Physics 8.861: Advanced Topics in Superfluidity

- My plan for this course is quite different from the published course description. I will be focusing on a very particular circle of ideas around the concepts: anyons, fractional quantum Hall effect, topological quantum computing.

- Although those topics might seem rather specialized, they bring in mathematical and physical ideas of great beauty and wide use.

- These topics are currently the subject of intense research. The goal of this course will be to reach the frontiers of research.

- There is no text, but I will be recommending appropriate papers and book passages as we proceed. If possible, links will be provided from the course home-page.

- Here are 5 references that cover much of the *theoretical* material we'll be studying:

F. Wilczek, ed. Fractional Statistics and Anyon Superconductivity (World Scientific, 1990)

A. Kitaev, A. Shen, M. Vyalyi Classical and Quantum Computation (AMS)

J. Preskill, Lecture Notes on Quantum Computing, especially chapter 9: Topological Quantum Computing, http://www.theory.caltech.edu/people/preskill/ph229/

A. Kitaev, quant-ph/9707021

A. Kitaev, cond-matt/050643

Lecture 1

Introductory Survey

Fractionalization

- Quantum mechanics is a young theory. We are still finding basic surprises.
- One surprising phenomenon - perhaps we should call it a “meta-phenomenon”, since it appears in different forms in many different contexts - is that the quantum numbers of *macroscopic physical states* often differ drastically from the quantum numbers of the underlying *microscopic degrees of freedom*.
- Reference: F. W., cond-mat/0206122

- The classic example is *confinement* of quarks. This was a big stumbling-block to accepting quarks, and to establishing the theory of the strong interaction.

- Now we "understand" confinement in simple ways. One way is through strong coupling lattice gauge theory. Another is by analogy to superconductivity and the Higgs phenomenon. Both are closely related to ideas we'll be developing in depth.

- Aside: Color-flavor locking.

Confinement From Superconductivity

- BCS: Nonperturbative effects in a tractable (weak coupling) context.
- Meissner effect ↔ Confinement
- Gap ~ Chiral symmetry breaking

Color-Flavor Locking

- $<q^{\alpha}_{a}\, q^{\beta}_{b}> = \kappa_1\, \delta^{\alpha}{}_{a} \delta^{\beta}{}_{b} + \kappa_2\, \delta^{\alpha}{}_{b} \delta^{\beta}{}_{a}$
- Color x Flavor_L x Flavor_R → Color + Flavor_L + Flavor_R
- $SU(3)_{local}$ x $SU(3)_{global}$ x $SU(3)_{global}$ → $SU(3)_{global}$
- The new symmetry generators are non-trivial combinations of old symmetry generators - including gauge parts!

- Quark fields produce states with the quantum numbers of baryons.
- Gluon fields produce states with the quantum numbers of vector mesons.
- Collective excitations associated with spontaneous chiral symmetry breaking produce states with the quantum numbers of pseudoscalar mesons.
- Glue - γ mixing imparts integer charges to the quarks!

- End of aside.

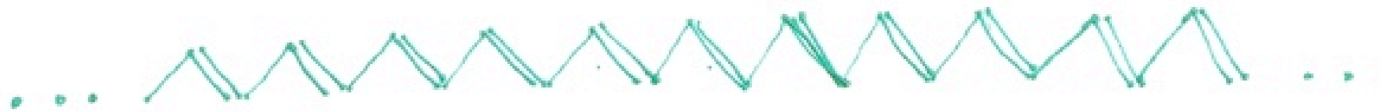
Fractionalization, in General

- In elementary QED, we are accustomed to the idea that the observed charge of particles directly reflects the charge of local fields that create the particles.
- This happens despite vacuum polarization, because the effect of renormalization is universal: $e_{ren} = (Z_3)^{1/2}\, e_{bare}$. Heuristically, this universality is a consequence of absence of any lasting "character" for a very low-resolution photon to sense.

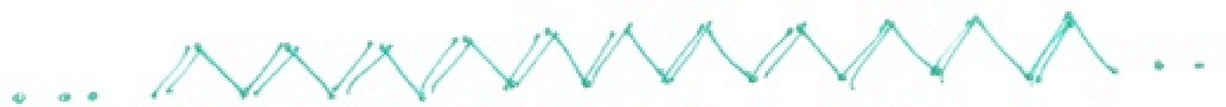
- If, however, there is additional long-range, stable structure associated with a state - e.g., a topological quantum number - then we can have $e_{ren} = (Z_3)^{1/2} e_{bare} + (Y_3)^{1/2} q_{top}$. If q_{top} is modular, the charge spectrum remains rational, but in general it needn't be.

- On first hearing all that might sound like an esoteric abstract possibility, but we will see many examples.

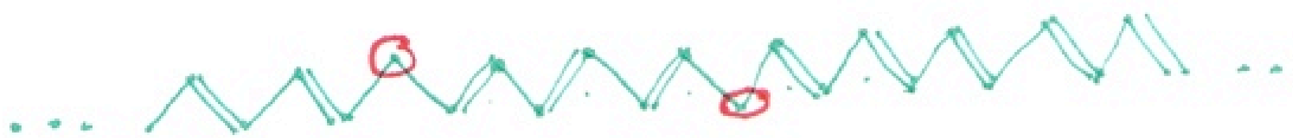
- There is even a nice pictorial realization: the Schrieffer counting argument.



A-vacuum: 33 bonds



B-vacuum: 33 bonds



2 "solitons" : A → B , B → A

32 bonds

Each solition lacks $\frac{1}{2}$ bond!

Locking Charge and Angular Momentum for Vortices

- Now I'd like to show you the simple example, that first started me thinking about fractional angular momentum and statistics.

- Consider an SU(2) "isospin" gauge theory broken down to nothing* by an isospin-3/2 scalar "Higgs" field, realized as a 3-index symmetric spinor. The vacuum expectation value is $<\eta_{\alpha\beta\gamma}> = v\, \delta_{\alpha 1}\delta_{\beta 1}\delta_{\gamma 1}$ in the ground state - or, of course, any SU(2) rotation thereof.

- Rotating around the (internal) z-axis, by θ induces a phase $e^{3i\theta/2}$ on this VEV.

- A minimal vortex makes a ϕ-dependent rotation around the *internal* z-axis through $2\phi/3$, where ϕ is the *spatial* azimuthal angle. Thus for $\phi=2\pi$ the VEV is unchanged, and everything is consistent. (Of course at the origin ϕ is ill-defined, but since $\eta(0)$ vanishes we don't need to know how to rotate it, and nothing is singular.)

- The asymptotics $<\eta(r, \Phi)> \rightarrow v\, e^{2i\,\Phi/3}$ (as $r \rightarrow \infty$) is not rotationally invariant: scalar fields shouldn't change with angle!

- However the "locked" combination $L_z^{modified} \equiv L_z - 2/3\, I_z$

  of naive rotations with gauge transformations does leave this condensate invariant.   It generates a genuine rotational symmetry in the presence of the vortex.

- Now suppose an isospin-1/2 particle is bound to the vortex. For it, $L_z^{\text{modified}} = L_z - 2/3\ \sigma_z$ has eigenvalues = integer ± 1/3!

- A pregnant question: What about the spin-statistics connection??

- Expect exchange factor = $e^{i2\pi J}$ based on ribbon argument of Finkelstein and Rubenstein. (Starting from path integrals in a relativistic theory.)

- *Actually the SU(2) gauge symmetry is not completely broken. There is a residual Z_3, generated by $e^{4\pi i I_3}$, that leaves the VEV invariant. The existence of vortices and their peculiar interaction with charges, which we'll be discussing in depth, is characteristic of a *discrete* gauge theory.

Fractional Statistics and Emergent Gauge Fields

- A charged particle acquires phase when transported around a flux tube (thin solenoid), according to $e^{iS} = e^{i\int L dt} = e^{i\int qA \cdot v\, dt} = e^{i\int qA \cdot dx}$. The phase affects quantum interference between alternative paths around the solenoid. The physical consequences are called Aharonov-Bohm effects.

- The phase is of a purely geometric character. It occurs despite the absence of classical forces - i.e., no B-field in the region outside the solenoid.

- In the case of flux tube-charge composites, the Aharonov-Bohm phase implements fractional statistics, and restores the expected spin-statistics connection.

- Nonabelian versions are also possible, obviously. They bring in a host of interesting complications, as we'll see.
- In the nonabelian case, braiding operations in physical space produce intricate motions in Hilbert space. This can support quantum information processing and quantum computing.
- Even the abelian case has rich possibilities, in complex topologies. Roughly, statistical flux can pass through the holes. (Note that embedding of 2d surfaces with boundaries into 3d can become *quite* complicated - one step beyond knots!)

Quantum Hall Effect; Experiments

The Quantum Hall Playground

- The quantum Hall complex of states occurs in effectively 2-dimensional electron gases at low T and large B. They are new states of matter, dominated by quantum mechanics, with striking phenomena and a very interesting theory.
- A powerful way of thinking about them - I think the most profound way - is as a new form of superconductivity, in which fictitious gauge fields implementing fractional statistics get intertwined with electromagnetism.

- The quasiparticles (and quasiholes) of the quantum Hall states are firmly predicted to be fractionally charged anyons.

- Most of the observed states are predicted to support abelian anyons, but one at filling fraction $\nu = 5/2$ almost surely, and possibly others, support nonabelian anyons.

- There are models and ideas for other realizations of anyons, as we'll discuss. But so far only the QHE is securely - though only theoretically - established.

Experimental Anyonics?

- There have been convincing direct observations of fractional charge through shot noise and through direct imaging (and more controversially through resonant tunneling).

- Fractional statistics may also have been observed, though not everyone is convinced. (We'll be examining the controversy.)

Direct observation of fractional statistics in two dimensions

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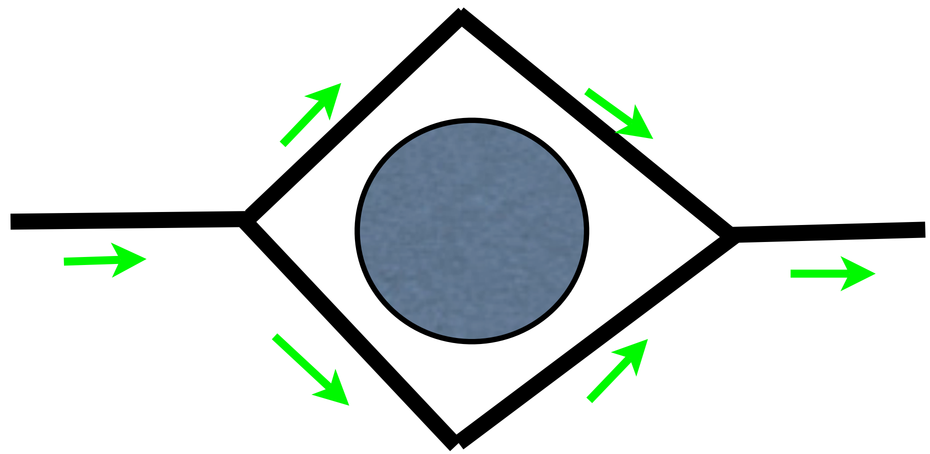
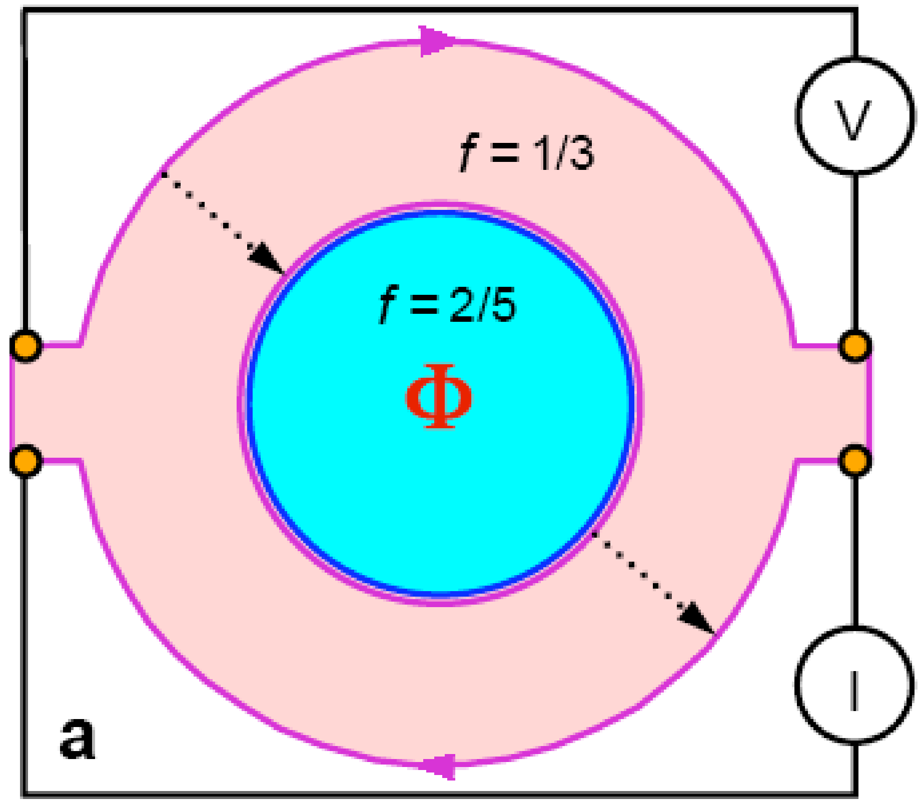
In two dimensions a closed loop enclosing another particle is topologically distinct from a loop enclosing no particles, the laws of physics permit existence of particles with fractional exchange statistics. The elementary excitations (Laughlin quasiparticles) of a fractional quantum Hall fluid have fractional electric charge and are expected to obey fractional statistics. Here we report experimental realization of a quasiparticle interferometer, where interference fringes are observed as conductance oscillations as a function of magnetic flux in an Aharonov-Bohm effect. When a quasiparticle of the 1/3 fluid executes a closed path around an island of the 2/5 fluid, we observe the interference shift by one fringe upon introduction of five magnetic flux quanta (5*h*/*e*) into the island. The corresponding 2*e* charge period is confirmed directly in calibrated gate experiments. These observations imply relative statistics of $\Theta_{2/5}^{1/3} = -1/15$ when an *e*/3, $\Theta_{1/3} = 2/3$ quasiparticle encircles one *e*/5, $\Theta_{2/5} = 2/5$ quasiparticle.

Keywords: fractional statistics, fractional charge, electron interferometer, Laughlin quasiparticle, quantum computation, quantum Hall effect, tunneling, quantum coherence

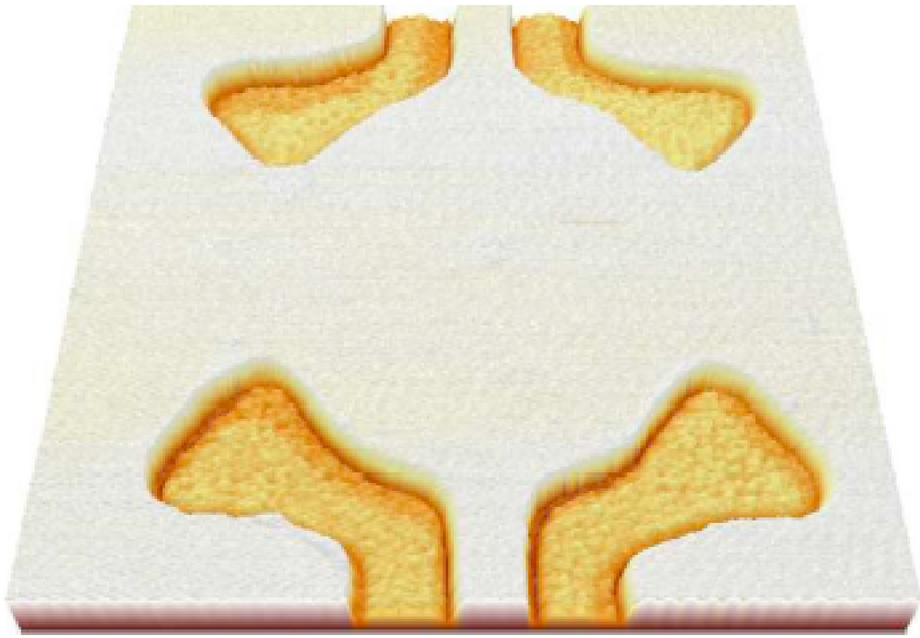


Fundamental idea: interference between paths depends on the phase-influence of anyons in between.

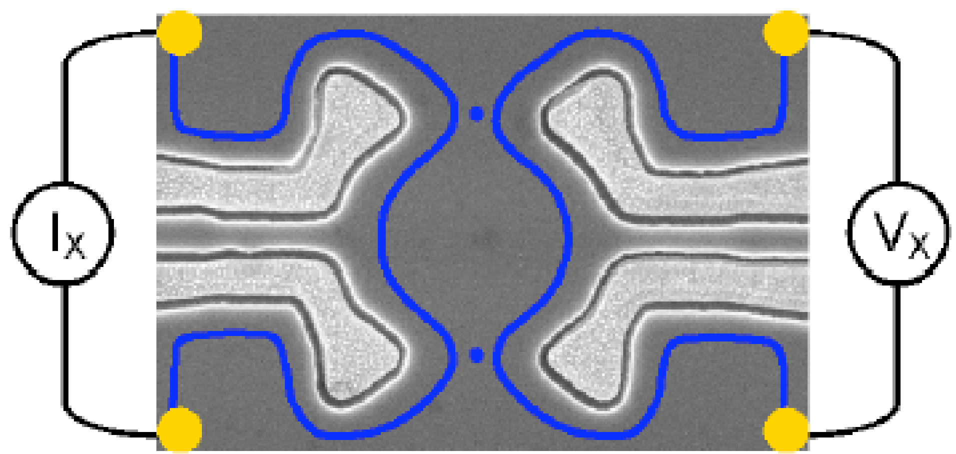


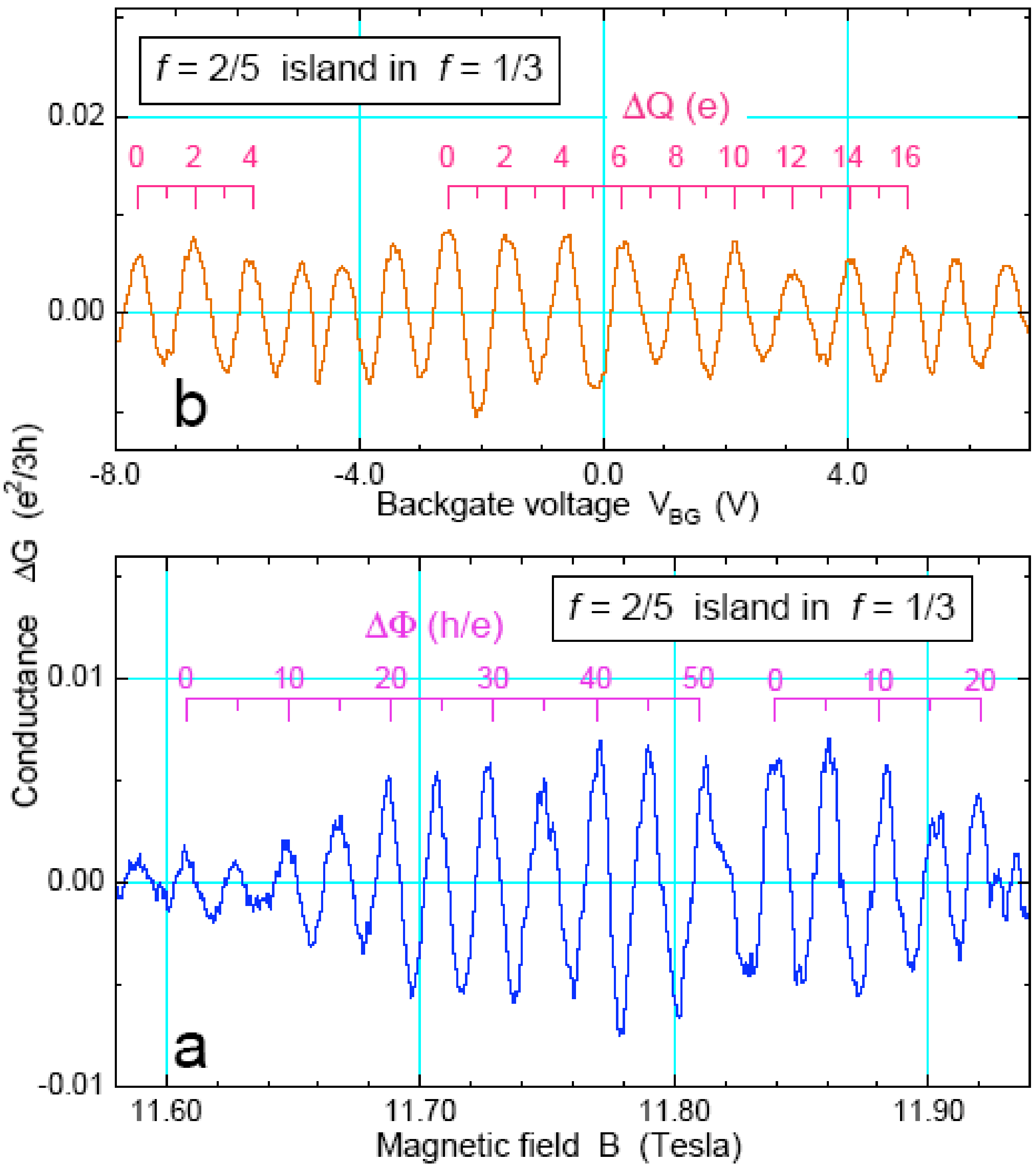
V
f = 1/3
f = 2/5
Φ
I
a

a



b





f = 2/5 island in f = 1/3
ΔQ (e)
0 2 4
0 2 4 6 8 10 12 14 16
b
0.02
0.00
-8.0
-4.0
0.0
4.0
Backgate voltage V_BG (V)
Conductance ΔG (e²/3h)
f = 2/5 island in f = 1/3
ΔΦ (h/e)
0 10 20 30 40 50
0 10 20
a
0.01
0.00
-0.01
11.60
11.70
11.80
11.90
Magnetic field B (Tesla)

$$2\pi \ \frac{e}{3} \ \frac{5}{e} + 10 \ \theta_{\frac{1}{3}|\frac{2}{5}} = 2\pi$$

$$\frac{\theta_{\frac{1}{3}|\frac{2}{5}}}{\pi} = \frac{1}{15}$$

- The periodicity in 5h/e is striking, regardless of interpretation.
- There are several issues, however:
  - Charge on island may not be accumulating by creation of 2/5 quasiparticles.
  - The independent-particle model of charging may be naive.

- In any case, the basic strategy seems sound: let currents flow in either of two ways around an island, and look for interference that is sensitive to putting charge on the island.

- A new generation of interference experiments is under development, aiming to confirm both abelian and nonabelian anyon statistics.

- There is also an interesting proposal to detect statistics through noise. (Kim et al. , PRL **95** 176402 (2005))

Quantum Computing

Quantum Computing: Toy Example and Tease

- Quantum computers attempt to exploit entanglement and the vast size of Hilbert space for parallelism and bandwidth.
- Classical bits are binary variables. Qubits are binary variables that can be superposed e.g., spin up or down with respect to a specified direction. Thus a qubit spans a 2-complex dimensional Hilbert space. And n qubits span a 2^n dimensional space!

- A quantum computer performs a unitary transformation on its input, and is read out by a measurement.

- *Reversible* classical computers are a special case, where the unitary transformation simply permutes the canonical basis vectors.

- A simple toy example will bring out some characteristic features:

Imagine we have a machine that calculates a function $f : \{0,1\} \to \{0,1\}$, and that we want to determine whether f is "constant" ($f(0) = f(1)$) or "balanced" (($f(0) \neq f(1)$). If the machine is a classical computer, we have no alternative but to compute $f(0)$ and $f(1)$ separately. But if our machine is a quantum computer, we can proceed differently.

To be concrete, suppose our quantum computer performs the unitary transformation

$$\begin{aligned}
|0\rangle|0\rangle &\to |0\rangle|f(0)\rangle \\
|1\rangle|0\rangle &\to |1\rangle|f(1)\rangle \\
|0\rangle|1\rangle &\to |0\rangle|1 \oplus f(0)\rangle \\
|1\rangle|1\rangle &\to |1\rangle|1 \oplus f(1)\rangle
\end{aligned}$$

Note that this takes an orthonormal basis to an orthonormal basis.

Now we observe

$$\begin{aligned}
|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) &\to (-1)^{f(0)}|0\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\
|1\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) &\to (-1)^{f(1)}|1\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)
\end{aligned}$$

and therefore

$$\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \to (-1)^{f(0)}\left(\frac{|0\rangle - (-1)^{f(0)+f(1)}|1\rangle}{\sqrt{2}}\right)\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

Now we've isolated $f(0) \oplus f(1)$ in the first bit (right?).

We've got

$$(\frac{|0\rangle - |1\rangle}{\sqrt{2}})(\frac{|0\rangle - |1\rangle}{\sqrt{2}}) \to (-1)^{f(0)}(\frac{|0\rangle - (-1)^{f(0)+f(1)}|1\rangle}{\sqrt{2}})(\frac{|0\rangle - |1\rangle}{\sqrt{2}})$$

To clean this up and put it in the standard format we apply the unitary transformation

$$|0\rangle(\frac{\langle 0| - \langle 1|}{\sqrt{2}}) + |1\rangle(\frac{\langle 0| + \langle 1|}{\sqrt{2}})$$

to the first bit of the result. Then we measure the first bit of the output. If it is $|0\rangle$, then f is constant. If it is $|1\rangle$, then f is balanced.

- That toy example (due to Deutsch) illustrates how global aspects of a function can be projected out by quantum superposition.

- More serious examples of the potential advantages of quantum computers are Shor's algorithm for factoring large numbers and Grover's algorithm for search.

- Probably the most important practical use for quantum computers would be to do quantum mechanics! (In particular, chemistry.)

- Quantum minds might really “think different”.
- They might want to be left alone. Their reticence could supply the answer to Fermi’s question: “Where are they?”