8.581 Entropy, information and the brain

Problems for Lecture 5.

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1. **Building the optimal filter.** In the lecture we discussed the problem of filtering a signal so as to transmit information at the maximum rate given a fixed dynamic range for the transmitted signal. At high signal to noise ratios, the result is the power transfer characteristics of the the filter should be matched to the spectrum of the input signals, $|\tilde{F}(\omega)|^2 \propto 1/S(\omega)$. For $S(\omega) \propto 1/|\omega|$, this predicts $|\tilde{F}(\omega)| \propto \sqrt{|\omega|}$, and though this seems strange it is observed in the experiments on the fly retina.

   - With two degrees of freedom (e. g., voltage and and the number of open channels) one can build a high pass filter. Specifically, if we write
   $$\begin{aligned} \frac{dx_1}{dt} &= -Ax_1 + Bx_2 + I && (1) \\ \frac{dx_2}{dt} &= Cx_1 - Dx_2, && (2) \end{aligned}$$
   find a choice of the coefficients so that signal transfer from I to x_1 vanishes at zero frequency (this is not hard!). The find the form of the whole frequency response and show that it is *not* the fractional power observed experimentally.
   - de Ruyter and Laughlin have suggested a diffusion model for the fractional power. Consider the diffusion equation with a source,
   $$\frac{\partial x}{\partial t} = D\nabla^2 c + I(t)\delta(\vec{x}), \qquad (3)$$
   and find the transfer function from $I(t)$ to $c(\vec{x}, t)$. In particular, look at the behavior for low frequencies.
   - Can you think of a way to incorporate a diffusion process into a phenomenological model so as to produce the observed frequency response? Think about feedback.

2. **Lateral inhibition.** The center surround organization of receptive fields in the retina is often modelled by a 'lateral inhibitory' network of the sort first described in the horseshoe crab retina. Imagine that neurons are on a lattice, each receives input from the outside world, and talks to its neighbors. In one dimension, the simplest linear network of this form is
$$\frac{dV_n}{dt} = -\frac{1}{\tau}V_n - J(V_{n-1} + V_{n+1}) + I_n, \qquad (4)$$
where positive J means that neighbors inhibit one another.

- Find the response of V to I, remembering that I can vary both in space and time. Express your result in the frequency and spatial frequency domain.
- What is the asymptotic behavior at low frequencies? Is this the sort of spatial filtering that would provide optimal encoding of signals with a spatial power spectrum $S \propto 1/|k|$?
- Can you build a more interesting filter (perhaps even the optimal filter?) by allowing for further neighbor interactions? That is, consider

$$\frac{dV_n}{dt} = -\frac{1}{\tau}V_n - \sum_m J(m)(V_{n-m} + V_{n+m}) + I_n, \tag{5}$$

  and see if you can choose the coupling $J(m)$ to produce a desired spatial response function. What are the important features of a $J(m)$ that would yield the optimal filter for $S \propto 1/|k|$?
- Generalize this discussion to two dimensions.