r}_i^a - \mathbf{r}_j^a) + \mu\sum_j \mathbf{F}(\mathbf{r}_i^a - \mathbf{r}_j^p) + \sqrt{2D_t}\eta_i^a(t) \quad (1)$$

$$\dot{\mathbf{r}}_i^p = \mu\sum_j \mathbf{F}(\mathbf{r}_i^p - \mathbf{r}_j^a) + \mu\sum_j \mathbf{F}(\mathbf{r}_i^p - \mathbf{r}_j^p) + \sqrt{2D_t}\eta_i^p(t) \quad (2)$$

$$\dot{\theta}_i = \sum_{|\mathbf{r}_i^a - \mathbf{r}_j^a| < r_0} \frac{\gamma}{n_i}\sin\left[2(\theta_j - \theta_i)\right] + \sqrt{2D_r}\xi_i(t). \quad (3)$$

Above, v_0 is the self-propulsion speed of the active particles, $\hat{\mathbf{u}}(\theta_i)$ is a unit vector pointing in the direction of self-propulsion, μ is the particles' mobility, $\mathbf{F}(\mathbf{r} - \mathbf{r}')$ is the repulsive force exerted by the particle at $\mathbf{r}'$ on the particle at $\mathbf{r}$, $D_t = \mu k_B T$ is the translational diffusion coefficient, γ is the strength of the nematic alignment interactions between the active particles, r_0 is the distance over which active particles exert nematic torques on each other, n_i is the number of particles exerting torques on the i^{th} active particle, and D_r is the rotational diffusion coefficient. The noise terms $\eta_i^a(t)$, $\eta_i^p(t)$, and $\xi_i(t)$ are zero-mean unit-variance Gaussian white noise.

To study the onset of phase instabilities in these active-passive mixtures, we will proceed by constructing the dynamics of the particles' density fields. To do so, we first derive the dynamics of the one-particle distribution functions $P_a^{(1)}(\mathbf{r}, \theta, t)$ and $P_p^{(1)}(\mathbf{r}, t)$ of the active and passive particles, which are given by

$$P_a^{(1)}(\mathbf{r}, \theta, t) = \left\langle \sum_i \delta(\mathbf{r} - \mathbf{r}_i^a(t))\delta(\theta - \theta_i(t)) \right\rangle \quad (4)$$

$$P_p^{(1)}(\mathbf{r}, t) = \left\langle \sum_i \delta(\mathbf{r} - \mathbf{r}_i^p(t)) \right\rangle. \quad (5)$$

Taking the time derivative of both equations above and using Ito's formula [7], we obtain the following evolution equations for $P_a(\mathbf{r},\theta,t)$ and $P_p(\mathbf{r},t)$

$$\dot{P}_p^{(1)}(\mathbf{r},t) = -\nabla\cdot\mathbf{J}_p(\mathbf{r},t) \quad (6)$$

$$\dot{P}_a^{(1)}(\mathbf{r},\theta,t) = -\nabla\cdot\mathbf{J}_a(\mathbf{r},\theta,t) - \frac{\partial J_\theta(\mathbf{r},\theta,t)}{\partial\theta}, \quad (7)$$

where the fluxes $\mathbf{J}_p(\mathbf{r},t)$, $\mathbf{J}_a(\mathbf{r},\theta,t)$, and $J_\theta(\mathbf{r},\theta,t)$ are given by

$$\mathbf{J}_p(\mathbf{r},t) = -D_t\nabla P_p^{(1)}(\mathbf{r},t) + \mu\int\mathbf{F}(\mathbf{r}-\mathbf{r}')\left(\int_0^{2\pi}P_{pa}^{(2)}(\mathbf{r},\mathbf{r}',\theta',t)d\theta' + P_{pp}^{(2)}(\mathbf{r},\mathbf{r}',t)\right) d^2\mathbf{r}' \quad (8)$$

$$\mathbf{J}_a(\mathbf{r},\theta,t) = v_0\hat{\mathbf{u}}(\theta)P_a^{(1)}(\mathbf{r},\theta,t) - D_t\nabla P_a^{(1)}(\mathbf{r},\theta,t) + \mu\int\mathbf{F}(\mathbf{r}-\mathbf{r}')\times \left(P_{pa}^{(2)}(\mathbf{r}',\mathbf{r},\theta,t) + \int_0^{2\pi}P_{aa}^{(2)}(\mathbf{r},\theta,\mathbf{r}',\theta',t)d\theta'\right)d^2\mathbf{r}' \quad (9)$$

$$J_\theta(\mathbf{r},\theta,t) = -D_r\frac{\partial P_a^{(1)}(\mathbf{r},\theta,t)}{\partial\theta} + \frac{\gamma}{n(\mathbf{r})}\int_0^{2\pi}\int\sin\left[2(\theta'-\theta)\right]\left(1-\Theta\left(|\mathbf{r}'-\mathbf{r}|-r_0\right)\right) P_{aa}^{(2)}(\mathbf{r},\theta,\mathbf{r}',\theta',t)d^2\mathbf{r}'d\theta'. \quad (10)$$

Above, $n(\mathbf{r}) = \int_0^{2\pi}\int\left(1-\Theta\left(|\mathbf{r}'-\mathbf{r}|-r_0\right)\right)\times P_a^{(1)}(\mathbf{r},\theta,t)d^2\mathbf{r}'d\theta$, and the two-particle distribution functions $P_{pp}^{(2)}(\mathbf{r},\mathbf{r}',t)$, $P_{pa}^{(2)}(\mathbf{r},\mathbf{r}',\theta',t)$, and $P_{aa}^{(2)}(\mathbf{r},\theta,\mathbf{r}',\theta',t)$ are given by

$$P_{pp}^{(2)}(\mathbf{r},\mathbf{r}',t) = \left\langle\sum_i\delta(\mathbf{r}-\mathbf{r}_i^p(t))\sum_j\delta(\mathbf{r}'-\mathbf{r}_j^p(t))\right\rangle \quad (11)$$

$$P_{pa}^{(2)}(\mathbf{r},\mathbf{r}',\theta',t) = \left\langle\sum_i\delta(\mathbf{r}-\mathbf{r}_i^p(t))\sum_j\delta(\mathbf{r}'-\mathbf{r}_j^a(t))\delta(\theta'-\theta_j(t))\right\rangle \quad (12)$$

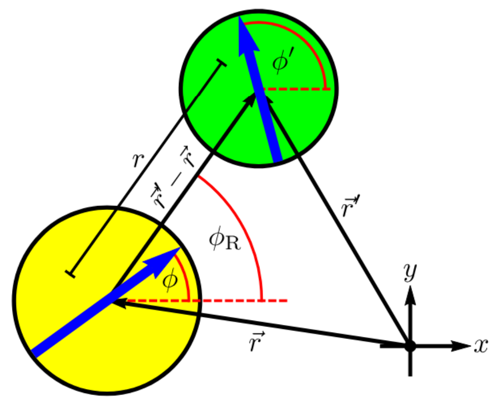

FIG. 1. Absolute and relative coordinates of two particles.

$$P_{aa}^{(2)}(\mathbf{r},\theta,\mathbf{r}',\theta',t) = \left\langle\sum_i\delta(\mathbf{r}-\mathbf{r}_i^a(t))\delta(\theta-\theta_i(t))\sum_j\delta(\mathbf{r}'-\mathbf{r}_j^a(t))\times \delta(\theta'-\theta_j(t))\right\rangle. \quad (13)$$

III. APPROXIMATE CLOSURES

Note that equations (6) and (7) are not closed equations for the one-particle distribution functions, which poses challenges for performing dynamical stability analysis. To obtain approximate closures for the dynamics of the one-particle distribution functions, we first decompose the two-particle distribution functions in terms of the pair distribution functions (PDFs) $g_{pp}(\mathbf{r},\mathbf{r}',t)$, $g_{pa}(\mathbf{r},\mathbf{r}',\theta',t)$, and $g_{aa}(\mathbf{r},\theta,\mathbf{r}',\theta',t)$ as

$$P_{pp}^{(2)}(\mathbf{r},\mathbf{r}',t) = P_p^{(1)}(\mathbf{r},t)P_p^{(1)}(\mathbf{r}',t)g_{pp}(\mathbf{r},\mathbf{r}',t) \quad (14)$$

$$P_{pa}^{(2)}(\mathbf{r},\mathbf{r}',\theta',t) = P_p^{(1)}(\mathbf{r},t)P_a^{(1)}(\mathbf{r}',\theta',t)g_{pa}(\mathbf{r},\mathbf{r}',\theta',t) \quad (15)$$

$$P_{aa}^{(2)}(\mathbf{r},\theta,\mathbf{r}',\theta',t) = P_a^{(1)}(\mathbf{r},\theta,t)P_a^{(1)}(\mathbf{r}',\theta',t)g_{aa}(\mathbf{r},\theta,\mathbf{r}',\theta',t). \quad (16)$$

A number of approximations can be made to reduce the number of variables that the PDFs depend on. We first assume that the PDFs $g(\mathbf{r},\theta,\mathbf{r}',\theta',t)$ depend only on $\mathbf{r}-\mathbf{r}'$ due to translational invariance of the system. Next, we reparameterize the pair distribution functions as $g(\mathbf{r}-\mathbf{r}', \theta, \theta', t) = g(|\mathbf{r}-\mathbf{r}'|, \theta_R, \theta, \theta', t)$, where θ_R is defined in Figure 1, which is taken from Ref. [6]. Assuming the system is also invariant under rigid rotations and angular inversion, we have $g(|\mathbf{r}-\mathbf{r}'|, \theta_R, \theta, \theta', t) = g(|\mathbf{r}-\mathbf{r}'|, \theta_R-\theta, \theta'-\theta, t) = g(|\mathbf{r}-\mathbf{r}'|, \theta_R-\theta, t)$. Finally, we assume the PDFs are time-independent, which is expected to hold when the system reaches a statistical steady-state. Using these approximations, the fluxes $\mathbf{J}_p(\mathbf{r}, t)$ and $\mathbf{J}_a(\mathbf{r}, \theta, t)$ can be simplified to

$$\mathbf{J}_p(\mathbf{r}, t) = -D_t \nabla P_p^{(1)}(\mathbf{r}, t) + \mu \int \mathbf{F}(\mathbf{r}-\mathbf{r}') \times \\ (g_{pa}(|\mathbf{r}-\mathbf{r}'|, \theta_R)\rho_a(\mathbf{r}', t) \\ + g_{pp}(|\mathbf{r}-\mathbf{r}'|, \theta_R) P_p^{(1)}(\mathbf{r}', t)\Big) d^2\mathbf{r}' P_p^{(1)}(\mathbf{r}, t) \quad (17)$$

$$\mathbf{J}_a(\mathbf{r}, \theta, t) = v_0 \hat{\mathbf{u}}(\theta) P_a^{(1)}(\mathbf{r}, \theta, t) - D_t \nabla P_a^{(1)}(\mathbf{r}, \theta, t) \\ + \mu \int \mathbf{F}(\mathbf{r}-\mathbf{r}') \Big(g_{pa}(|\mathbf{r}-\mathbf{r}'|, \theta_R-\theta) P_p^{(1)}(\mathbf{r}', t) \\ + g_{aa}(|\mathbf{r}-\mathbf{r}'|, \theta_R-\theta)\rho_a(\mathbf{r}', t)) \, d^2\mathbf{r}' P_a(\mathbf{r}, \theta, t), \quad (18)$$

where $\rho_a(\mathbf{r}', t) = \int_0^{2\pi} P_a^{(1)}(\mathbf{r}', \theta', t) d\theta'$ is the ensemble-averaged density field of the active particles.

Because the repulsive forces $\mathbf{F}(\mathbf{r}-\mathbf{r}')$ are short-range, we expect that they exhibit variations on length scales much shorter than those of the one-particle distribution functions $P_p^{(1)}(\mathbf{r}', t)$ and $\rho_a(\mathbf{r}', t)$. Thus, we will perform Taylor expansions of the one-particle distribution functions inside the integrals in equations (17) and (18) to first order in gradients. Additionally, to approximate the angular dependence of the one-particle distribution functions outside of the integrals in equations (17) and (18), we will expand them in spherical harmonics up to first order. Using these approximations, discarding terms in the fluxes $\mathbf{J}_p(\mathbf{r}, t)$ and $\mathbf{J}_a(\mathbf{r}, \theta, t)$ that are second order or higher in gradients, and integrating both sides of equation (7) with respect to θ, we obtain the following evolution equations for the ensemble-averaged density fields $\rho_a(\mathbf{r}, t)$ and $\rho_p(\mathbf{r}, t) = P_p^{(1)}(\mathbf{r}, t)$ of the active and passive particles after simplifying.

$$\dot{\rho}_p(\mathbf{r}, t) = -\nabla \cdot \mathbf{j}_p(\mathbf{r}, t) \quad (19)$$

$$\dot{\rho}_a(\mathbf{r}, t) = -\nabla \cdot \mathbf{j}_a(\mathbf{r}, t) \quad (20)$$

The fluxes $\mathbf{j}_p(\mathbf{r}, t)$ and $\mathbf{j}_a(\mathbf{r}, t)$ are given by

$$\mathbf{j}_p(\mathbf{r}, t) = \left(-D_t - c_{pp}\rho_p(\mathbf{r}, t)\right) \nabla \rho_p(\mathbf{r}, t) \\ - c_{pa}\rho_p(\mathbf{r}, t) \nabla \rho_a(\mathbf{r}, t) \quad (21)$$

$$\mathbf{j}_a(\mathbf{r}, t) = \left(-D_t - c_{aa}\rho_a(\mathbf{r}, t)\right) \nabla \rho_a(\mathbf{r}, t) \\ - c_{pa}\rho_a(\mathbf{r}, t) \nabla \rho_p(\mathbf{r}, t) \\ + v_0(1 - b_a\rho_a(\mathbf{r}, t)/v_0 - b_p\rho_p(\mathbf{r}, t)/v_0)\mathbf{m}(\mathbf{r}, t). \quad (22)$$

Above, $\mathbf{m}(\mathbf{r}, t) = \int_0^{2\pi} \hat{\mathbf{u}}(\theta) P_a^{(1)}(\mathbf{r}, \theta, t) d\theta$ is the ensemble-averaged orientation field of the active particles, and the coefficients c_{pp}, c_{pa}, c_{aa}, b_p, and b_a are

$$c_{pp} = \frac{1}{2}\mu \int_0^\infty r^2 F(r) \int_0^{2\pi} g_{pp}(r, \theta) d\theta dr \quad (23)$$

$$c_{pa} = \frac{1}{2}\mu \int_0^\infty r^2 F(r) \int_0^{2\pi} g_{pa}(r, \theta) d\theta dr \quad (24)$$

$$c_{aa} = \frac{1}{2}\mu \int_0^\infty r^2 F(r) \int_0^{2\pi} g_{aa}(r, \theta) d\theta dr \quad (25)$$

$$b_p = \mu \int_0^\infty r F(r) \int_0^{2\pi} g_{pa}(r, \theta) \cos(\theta) d\theta dr \quad (26)$$

$$b_a = \mu \int_0^\infty r F(r) \int_0^{2\pi} g_{aa}(r, \theta) \cos(\theta) d\theta dr. \quad (27)$$

Note that the coefficients above are independent of $\mathbf{r}$ due to translational invariance. Although the PDFs appearing in the expressions above are unknown at this stage, they can be computed from numerical simulations.

Equations (19) and (20) are still not closed due to the appearance of the orientation field $\mathbf{m}(\mathbf{r}, t)$ in equation (20). To complete the closure, we first obtain the dynamics of $\mathbf{m}(\mathbf{r}, t)$ by multiplying both sides of equation (7) by $\hat{\mathbf{u}}(\theta)$ and integrating over θ. Then, discarding terms that are second order or higher in gradients, we get

$$\dot{\mathbf{m}}(\mathbf{r}, t) = -D_r \mathbf{m}(\mathbf{r}, t) \\ - \int_0^{2\pi} \hat{\mathbf{u}}(\theta) \frac{\partial}{\partial \theta} \left(\frac{\gamma}{n(\mathbf{r})} \int_0^{2\pi} \int \sin[2(\theta'-\theta)] \times \right. \\ \left. \left(1 - \Theta\left(|\mathbf{r}'-\mathbf{r}| - r_0\right)\right) P_{aa}^{(2)}(\mathbf{r}, \theta, \mathbf{r}', \theta', t) d^2\mathbf{r}' d\theta' \right) d\theta \\ -\nabla \left(\frac{1}{2} v_0 \left(1 - b_a\rho_a(\mathbf{r}, t)/v_0 - b_p\rho_p(\mathbf{r}, t)/v_0\right) \rho_a(\mathbf{r}, t) \right). \quad (28)$$

Results from Ref. [5] suggest that, at high temperatures, the effect of the alignment interactions on the dynamics of $\mathbf{m}(\mathbf{r},t)$ can be accounted for by a renormalization of the coefficient D_r which controls the decay of orientational order. The new damping coefficient $\tilde{D}_r$ is defined such that

$$\dot{\mathbf{m}}(\mathbf{r},t) = -\tilde{D}_r \mathbf{m}(\mathbf{r},t) \\ -\nabla\left(\frac{1}{2}v_0\left(1 - b_a\rho_a(\mathbf{r},t)/v_0 - b_p\rho_p(\mathbf{r},t)/v_0\right)\rho_a(\mathbf{r},t)\right), \tag{29}$$

and, to leading order, is given by

$$\tilde{D}_r = D_r - \frac{4\gamma D_r(\gamma - D_r)}{n(5D_r-\gamma)(13D_r-\gamma)}. \tag{30}$$

Thus, for $D_r < \gamma < 5D_r$, the damping of $\mathbf{m}(\mathbf{r},t)$ is reduced by nematic alignment. Simulations from Ref. [5] show that this effect also holds non-perturbatively as γ increases.

Looking at the dynamics of $\mathbf{m}(\mathbf{r},t)$, we can see that it relaxes on a finite time scale $\tilde{D}_r^{-1}$. In contrast, because the total number of particles is conserved, the relaxation time of the density field $\rho_a(\mathbf{r},t)$ will diverge as the system size becomes very large. Thus, we expect that $\mathbf{m}(\mathbf{r},t)$ will reach a steady-state configuration much faster than $\rho_a(\mathbf{r},t)$, so we can substitute the steady-state for $\mathbf{m}(\mathbf{r},t)$ into equation (20) to close the dynamics of $\rho_a(\mathbf{r},t)$. Doing so, we get

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_p(\mathbf{r},t)\\ \rho_a(\mathbf{r},t)\end{pmatrix} = \nabla\cdot\left(\begin{pmatrix}D_{pp}(\rho_p) & D_{pa}(\rho_p)\\ D_{ap}(\rho_p,\rho_a) & D_{aa}(\rho_p,\rho_a)\end{pmatrix}\times \nabla\begin{pmatrix}\rho_p(\mathbf{r},t)\\ \rho_a(\mathbf{r},t)\end{pmatrix}\right), \tag{31}$$

where

$$D_{pp} = D_t + c_{pp}\rho_p(\mathbf{r},t) \tag{32}$$

$$D_{pa} = c_{pa}\rho_p(\mathbf{r},t) \tag{33}$$

$$D_{ap} = \Big(c_{pa} \\ -\frac{1}{2\tilde{D}_r}b_p v_0\left(1 - b_a\rho_a(\mathbf{r},t)/v_0 - b_p\rho_p(\mathbf{r},t)/v_0\right)\Big)\rho_a(\mathbf{r},t) \tag{34}$$

$$D_{aa} = D_t + \frac{\left(v_0(1 - b_a\rho_a(\mathbf{r},t)/v_0 - b_p\rho_p(\mathbf{r},t)/v_0)\right)^2}{2\tilde{D}_r} \\ +\left(c_{aa} - \frac{1}{2\tilde{D}_r}b_a v_0\left(1 - b_a\rho_a(\mathbf{r},t)/v_0 - b_p\rho_p(\mathbf{r},t)/v_0\right)\right) \\ \rho_a(\mathbf{r},t). \tag{35}$$

We can see that, at leading order in gradients, the dynamics of the density fields take the form of a diffusion equation with a density-dependent diffusivity tensor. The flux of passive particles contains contributions from the translational noise and the repulsive interactions. The flux of active particles contains these same contributions, but the translational diffusion is enhanced by the effective self-propulsion speed $v_0(1 - b_a\rho_a(\mathbf{r},t)/v_0 - b_p\rho_p(\mathbf{r},t)/v_0)$, and the contribution from the interactions is reduced due to self-propulsion. Note the appearance of the effective speed $v_0(1-b_a\rho_a(\mathbf{r},t)/v_0-b_p\rho_p(\mathbf{r},t)/v_0)$ everywhere rather than the bare speed v_0 due to slowing down of the active particles from collisions experienced in the direction opposite of self-propulsion.

IV. LINEAR STABILITY ANALYSIS

Having obtained approximate closures for the dynamics of the particles' density fields, we can now perform linear stability analysis to obtain criteria for the active-passive mixture to become unstable. From Ref. [6], it is known that two types of instabilities can occur in such mixtures. The first kind, which is a stationary bifurcation, is the same instability that causes motility-induced phase separation in one-component active systems; the effect of nematic alignment on phase separation in such systems has been studied in Ref. [5]. Thus, we will instead focus on how alignment interactions affect the Hopf bifurcation, the second type of instability which is only possible in multi-component systems.

We start by considering perturbations to the density fields $\rho_p(\mathbf{r},t) = \bar{\rho}_p + \delta\rho_p(\mathbf{r},t)$ and $\rho_a(\mathbf{r},t) = \bar{\rho}_a + \delta\rho_a(\mathbf{r},t)$, where $\bar{\rho}_p$ and $\bar{\rho}_a$ are the spatial averages of the density fields. Substituting into equation (31) and only retaining terms up to linear order in the perturbations, we get

$$\frac{\partial}{\partial t}\begin{pmatrix}\delta\rho_p(\mathbf{r},t)\\ \delta\rho_a(\mathbf{r},t)\end{pmatrix} = \nabla\cdot\left(\begin{pmatrix}D_{pp}(\bar{\rho}_p) & D_{pa}(\bar{\rho}_p)\\ D_{ap}(\bar{\rho}_p,\bar{\rho}_a) & D_{aa}(\bar{\rho}_p,\bar{\rho}_a)\end{pmatrix}\times \nabla\begin{pmatrix}\delta\rho_p(\mathbf{r},t)\\ \delta\rho_a(\mathbf{r},t)\end{pmatrix}\right). \tag{36}$$

The stability of the perturbations is thus determined by the eigenvalues of the diffusivity tensor in equation (36) that is evaluated at the average density values. The eigenvalues of this diffusivity tensor are

$$\lambda_1 = \frac{1}{2}\left(D_{pp} + D_{aa} - \sqrt{(D_{pp} - D_{aa})^2 + 4D_{pa}D_{ap}}\right) \tag{37}$$

$$\lambda_2 = \frac{1}{2}\left(D_{pp} + D_{aa} + \sqrt{(D_{pp} - D_{aa})^2 + 4D_{pa}D_{ap}}\right). \tag{38}$$

If $\mathrm{Re}(\lambda_i) > 0\ \forall i$, then the perturbations decay to zero, and the system is stable. If $\mathrm{Re}(\lambda_i) < 0$ for some i, then some perturbations are amplified, and the system is unstable. The spinodal, which separates the stable and unstable regions, is defined by imposing the condition $\mathrm{Re}(\lambda_i) = 0\ \forall i$. The Hopf bifurcation, which only occurs in multi-component systems, also requires that $\mathrm{Im}(\lambda_i) \neq 0\ \forall i$. Thus, the section of the spinodal corresponding to the Hopf bifurcation satisfies

$$D_{pp} = -D_{aa} \tag{39}$$

$$(D_{pp})^2 < D_{pa}D_{ap}. \tag{40}$$

If $D_{pp} < -D_{aa}$ and $(D_{aa} - D_{pp})^2 < -4D_{pa}D_{ap}$, the system is unstable and moving clusters will form. Because the alignment interactions enhance the persistence of the active particles, our theory predicts that increasing γ may be able to decrease the threshold for such moving clusters to form, provided that the imaginary parts of λ_1 and λ_2 remain non-zero. This also means that a mixture that would be stable in the absence of nematic alignment may become unstable and form moving clusters in the presence of alignment interactions.

V. CONCLUSION

Through coarse-graining microscopic equations of motion for active-passive mixtures with nematic alignment, we have derived approximate closed equations for the density fields in such mixtures. Using linear stability analysis, we have obtained conditions for the mixture to destabilize and form moving clusters. There are still many ways in which one could supplement the calculations presented here. Although we obtained the instability criteria analytically, these criteria involve integrals of the pair distribution functions, which were not computed explicitly. Thus, it would be interesting to estimate the PDFs from numerical simulations to calculate numerical values of the coefficients appearing in the instability criteria. The question of how accurate the closure assumptions are also remains. This could be answered by generating a phase diagram of the mixture using simulations and comparing it to the phase diagram obtained from the linear stability analysis.

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