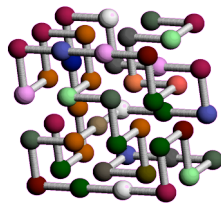


# Protein Folding 2

Thermodynamics  
Lattice Model  
Design of foldable proteins  
Calculations of Z-score

## Lattice Model



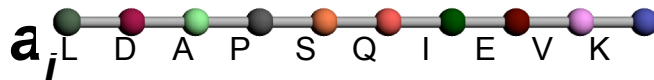
- Amino acids sit in the nodes of the cubic lattice.
- They interact with their neighbors on the lattice.
- It's very hard to fold such protein:  
**LEVINTHAL PARADOX**

## Energy function

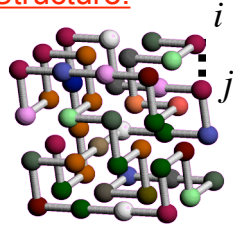
Energy function:

$$E = \sum_{i+1 < j} \Delta_{ij} U(a_i, a_j)$$

Sequence:



Structure:



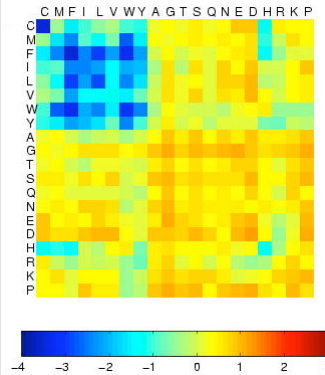
$$\Delta_{ij} = 1$$

if  $i$  and  $j$  are in contact

$$\Delta_{ij} = 0$$

otherwise

Potential:  $U(\mathbf{x}, \mathbf{y})$

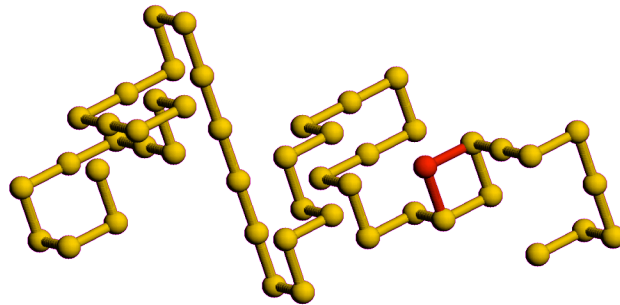


## Monte Carlo Dynamics

1. Pick a fragment.
2. Try a move
3. Compute  $\Delta E$
4. Accept the move with probability  $P$

$$P = 1 \quad \text{if } \Delta E \leq 0$$

$$P = \exp(-\Delta E/T) \quad \text{if } \Delta E > 0$$



• What do you expect to see at low  $T$ ?  
high  $T$ ?

# Demo

## Evolving a foldable sequence = sequence with a large energy gap

Three approaches to obtain foldable sequences:

1. Maximize  $\frac{E_n - \langle E \rangle}{E_c - \langle E \rangle}$
2. Minimize  $z = \frac{E_n - \langle E \rangle}{\sigma(E)}$
3. Minimize  $E_n$ , while constraining amino acid composition (i.e.  $\sigma$  and  $\langle E \rangle$ )

$$E = E(\vec{a}, \Delta, U); \quad E_n = E(\vec{a}, \Delta_n, U);$$

Algorithm: Metropolis Monte Carlo (for case 3: minimize the energy in the native structure)

1. Pick the Native Structure, a starting (random) sequence, and a potential  $U(x,y)$
2. Mutate the sequence and calculate the change in  $E_n$ :  $\Delta E_n = E_n(\vec{a}') - E_n(\vec{a})$
3. Accept or reject the mutation: accept with  $\text{Prob} = \max\{1, \exp(-\Delta E_n / T_{\text{seq}})\}$
4. Go to step 2.

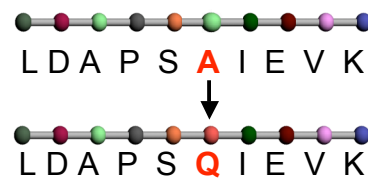


This satisfies Detailed Balance:

$$\frac{P(\vec{a} \rightarrow \vec{a}')}{P(\vec{a}' \rightarrow \vec{a})} = \exp\left[-\frac{E(\vec{a}') - E(\vec{a})}{T_{\text{seq}}}\right]$$

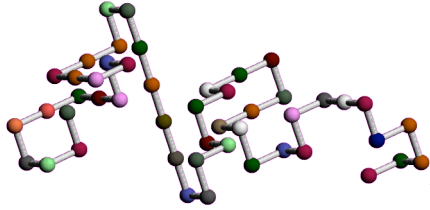
Converging to equilibrium

$$P(\vec{a}) = \exp\left[-\frac{E(\vec{a})}{T_{\text{seq}}}\right]$$



## Evolved proteins can fold!

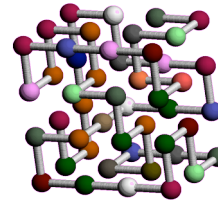
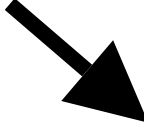
Random conformation



Folding Time:

$\langle t \rangle = 10^6 - 10^8$  for 48-mer

$\langle t \rangle = 10^5 - 10^7$  for 27-mer



Conformation of minimal energy

Folding time depends on the sequence  
and structure of the protein!