# 14.3 Relativistic Hall Effect by Mode Analysis

14.3.1 For this section we will fix conventions as

$$H = \alpha_i (i\partial_i - qA_i) + \beta m + q\phi$$

$$\alpha_i \equiv \gamma_0 \gamma_i \quad \beta \equiv \gamma_0$$

$$\alpha_i^2 = \beta^2 = 1; \{\alpha_1, \alpha_2\} = 0 = \{\beta, \alpha_i\}$$

$\phi$ is the electric potential, which we'll use only later (14.3. ).

14.3.2 We saw that this equation describes the low-energy modes of graphene near half filling. $m=0$ in "raw" graphene; and there are two copies, each described by a 2-component spinor. With the Haldane terms turned on (violating T and P) there ~~are~~ are effective masses ~~of opposite sign~~ at the two relevant "Dirac points" $\pm k^*$. The momenta in the effective Dirac equations refer of course to small deviations from $\pm k^*$, i.e. slow spatial modulations of these plane waves.

14.3.3 When necessary for concreteness we will ~~may~~ take $\gamma_0 = \sigma_3$, $\gamma_1 = i\sigma_1$, $\gamma_2 = i\sigma_2$.

14.3.2 We consider applying a magnetic field. Work in Landau gauge: $A_y = Bx$ $(+ A_x = 0)$. [14.16]

With $\Psi \equiv e^{-ikx}\phi$, the Schrödinger-Dirac equation $H\Psi = E\Psi$

(not the electric potential.)

becomes

$$E\phi = \left[\alpha_x(k - qBy) + \alpha_y(i\partial_y) + \beta m\right]\phi \equiv \mathcal{O}\phi$$

Applying $\mathcal{O}$ again, and using the properties of $\alpha + \beta$, we find

$$E^2\phi = \left[(k-qBy)^2 - \partial_y^2 - m^2 + \alpha_y\alpha_x \underbrace{i\partial_y(-qBy)}_{-iqB + (-qBy)\,i\partial_y} + \alpha_x\alpha_y(-qBy)\,i\partial_y\right]\phi$$

$$= \left[(k-qBy)^2 - \partial_y^2 - m^2 - iqB\,\alpha_y\alpha_x\right]\phi$$

$$= \left[(k-qBy)^2 - \partial_y^2 - m^2 - qB\sigma_3\right]\phi$$

Letting $\sigma_3 \phi_+ = \phi_+$ (upper component!)

$\sigma_3 \phi_- = -\phi_-$

we have

$$E^2 \phi_\pm = [(k - qBy)^2 - \partial_y^2 + m^2 \mp qB]\, \phi_\pm$$

Apart from a shift in y by $\frac{k}{qB}$ and ~~the~~ the additive factors $m^2 \mp qB_z$ ~~we recognize~~ (and the appearance of $E^2$ rather that E!) we recognize this as the Schrödinger equation for a nonrelativistic harmonic oscillator with $\frac{1}{2m} = 1$ and spring constant $K/2 = (qB)^2$. Thus for

$\uparrow$ kappa, not k

the frequency we have $\omega^2 = K/m = 4(qB)^2$ and for the

$\hookrightarrow$ so $\omega = 2|qB|$

eigenvalues $E^2 = |qB| + n\, 2|qB| + m^2 \mp qB$

$\uparrow$ no | | here!

[Note: $E^2$ for $\phi_+$ with n matches $E^2$ for $\phi_-$ with n-1, $n \geq 1$.]

14.3.3 From the solutions ~~with~~ of $\mathcal{O}^2 \phi = E^2 \phi$ we get solutions of the original equation $\mathcal{O}\tilde{\phi} = \pm E \tilde{\phi}$ using $\tilde{\phi} = \phi \pm \frac{\mathcal{O}\phi}{E}$

$$\left[\mathcal{O}\tilde{\phi} = \mathcal{O}\phi \pm \frac{\mathcal{O}^2\phi}{E} = \mathcal{O}\phi \pm E\phi = \pm E\left(\phi \pm \frac{\mathcal{O}\phi}{E}\right) = \pm E\tilde{\phi}\right]$$

14.3.4 However this won't work if $E=0$; for that case we must go back to the original equation.

We now ~~take~~ assure $qB>0$ for concreteness.

$E=0$ solutions arise for $\phi_+$ with $n=0$ (and $m^2=0$).

Our equation is

$$0 = [\alpha_x(k-qBy) + \alpha_y(i\partial_y)]\phi_+$$

or

$$0 = \Big[(k-qBy) + \underbrace{\underbrace{\alpha_x\alpha_y}_{+i\sigma_3}\,(i\partial_y)}_{-\partial_y}\Big]\phi_+ \qquad [-(i\sigma_x)(i\sigma_y)] \text{ from } \gamma_0$$

solved by $\phi_+(y) \propto e^{-\frac{qB}{2}\left(y-\frac{k}{qB}\right)^2}$. N.B. normalizable ($qB>0$).

14.3.5 If $qB<0$ the roles of $\phi_+$ and $\phi_-$ are reversed, ...

14.3.6 Adding $\sigma_3 m$ to $\mathcal{O}$ in 14.3.4, we see that the 0 mode

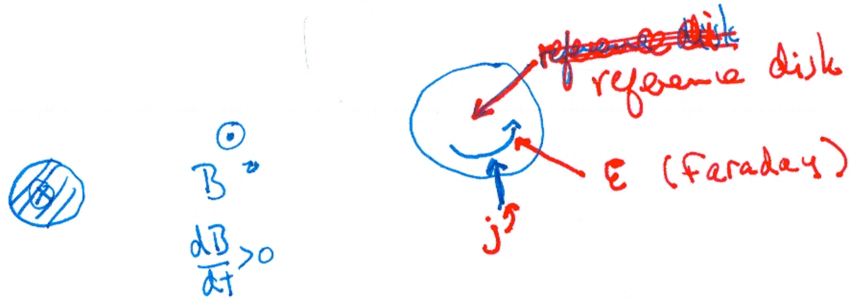
$\sigma_3 \to m$, for $\phi_+$

is shifted to positive energy (unoccupied) for $m>0$, negative energy (occupied) for $m<0$.

**14.3.7** ~~The~~ When the $m_s$ at the two critical ~~$\pm k_*$~~ momenta $\pm k_*$ have the same sign, the occupancy conditions agree. Thus a small magnetic field will induce a charge density – the two-nodes get "denser". ~~As~~ Opposite $m_s$ occur for sufficiently large values of $t_2 \sin\phi$ in the Haldane model.

[N.B.: Haldane uses a parity-reversing convention at the two critical momenta, so he says ~~the~~ the $m_s$ have "opposite" signs where I say "same", and vice-versa.]

**14.3.8** When a small B induces charge, there must be transverse currents flowing in to supply it.




Thus there is a Hall effect (transverse conductivity).

This is the philosophy of the "Streda formula".

It also follows from the covariant relation $j_\mu \propto \epsilon_{\mu\nu\rho} F_{\nu\rho}$