• This model also supports a simple and instructive model of confinement.

• The ground state(s) can be constructed in another way, by projecting onto the +1 eigenstates of the As and Bs directly:

$$|\text{vacuum}\rangle = \prod \frac{1 + A}{\sqrt{2}} \prod \frac{1 + B}{2} |0\rangle$$

where $|0\rangle$ is the “all down” state. The second factor is redundant, but serves to remind us that the vacuum contains a magnetic as well as an electric condensate. Also, with this factor present, the operator product projects any state that it doesn’t annihilate to a ground state. The square root insures proper normalization. In

$$(1 + A)^2 = 1 + 2A + A^2 = 2 + 2A$$

the term with $A$ flips spins, and has zero matrix element. (There’s a small loophole here, since in principle a whole bunch of As could conspire to undo one another’s flips – but that doesn’t happen.)
Of course, we could choose instead to let $|0\rangle$ be the states of all $x$-components of spins down; then the first factor is redundant.
Now consider an electric source with a small charge $q$, rather than “unit” charge. We produce a pair by having a modified open $Z$-type string, in which we have a product of operators $e^{2\pi i q \sigma_z}$ instead of simply $\sigma_z$ (which corresponds to $q = 1/2$). Note that now the orientation of the string matters.
• We want to evaluate the energy of a separated charge-anticharge pair. The Hamiltonian has $A$ terms and $B$ terms. The matrix element of the $B$ terms is unaffected by the presence of the pair and its string.

• But the matrix elements of the $A$ terms are affected, since the string does NOT anticommute with the $A$ operators; indeed $e^{2\pi i q \sigma^z} A = A e^{-2\pi i q \sigma^z}$ if $A$ contains the link where $\sigma^z$ lives.

• For an $A$ term that intersects a given bit of the string we have:
\( \langle 0 | \prod \frac{1 + A}{\sqrt{2}} e^{-2\pi i q \sigma^z} A \prod e^{2\pi i q \sigma^z} \prod \frac{1 + A}{\sqrt{2}} | 0 \rangle = \langle 0 | \prod \frac{1 + A}{\sqrt{2}} A \prod e^{4\pi i q \sigma^z} \prod \frac{1 + A}{\sqrt{2}} | 0 \rangle \\
= e^{-4\pi i q} \langle 0 | \prod \frac{1 + A}{\sqrt{2}} A \prod \frac{1 + e^{8\pi i q} A}{\sqrt{2}} | 0 \rangle \\
= e^{-4\pi i q} \langle 0 | \prod \frac{1 + A}{\sqrt{2}} \prod \frac{1 + e^{8\pi i q} A}{\sqrt{2}} | 0 \rangle \\
= \cos 4\pi q \)
Each string bit gives us two such factors, since there are two A-operators that it hits; likewise, there A-operator that intersects the string intersects it twice. So we get a contribution of $(\cos 4\pi q)^2$, instead of $-1$, for each of $2L$ terms in $H$, where $L$ is the length of the string.

Thus the energy of the string is proportional to its length, and the charges are confined.

An exactly similar argument works for magnetic charge.
• We can say, in a suggestive language, that the $(1+A)$ factors represent a magnetic condensate. Indeed, we associate products of As with X-type operators, which when cut reveal magnetic charges.

• Heuristically, then, the “fractional” electric charge frustrates the magnetic condensate.